3 Percent and Percent Change

**Concept Questions (Section 3.1)**

1. If the portion is 4 times the size of the base then *rate* = 400%

3. If the percent rate is 1000% and the base is 1 then the portion is 10 times the base.

**Exercise 3.1**

1. *Rate* =  ⇒ 

*Portion* = 0.0175 × $350 = $6.13

3. *Rate* = 

5. *Rate* =  ⇒ 

0.60(*Base*) = $45

*Base* = 

7. *Rate* =  ⇒ 

*Portion* = 2.333 × $75 = $174.98

9. *Rate* =  = 2.00 = 200%

11. *Rate* =  ⇒ 

*Portion* = 1.50 × $60 = $90.00

13. *Rate* =  ⇒ 

0.075(*Base*) = $1.46

*Base* =  = $19.47

15. *Rate* =  = 0.621 = 62.1%

17. *Rate* =  ⇒ 

*Base* =  = $105.26

19. *Rate =* * =* 0.0100 = 1.00%

21. *Rate* =  ⇒ 

*Portion* = 0.005 × $10.00 = $0.05

23. *Rate* =  ⇒ 

*Base* =  = $150.00

25. *Rate* =  ⇒ 

*Portion* = 1.305 × $455 = $593.78

27. *Rate* =  ⇒ 

*Base* =  = $125.00

29. *Rate* =  ⇒ 

*Base* =  = $2000.00

31. *a.* *Rate* =  = = 1.684 = 168%

The trip costs 168% of their gross monthly income.

*b.* Disposable income = 100% – 72% = 28% of gross income.

Annual disposable income = 12(0.28 × $8775) = $29,484

The cost of the trip is



of Cecilia’s and Nathan’s annual disposable income.

33. Given: *Base* = 540 ml

a) *Rate =* 100 - (28 + 15.5 + 6) = 50.5% other ingredients

*Rate* =  ⇒ 0.505 = = 272.7 ml

b) *Rate =* 28%

*Base*  = x 540 ml = 337.5 ml

0.28 = = 94.5 ml

35. The budgeted expenses are the *Base* while the actual expenses are the *Portion.*

*Rate* =  ⇒ 

Budget =  = $252,100 (to the nearest $100)

Brockton budgeted $252,100 for snow clearance.

37. Total hours in a year = 52 weeks × 7 days/week × 24 hours/day = 8736 hours

Number of days worked = [(52 – 2) weeks × 5 days/week] – 7 holidays = 243 days

Total hours worked = 243 × 7.5 = 1822.5

Percentage of total hours that are worked =  = 20.9%

39. Percentage of impurities = 100% − 99% = % = 0.56%

Amount of impurities in a 150-g bar of soap = *Rate* × *Base* = 0.0056 × 150 g = 0.840 g = 840 mg

41. Discount broker would charge $25 + 800($0.05) = $65.00

Full-service broker would charge 0.022(800 × $21.75) = $382.80

The discount broker charges $382.80 − $65.00 = $317.80 less

The percent saved is  = 83.0%

43. Canada’s population density = = 4.064 people per square km.

Japan’s population density = = 336.62 people per square km.

Canada’s population density is only = 1.21% of Japan’s population density.

45. 13,500 seats represent 67.50% of capacity.

That is, 13,500 = 0.6750(Capacity)

Capacity =  = 20,000 seats

Seats not sold to season-ticket holders = 20,000 – 13,500 = 6500

6500 seats were not sold to season-ticket holders.

47. Portion of commission retained = *Rate* × *Base* = 0.60 × 4.8% = 2.88%

Income of $150,480 (*Portion*) is 2.88% of sales (*Base*).

That is, $150,480 = 0.0288 × Sales

Stan’s sales volume was $5,225,000.

49. *a.* The expected number of deaths (*Portion*) among 50,000 males (*Base*) is

 × 50,000 = 170

*b.* The number of 35-year-old males in the city of 1.45 million is

0.0083 × 1,450,000 = 12,035.

The expected number of deaths in this group in a year is 0.0034 × 12,035 = 41.

**Exercise 3.2**

a)  10%

c)  200%

e)  $150

g) 900 cm

i)  $25

1. 

