6 Applications of Linear Equations

**Exercise 6.1**

1.

|  |  |  |  |
| --- | --- | --- | --- |
| ***x:*** | –3 | 0 | 6 |
| ***y:*** | –6 | 0 | 12 |

3.

|  |  |  |  |
| --- | --- | --- | --- |
| *x:* | –3 | 0 | 6 |
| *y:* | 10 | 4 | –8 |

5.

|  |  |  |  |
| --- | --- | --- | --- |
| *x:* | –8 | 0 | 12 |
| *y:* | –3 | 3 | 12 |





7.

|  |  |  |  |
| --- | --- | --- | --- |
| *x:* | 0 | 3000 | 6000 |
| *y:* | 5000 | 18,500 | 32,000 |



9. In each part, rearrange the equation to render it in the form *b =* (slope)*a +* (intercept)

*a.* 2*b* + 3 = 5*a*

2*b* = 5*a* – 3

*b* = *a* – 

The slope is  and the *b-*intercept is  .

*b.* 3*a* – 4*b* = 12

– 4*b* = –3*a* + 12

*b* = *a* – 3

The slope is  and the *b-*intercept is –3

*c.* 0 = 2400 – 4*a* – 5*b*

5*b* = – 4*a* + 2400

*b* = *a* + 480

The slope is  and the *b-*intercept is 480

*d.* 7*a* = – 8*b*

8*b* = –7*a*

*b* = *a*

The slope is  and the *b-*intercept is 0

11. a. 5(0) + 25*y* = 15

25*y* = 15

The y-intercept is

5*x* + 25(0) = 15

5*x* = 15

*x* = 3 The *x*-intercept is 3

b. 0 = 63 – 7(0) + 21*y*

-21*y* = 63

*y* = -3 The y-intercept is -3

0 = 63 – 7*x* + 21(0)

7*x* = 63

*x* = 9 The x-intercept is 9

c. 13(0) = 35*y*

*y* = 0 The y-intercept is 0

13*x* = 35(0)

*x* = 0 The *x*-intercept is 0

d. 11(0) + 110 = 22*y*

110 = 22*y*

*y* = 5 The y-intercept is 5

11*x* + 110 = 22(0)

11*x* = -110

*x* = -10 The x-intercept is -10

13. Ehud earns $1500 per month plus 5% of sales revenue. Then gross earnings

*E* = $1500 + 0.05*R*

Expressing this equation in the form *y = mx + b*

*E* = 0.05*R* + $1500

On a plot of *E* vs. *R*, slope = 0.05 and *E*-intercept = $1500.

15. Randolph charges an $85 service call fee plus 2($15) per hour

Then *C* = $85 + $30*H*

In the *y* = *mx* + *b* form the equation is:

*C* = $30*H* + $85

On a plot of *C* vs. *H*, slope = $30 and *C*-intercept = $85

***x***

***y***

*y* = –2

*x* – 3*y* = 3

17.*x* –3*y* = 3

|  |  |  |
| --- | --- | --- |
| *x:* | –6 | 3 |
| *y:* | –3 | 0 |

*y* = –2

|  |  |  |
| --- | --- | --- |
| *x:* | –6 | 3 |
| *y:* | –2 | –2 |

The solution is

(*x, y*) = (–3, –2).

19.

*x* – 3*y* = 0

|  |  |  |
| --- | --- | --- |
| *x:* | –6 | 3 |
| *y:* | –2 | 1 |

*x* + 2*y* = –5

|  |  |  |
| --- | --- | --- |
| *x:* | –6 | 3 |
| *y:* | 0.5 | –4 |

The solution is

(*x, y*) = (–3, –1).

***x***

***y***

*x –* 3*y* = 0

*x* + 2*y* = –5

21.

*y –* 3*x =* 11

|  |  |  |
| --- | --- | --- |
| *x:* | – 4 | 2 |
| *y:* | –1 | 17 |

5*x* + 30 = 4*y*

|  |  |  |
| --- | --- | --- |
| *x:* | – 4 | 2 |
| *y:* | 2.5 | 10 |

The solution is

(*x, y*) = (–2, 5).

23.

7*p* – 3*q* = 23

|  |  |  |
| --- | --- | --- |
| *p:* | 0 | 6 |
| *q:* | –7.67 | 6.33 |

–2*p* – 3*q* = 5

|  |  |  |
| --- | --- | --- |
| *p:* | 0 | 6 |
| *q:* | –1.67 | –5.67 |

The solution is

(*p, q*) = (2, –3).