3. 

5. 

7. = $134.39[1 + (–0.12)] = $134.39(0.88) = $118.26

9.  = (26.3 cm)(1 + 3.00) = 105.2 cm

11. 

13. Given: *Vi* = $90, *Vf* = $100



$100 is 11.11% more than $90.

15. Given: *c =* 25%, *Vf* = $100



$80.00 increased by 25% equals $100.00.

17. Given: *Vf* = $75, *c* = 75%



$75 is 75% more than $42.86.

19. Given: *Vi* = $759.00, *Vf* = $754.30



$754.30 is 0.62% less than $759.00.

21. Given: *Vi* = $75, *c* = 75%

*Vf = Vi* (1 + *c*) = $75(1 + 0.75) = $131.25

$75.00 becomes $131.25 after an increase of 75%.

23. Given: *Vf* = $100, *c* = – 20%



$125 after a reduction of 20% equals $100.

25. Given*: Vf* = $549, *c* = 



$658.80 after a reduction ofequals $549.

27. Given: *Vi* = $102, *c* = –2%

*Vf = Vi*(1 + *c*) = $102(1 – 0.02) = $99.96

$102 after a decrease of 2% is $99.96.

29. Given: *Vi* = $250, *Vf* = $750



$750 is 200.00% more than $250.

31. Given: *c* = 0.75%, *Vi* = $10,000

*Vf = Vi* (1 + *c*) = $10,000(1 + 0.0075) = $10,075.00

$10,000 after an increase of % is $10,075.00.

33. Given: *c* = 150%, *Vf* = $575

 = $230.00

$230.00 when increased by 150% equals $575.

35. Given: *Vi*= $150, *c* = 150%

*Vf = Vi* (1 + *c*) = $150(1 + 1.5) = $375.00

$150 after an increase of 150% is $375.00.

37. Let the TV’s pre-tax price be *p*. Then

*p* + 0.05*p* + 0.07*p* = $2797.76

*p* =  = $2498.00

Then, GST = 0.05*p* = 0.05($2498) = $124.90

and PST = 0.07*p* = 0.07($2498) = $174.86

39. *a.* . Given: *Vi* = 32,400, *Vf* = 27,450

 = –15.28%

The number of hammers sold declined by 15.28%.

*b.* Given: *Vi* = $15.10, *Vf* = $15.50

 = 2.65%

The average selling price increased by 2.65%.

*c.* Year 1 revenue = 32,400($15.10) = $489,240

Year 2 revenue = 27,450($15.50) = $425,475

 = –13.03%

The revenue decreased by 13.03%.

41. Pick an arbitrary price, say $1.00, for a bar of the soap.

The former unit price was  per gram.

The new unit price is  per gram.

The percent increase in unit price is

 = 11.11%

43. Initial unit price = = $2.2167 per kg

Final unit price = = $2.3267 per kg

The percent increase in unit price is

 = 4.96%

45. Given: *Vf* = $338,500, *c* = 8.7%

 = $311,400

The average price one year ago was $311,400.

47. Canada’s exports to US exceeded imports from the US by 23%.

That is, Exports = 1.23(Imports)

Therefore, Imports =  = 0.8130(Exports)

That is, Canada’s imports from US (= US exports to Canada) were

1 − 0.8130 = 0.1870 = 18.70%

less than Canada’s exports to US (= US imports from Canada.)

49. Given: For Apple iPhones, *Vf* = 61.0 million, *c* = 40%

 = 43.5714 million = 43,571,400

Rounded to the nearest 10,000, Apple sold 43,570,000 iPhones for the same quarter in the previous year.

51. Percent change in the daily active Snapchat users



The number of active Snapchat users decreased by 2.62%.

53. Given: *Vf* = $0.45, *c* = 76%



Price decline = *Vi – Vf* = $1.88 – $0.45 = $1.43

The share price dropped by $1.43.

55. If General Paint’s prices are marked down by 30%, then

General Paint’s prices = 0.70(Cloverdale Paint’s prices)

Hence, Cloverdale’s prices =  =

= 1.4286(General Paint’s prices)

Therefore, you will pay 42.86% more at Cloverdale Paint.

57. Given: Operating expenses = 0.40(Revenue)

Then Revenue =  =

= 2.5(Operating expenses)

That is, Revenue is 250% of Operating expenses, or

Revenue exceeds Operating expenses by 250% − 100% = 150%.

59. Current unit price = = 0.8980 cents per ml

New unit price = 1.10(0.8980 cents per ml) = 0.9878 cents per ml

Price of a 425-ml container = (425 ml) × (0.9878 cents per ml) = 419.8 cents = $4.20

61. For Year 1, *Vf*  = $6 and *Vf – Vi* = – $4

Therefore, *Vi* = *Vf +* $4 = $6 + $4 = $10



For Year 2, *Vi* = $6 and *Vf – Vi* = $4

Therefore, 

The percent change was −40.00% in Year 1 and 66.67% in Year 2.

63. Given: For the appreciation, *Vi* = Purchase price, *c* = 140%, *Vf* = List price

For the price reduction, *Vi* = List price, *c* = –10%, *Vf* = $172,800

List price = $192,000

Original purchase price = 

The owner originally paid $80,000 for the property.

65. Suppose the initial ratio is .

If the denominator is reduced by 20%, then

Final ratio = 

That is, the value of the ratio increases by 25%.

67. Use ppm as the abbreviation for “pages per minute”.

Given: Lightning printer prints 30% more ppm than the Reliable printer.

That is, the Lightning’s printing speed is 1.30 times faster than Reliable’s printing speed.

Therefore, the Reliable’s printing speed is

 = 0.7692 = 76.92% of the Lightning’s printing speed

If Lightening can print 100 pages per minute then Reliable can print 76.92 pages per minute.

or Reliable is 100% − 76.92% = 23.08% slower than the Lighting’s speed.

The Lightning printer will require 23.08% less time than the Reliable for a long print job.

69. Let us use OT as an abbreviation for “overtime”.

The number of OT hours permitted by this year’s budget is

OT hours (this year) = 

The number of overtime hours permitted by next year’s budget is

OT hours (next year) = 

= 0.980952

= 98.0952% of this year’s OT hours

The number of OT hours must be reduced by 100% − 98.0952% = 1.90%.

**Point of Interest (Section 3.3)**

**A 20% Gain Doesn’t Offset a 20% Loss**

1. Let *Vi* = $100, then *Vf* = $100 to break even

$100 = $100[1+(−50%)][1+(−60%)][1+(−80%)][1+(−90%)](1+*C5*)

$100 = $100(0.50)(0.40)(0.20)(0.10)(1 +*C5*)

$100 = 0.40(1+*C5*)

*C5* = 249 or 24,900%

**Concept Questions (Section 3.3)**

1. Yes. If the expenses associated with an investment exceed the income from the investment, then the net income and the income yield will be negative. For example, if you hold a piece of raw land as an investment, you will have no income from the property but you must pay property taxes each year. The net income and income yield are then negative.

3. Yes. Suppose, for example, you bought a $160,000 condominium as an investment property using $40,000 of your own money and $120,000 borrowed on a mortgage loan. Subsequently, the condo’s market value fell to $100,000 because “leaky condo” problems were discovered in the building. At that point, you have lost more than 100% of your initial $40,000 investment because the condo’s market value is less than the amount owed on the mortgage loan. You must still repay the balance on the loan after the proceeds of the sale are applied to the loan.

5. The magnitude of the overall percent change is smaller than the sum. To illustrate, consider two successive 10% decreases from a beginning value of $1000. The first 10% decrease causes a $100 decrease to $900. The second 10% decrease acts on $900 rather than on the initial $1000. The dollar amount of the second reduction is only $90. The overall reduction is $190, which is only 19% (not 20%) of the original $1000.