***p***

***q***

7*p –* 3*q =* 23

*–*2*p –*3*q =* 5

**Exercise 6.2**

1. 3x – 2y = 6 ➀

y = x – 1 ➁

Substitute equation ➁ into equation ➀:

3x – 2(x – 1) = 6

3x – 2x + 2 = 6

x = 4

Substitute into equation ➁:

y = 4 – 1

y= 3

(x, y) = (4, 3)

Check: LHS of ➀ = 3(4) – 2(3) = 12 - 6 = 6 = RHS of ➀

3. x – y = 2 ➀

3x + 4y = 20 ➁

➀  3: 3x – 3y = 6

Subtract: 7y = 14

y = 2

Substitute into equation ➀:

x – 2 = 2

x = 4

(x, y) = (4, 2)

Check: LHS of ➁ = 3(4) + 4(2) = 20 = RHS of ➁

5. 4a – 3b = –3 ➀

5a – b = 10 ➁

➀ 1: 4a – 3b = –3

➁ 3: 15a – 3b = 30

Subtract: –11a = –33

a = 3

Substitute into equation ➁:

5(3) – b = 10

b = 5

(a, b) = (3, 5)

Check: LHS of ➀ = 4(3) – 3(5) = –3 = RHS of ➀

7. y = 2x ➀

7x – y = 35 ➁

Add: 7x = 2x + 35

5x = 35

x = 7

Substitute into ➀:

y = 2(7) = 14

(x, y) = (7, 14)

Check: LHS of ➁ = 7(7) – 14 = 49 – 14 = 35 = RHS of ➁

9. d = 3c – 500 ➀

0.7c + 0.2d = 550 ➁

Substitute equation ➀ into equation ➁:

0.7c + 0.2(3c – 500) = 550

0.7c + 0.6c – 100 = 550

1.3c= 650

c = 500

Substitute into ➀: d = 3(500) – 500 = 1000

(c, d) = (500, 1000)

Check: LHS of ➁ = 0.7(500) + 0.2(1000) = 550 = RHS of ➁

11. 2v + 6w = 1 ➀

10v – 9w = 18 ➁

To eliminate v,

➀ 10: 20v + 60w = 10

➁  2: 20v – 18w = 36

Subtract: 0 + 78w = –26

w = 

Substitute into ➀:

2v + 6= 1

2v = 1 + 2

v = 

(v, w) = 

Check: LHS of ➁ =  = 18 = RHS of ➁

13. 37x – 63y = 235 ➀

18x + 26y = 468 ➁

To eliminate x,

➀18: 666x –1134y = 4230

➁37: 666x + 962y = 17,316

Subtract: 0 – 2096y = –13,086

y = 6.243

Substitute into ➀:

37x – 63(6.243) = 235

37x = 628.3

x = 16.98

(x, y) = (17.0, 6.24)

Check: LHS of ➁ = 18(16.98) + 26(6.243) = 468.0 = RHS of ➁

15. 0.33e + 1.67f = 292 ➀

1.2 e + 0.61f = 377 ➁

To eliminate e,

➀0.33: e + 5.061f = 884.8

➁ 1.2: e + 0.508f = 314.2

Subtract: 0 + 4.553f = 570.6

f = 125.3

Substitute into ➀:

0.33e + 1.67(125.3) = 292

0.33e = 82.75

e = 250.8

(e, f) = (251, 125)

Check: LHS of ➁ = 1.2(250.8) + 0.61(125.3) = 377.4 = RHS of ➁

17. Let *r* represent the number of regular members and *s* the number of student members.

Then *r* + *s* = 583 ➀

Total revenue: $2140*r* + $856*s* = $942,028 ➁

➀$856: $856*r* + $856*s* = $499,048

Subtract: $1284*r* + 0 = $442,980

*r* = 345

Substitute into ➀: 345 + *s* = 583

*s* = 238

The club had 238 student members and 345 regular members.

19. Let *s* represent the distance travelled at the slower speed (50 km/h).

Let *h* represent the distance travelled at the higher speed (100 km/h).

Since the total distance = 1000 km,

then *s* + *h* = 1000 ➀

Since travelling time = ,

then Time at slower speed =  and Time at higher speed = 

Since the total time = 12.3 hours,

then  + = 12.3 ➁

➁ × 100: 2*s* + *h* = 1230

Repeat ➀: *s* + *h* = 1000 ➀

Subtract: *s* + 0 = 230

Hence, Tina drive 230 km at 50 km/h and 1000 − 230 = 770 km at 100 km/h.