# Exercise 3.3

1. Given:  = $100,** = $110, Income = $10

*a.* Income yield =  = 

*b.* Capital gain yield = × 100% = × 100% = 10.00%

*c.* Rate of total return = Income yield + Capital gain yield = 10.00% + 10.00% = 20.00%

3. Given:  = $8790,** = $15,390, Income = $280

*a.* Income yield =  =  = 3.185% = 3.19%

*b.* Capital gain yield = × 100% = × 100% = 75.085% = 75.09%

*c.* Rate of total return = Income yield + Capital gain yield = 3.185% + 75.085% = 78.27%

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 5. | Security | Income yield | Capital gain yield | Rate of total return |
|  |  |  |  | Income yield  + Capital gain yield |
|  | BCE Inc. shares  (2018) | = 1.26% | = -6.00% | 1.26% + (-6.00%)  = -4.74% |
|  | Mawer New Canada Fund  (2018) | = 0.00% | = -1.20% | 0.00% + (-1.20%)  = -1.20% |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 7. | Security | Income yield | Capital gain yield | Rate of total return |
|  |  |  |  | Income yield  + Capital gain yield |
|  | Blackberry  shares  (2017) | 0.0% | = 51.95% | = 51.95% |
|  | Blackberry  shares  (2018) | 0.0% | = -16.95% | = -16.95% |
|  | Scotia Can.  Bond Fund (2017) | = 2.35% | = -0.52% | 2.35% + (-0.52%)  = 1.83% |
|  | Scotia Can.  Bond Fund (2018) | = 2.01% | = -3.58% | 2.01% + (-3.58%)  = -1.57% |

9. Given:  = $24.10,** = $25.50, Income = $0.83

Income yield =  =  = 3.44%

Capital gain yield =  × 100% =  × 100% = 5.81%

Rate of total return = Income yield + Capital gain yield = 3.44% + 5.81% = 9.25%

11. Given:  = $36.75,** = $32.25, Income = $0.50

Income yield =  =  = 1.36%

Capital gain yield =  × 100% =  × 100% = –12.24%

Rate of total return = Income yield + Capital gain yield = 1.36% + (– 12.24%) = –10.88%

13. Let *Vi*  represent the share price at the beginning of 2013.

Let *Vf*  represent the share price at the end of 2017.

*a.* 

= *Vi* (1.4006)(1.2416)(1.1944)(1.1200)(1.3766)

= *Vi* (3.2024)

= *Vi* (1 + 2.2024)

Therefore, Microsoft shares rose 220.24% over the five years.

*b.* If *Vf*  = $85.54,

then = $26.71

15. Given: *c*1 = *c*2 = 0.25

*Vf = Vi* (1 + *c*1)(1 + *c*2) = *Vi* (1.25)2 = 1.5625*Vi*

The percent increase over the entire two years was 56.25%.

17. Given: *c*1 = 0.50, *Vf = Vi*

Substitute in *Vf* = *Vi* (1 + *c*1)(1+ *c*2)

*Vi = Vi*(1.50)(1 + *c*2)

1 = 1.50(1 + *c*2)

*c*2 =  =  = –33.33%

A 33.33% decline in the second year will wipe out a 50% gain in the first year.

19. Given: *c*1 = –0.50, *Vf = Vi*

Substitute in *Vf = Vi*(1 + *c*1)(1 + *c*2)

*Vi*= *Vi*(1 – 0.5)(1 + *c*2)

*c*2 =  = 1.0000

A 100.00% gain in the second year is required to break even.

21. Given: *c*1 = *c*2 = 0.10, *Vf* = 1.30*Vi*

Substitute in *Vf = Vi*(1 + *c*1)(1 + *c*2)(1 + *c*3)

1.30*Vi = Vi* (1.10)(1.10)(1 + *c*3)

1.30 = 1.21(1 + *c*3)

*c*3 =  = 0.0744

A gain of 7.44% in the third year will produce a cumulative gain of 30%.

23. The value at the end of 2017 of an initial investment of *Vi* in the Canopy Growth Corp. Fund was

*Vf = Vi*(1 + (-0.9000))(1 + 81.4000)(1 + 0.4417)(1 + 2.0774)(1 + 2.2538) = 118.9534*Vi*

Therefore, the percent increase in the value of the fund was 11,795.34%.

Over the same period the S&P/TSX Total Return Index increased from *Vi* to

*Vf = Vi*(1 + 0.0955)(1 + 0.0742)(1 – 0.1109)(1 + 0.1751)(1 + 0.0603) = 1.3036*Vi*

Therefore, the percent increase in S&P/TSX Total Return Index was 30.36%.

The Canopy Growth Corp. Fund outperformed the S&P/TSX Total Return Index by

11,795.34% – 30.36% = 11,764.98%

25. The value at the end of 2017 of an initial investment of *Vi* in the Fidelity Canadian Asset Allocation Fund was

*Vf = Vi*(1 +0.0978)(1 + 0.0953)(1 – 0.0097)(1 + 0.0685)(1 + 0.0470) = 1.3321*Vi*

Therefore, the percent increase in the value of the fund was 33.21%.

Over the same period the S&P/TSX Total Return Index increased from *Vi* to

*Vf = Vi*(1 + 0.0955)(1 + 0.0742)(1 – 0.1109)(1 + 0.1751)(1 + 0.0603) = 1.3036*Vi*

Therefore, the percent increase in S&P/TSX Total Return Index was 30.36%.

The Fidelity Canadian Asset Allocation Fund out-performed the S&P/TSX Total Return Index by

33.21% – 30.36% = 2.85%

27. The income yield will be  =  = 0.93%

For a rate of total return = 10%,

Capital gain yield = Rate of total return − Income yield = 10% − 0.93% = 9.07%

The end-of-2019 Suncor Energy share price must be

*Vi* (1 + *c*) = $42.93(1 + 0.0907) = $46.82

29. Given:  = $5000  400 = $12.50 per unit, Rate of total return = 22%,  = $13.75

Income + Capital gain =  = 0.22 × $12.50 = $2.75

Since Capital gain =  –  = $13.75 – $12.50 = $1.25

Then Income = $2.75 – $1.25 = $1.50

The fund distributed $1.50 per unit during the year.

31. Given: *Vf* = $47,567, *c*1 = 0.154, *c*2 = 0.243, *c*3 = 0.321, *c*4 = –0.033

*a.* 

Vi 

Victor’s original investment was $25,959.69.

*b.* The dollar amount of the increase in the third year was

*c*3*Vf* (after 2 years) = *c*3*Vi* (1 + *c*1)(1 + *c*2) = 0.321($25,959.69)(1.154)(1.243) = $11,953.13

The value of the fund increased by $11,953.13 in Year 3.

**Exercise 3.4**

1. Total Market Sales for 2018 = $56.18M

Total Market Sales for 2019 = $58.95M

|  |  |  |
| --- | --- | --- |
| **Brand** | **2018 Market Share** | **2019 Market Share** |
| A |  |  |
| B |  |  |
| C |  |  |
| D |  |  |
| E |  |  |
| F |  |  |

3. Coke consumed = 0.1195 x 124.56B litres = 14.88B litres consumed

Pepsi consumed = 0.0612 x 124.56B litres = 7.62B litres consumed

5. Step 1: calculate the sales for all other brands in 2019

Step 2: calculate total sales for 2020

Step 3: calculate the sales for all other brand in 2020

|  |  |  |
| --- | --- | --- |
| **Brand** | **2019 Sales $M** | **2020 Sales $M** |
| Core Buster | 103.49 | 102.61 |
| Life Fit | 60.37 | 63.70 |
| Home Fit | 43.12 | 42.55 |
| Torrent | 155.23 | 160.87 |
| All Other Brands | = 862.4 – 103.49 -60.37 -43.12 -155.23 = 500.19 | = 840.84 – 102.61 – 63.70 – 42.55 – 160.87 = 471.11 |
| Total | 862.4 | = 862.4(1 – 2.5%) = 840.84 |

a) Market Share for each brand

|  |  |  |
| --- | --- | --- |
| **Brand** | **2019 Market Share** | **2020 Market Share** |
| Core Buster |  |  |
| Life Fit |  |  |
| Home Fit |  |  |
| Torrent |  |  |

b) Market Growth for each brand

|  |  |
| --- | --- |
| **Brand** | **Market Growth** |
| Core Buster |  |
| Life Fit |  |
| Home Fit |  |
| Torrent |  |