21. Let *h* represent the rate per hour and *k* represent the rate per km.

Vratislav’s cost: 2*h* + 47*k* = $54.45 ➀

Bryn’s cost: 5*h* + 93*k* = $127.55 ➁

To eliminate h,

➀5: 10*h* + 235*k* = $272.25 ➀

➁2: 10*h* + 186*k* = $255.10 ➁

Subtract: 0 + 49*k* = $ 17.15

*k* = $0.35 per km

Substitute into ➀:

2*h* + 47($0.35) = $54.45

2*h* = $54.45 − $16.45

= $38.00 per hour

*h* = $19.00 per hour

Budget Truck Rentals charged $19.00 per hour plus $0.35 per km.

23. Let *C* represent the interest rate on Canada Savings Bonds.

Let *O* represent the interest rate on Ontario Savings Bonds.

Year 1 interest: 4($1000)*C* + 6($1000)*O* = $438 ➀

Year 2 interest: 3($1000)*C* + 4($1000)*O* = $306 ➁

➀3: $12,000*C* + $18,000*O* = $1314 ➀

➁4: $12,000*C* + $16,000*O* = $1224 ➁

Subtract: 0 + $2000*O* = $ 90

*O* == 0.045 = 4.5%

Substitute into ➁: $3000*C* + $4000(0.045) = $306

*C* == 0.042 = 4.2%

The Canada Savings Bonds earn 4.2% per annum and

the Ontario Savings Bonds earn 4.5% per annum.

25. Let *x* represent the number of units of product X and

*y* represent the number of units of product Y. Then

*x* + *y* = 93 ➀

0.5*x* + 0.75*y* = 60.5 ➁

➀0.5: 0.5*x* + 0.5*y* = 46.5

Subtract: 0 + 0.25*y* = 14

*y* = 56

Substitute into ➀: *x* + 56 = 93

*x* = 37

Therefore, 37 units of X and 56 units of Y were produced last week.

27. Let M be the number of litres of milk and J be the number of cans of orange juice per week.

$1.50M + $1.30J = $57.00 ➀

$1.60M + $1.37J = $60.55 ➁

To eliminate M,

➀1.6: $2.40M + $2.080J = $91.200

➁1.5: $2.40M + $2.055J = $90.825

Subtract: 0 + $0.025J = $0.375

J = 15

Substitution of J = 15 into either equation will give M = 25. Hence, 25 litres of milk

and 15 cans of orange juice are purchased each week.

29. Let S represent the number of people who bought single tickets and T represent

the number of people who bought at three-for-$5. Then

S + 3T = 3884 ➀

$2S + $5T = $6925 ➁

To eliminate S,

➀$2: $2S + $6T = $7768

➁: $2S + $5T = $6925

Subtract: 0 + $1T = $843

T = 843

Hence, 843 people bought tickets at the three-for-$5 discount.

31. Let P represent the annual salary of a partner and T represent the annual salary

of a technician. Then

7P + 12T = $1,629,000 ➀

1.05(7P) + 1.08(12T) = $1,734,750 ➁

➀1.05: 1.05(7P) + 1.05(12T) = $1,710,450

Subtract: 0 + 0.03(12T) = $24,300

T = $67,500

Substitute into ➀: 7P + 12($67,500) = $1,629,000

P = $117,000

The current annual salary of a partner is $117,000 and of a technician is $67,500.