c) Market growth for all other brands in the industry

=

7. Given 2019 sales of $146.27M equivalent to 3.6% market share

Athletic wear 2019 industry Sales =

Given 2020 sales of $239.11M equivalent to 5.5% market share

Athletic wear 2020 industry sales =

Industry growth = 7.0% growth

9. 2019 market share = 28.07%

11. Google brand 2017 sales = 0.193 x $2.9M = $0.5597M

Google brand 2018 sales = 0.362 x 9.0M = $3.258M

Market Growth of Google brand = 482.1% growth

**Review Problems**

1. *Rate* =  x 100% = 256.49%

3. *Base* =  =  = $133.33

5. *Base* =  =  = $75.00

7. *Portion = Rate x Base* = 3.50 x $0.62 = $2.17

9. Portion = 17 members, Rate = 11% of 42% = 0.11 x 42% = 4.62%

*Base* =  =  = 367.965

Rounded to the nearest whole person, the building has 368 employees.

11. a. Vi = 1970 price = $1686

Vf = 2019 price = $16,790

`  = 895.85%

b. Vi = 1970 price + tax = $1686(1 + 0.05) = $1770.30

Vf = 2019 price + tax = $16,790(1 + 13%) = $18,972.70

= 971.72%

13. Given: For the first year, *Vi* = $3.40, *Vf* = $11.50.

For the second year, *Vi* = $11.50, *c* = – 35%.

*a.* 

The share price increased by 238.24% in the first year.

*b.* Current share price, *Vf = Vi* (1 + *c*) = $11.50(1 – 0.35) = $7.48.

15. Given: Last year’s revenue = $2,347,000

Last year’s expenses = $2,189,000

*a.* Given: Percent change in revenue = 10%; Percent change in expenses = 5%

Anticipated revenues, *Vf = Vi* (1 + *c*) = $2,347,000(1.1) = $2,581,700

Anticipated expenses = $2,189,000(1.05) = $2,298,450

Anticipated profit = $283,250

Last year’s profit = $2,347,000 – $2,189,000 = $158,000

Percent increase in profit = 

*b.* Given: *c*(revenue) = –10%; *c*(expenses) = – 5%

Anticipated revenues = $2,347,000(1 – 0.10) = $2,112,300

Anticipated expenses = $2,189,000(1 – 0.05) = $2,079,550

Anticipated profit $32,750

Percent change in profit = 

The operating profit will decline by 79.27%.

17. Given: *c*1 = 0.23, *c*2 = 0.10, *c*3 = – 0.15, *c*4 = 0.05, *Vf* = $30.50

*a.* Substitute in *Vf = Vi*(1 + *c*1).....(1 + *c*4) and solve for *Vi*

$30.50 = *Vi*(1.23)(1.10)(0.85)(1.05)

*Vi* = 

A share’s initial price was $25.26.

*b.* The decline in the third year was 15% of the price at the end of the second year.

That is,

Price decline = 0.15×*Vi* (after 2 years) = 0.15($25.26)(1.23)(1.10) = $5.13

The share price declined by $5.13 in the third year.

19. Suppose the initial investment in the AGF China Focus fund was $100.

Its value after 6 years was

$100(1 + *c*1).....(1 + *c*6) = $100(1.3053)(1 – 0.4167)(1.2775)(1.0127)(1 – 0.2071)(1.170)

= $91.38

The overall rate of return is = -8.62%

*21.* a) Market Shares based on total location sales of $2120.18M for 2018 and $2113.13 for 2019

|  |  |  |
| --- | --- | --- |
| **Location** | **2018 Market Share** | **2019 Market Share** |
| British Columbia |  |  |
| Alberta |  |  |
| Ontario |  |  |
| Quebec |  |  |
| Nova Scotia |  |  |

b) Market Growth

|  |  |
| --- | --- |
| **Location** | **Market Growth** |
| British Columbia |  |
| Alberta |  |
| Ontario |  |
| Quebec |  |
| Nova Scotia |  |

*23.* Given: Income = $200; Income yield = 4%

Therefore, ×100% = 4%

and *Vi* =  = $5000

Since the capital gain yield was 10%,

*Vf = Vi(1 + c)* = $5000(1 + 0.10) = $5500.00

The investment’s value after 1 year was $5500.00.