**Exercise 6.3**

1. *a.* Variable cost *e*. Fixed cost

*b.* Fixed cost *f.* Mixed cost

*c.* Mixed cost *g.* Variable cost

*d.* Variable cost *h.* Fixed cost

3. Total variable costs = $1,600,000 – $400,000 = $1,200,000

If production increases by 20% (from 50,000 to 60,000 units), total

variable costs will also increase by 20% from

$1,200,000 to 1.2($1,200,000) = $1,440,000

and total costs will increase from

$1,600,000 to $400,000 + $1,440,000 = $1,840,000

**Exercise 6.4**

1. Given: *S* = $2.50 per CD; *VC* = $1.00 per CD; *FC* = $60,000 per month.

*TR* = (*S*)*X*

= $2.50*X*

*TC* = (*VC*)*X* + *FC*

= $1.00*X* + $60,000

|  |  |  |
| --- | --- | --- |
| *X:* | 20,000 | 60,000 |
| *TR:* | $50,000 | $150,000 |
| *TC:* | $80,000 | $120,000 |

*a.* 40,000 CDs/month

*b.* 45,000 CDs/month

3. Given: *S* = $0.10 per copy; *FC* = $300 per month;

*VC* = $0.015 + +$0.005 = $0.05 per copy

*TR* = (*S*)*X*

= $0.10*X*

*TC* = (*VC*)*X* + *FC*

= $0.05*X* + $300

|  |  |  |
| --- | --- | --- |
| *X:* | 4000 | 8000 |
| *TR:* | $400 | $800 |
| *TC:* | $500 | $700 |

*a.* 6000 copies/month

*b.* $50 per month

5. Given: *S* = $20 per unit; *VC* = $12 per unit; *FC* = $1200 per week.

*TR* = (*S*)*X*

= $20*X*

*TC* = (*VC*)*X* + *FC*

= $12*X* + $1200

|  |  |  |
| --- | --- | --- |
| *X:* | 0 | 250 |
| *TR:* | $0 | $5000 |
| *TC:* | $1200 | $4200 |

*a.* 150 units/week

*b.* (i) $240 loss

(ii) $800 profit

*c.* 200 units/week

7. Given: *S* = $70 per composter; *VC* = $43 per composter;

*FC* = $648,000 per year = $54,000 per month.

*TR* = (*S*)*X*

= $70*X*

*TC* = (*VC*)*X* + *FC*

= $43*X* + $54,000

|  |  |  |
| --- | --- | --- |
| *X:* | 1000 | 3200 |
| *TR:* | $70,000 | $224,000 |
| *TC:* | $97,000 | $191,600 |

*a.* 2000 composters

per month

*b.* $13,500/month

*c.* $10,800/month loss

(**c**)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Problem | Effect of the change on the: | | | |
| Number | *FC* line | *TC* line | *TR* line | Break-even point |
| 8. | No effect | Slope increases | No effect | At higher volume |
| 9. | No effect | No effect | Slope increases | At lower volume |
| 10. | Upward shift | Upward shift | No effect | At higher volume |

**Exercise 6.5**

1. Given: *S* = $30; *VC* = $10; *FC* = $100,000 per year

*a.* *TR* = (*S*)*X* = $30*X*

*TC* = (*VC*)*X* + *FC* = $10*X* + $100,000

To break even, *TR = TC*

Then, $30*X* = $10*X* + $100,000

$20*X* = $100,000

*X* = 5000 toys per year

*b.* Break-even revenue = (*S*)*X* = $30(5000) = $150,000 per year

c. Break-even volume = = 62.5% of capacity

d. NI = ($30 - $10)(6200) - $100,000

= ($20)(6200) - $100,000

NI = $24,000 profit

e. $10,000 = $20X - $100,000

$20X = $110,000

X = 5500 toys per year

3. Given: *S* = $3.20; *VC* = $1.20; *FC* = $250 per month

*a.* *FC* = 12($250) = $3000 per year

*TR* = (*S*)*X* = $3.20*X*

*TC* = (*VC*)*X* + *FC* = $1.20*X* + $3000

To break even, *TR = TC*

Then, $3.20*X* = $1.20*X* + $3000

$2.00*X* = $3000

*X* = 1500 jars per year

*b.* If Ingrid sells 3000 jars per year,

*NI =* (*S – VC*)*X* – *FC* = ($3.20 – $1.20)3000 – $3000 = $3000 per year

c. A loss of $1200 would be a net income of -$1200

-$1200 = ($3.20 - $1.20)*X* - $3000

$1800 = $2.00*X*

X = 900 jars per year

5. Given: *S* = $2.50; *VC* = $1.00; *FC* = $60,000 per month

*a.* *TR* = (*S*)*X* = $2.50*X*

*TC* = (*VC*)*X* + *FC* = $1.00*X* + $60,000

To break even, *TR* = *TC*

Then, $2.50*X* = $1.00*X* + $60,000

$1.50*X* = $60,000

*X* = 40,000 CDs per month

*b.* *NI =* (*S – VC*)*X* – *FC*

If *NI* = $7500 per month, then

$7500= ($2.50 − $1.00)*X* – $60,000

45,000 CDs per month

c. If total revenue = $130,000 then CDs per month = = 52,000 CDs

*NI* = ($2.50 - $1.00)(52,000) - $60,000

= $18,000 profit per month

d. *TR* = *S*(*X*) = $3.50*X*

To breakeven, *TR* = *TC*

$3.50*X* = $1.00*X* + $60,000

$2.50*X* = $60,000

*X* = 24,000 CDs per month to break even

7. Given: *S* = 10¢ per copy; *FC* = $300 per month;

*VC* = 1.5¢ + + 0.5¢ = 5¢ per copy

*a.* *TR* = (*S*)*X* = $0.10*X*

*TC* = (*VC*)*X* + *FC* = $0.05*X* + $300

To break even, *TR = TC*

Then, $0.10*X* = $0.05*X* + $300

$0.05*X* = $300

*X* = 6000 copies per month

*b.* If *X* = 6000 + 1000 = 7000 copies per month.

*NI =* (*S – VC*)*X – FC* = ($0.10 − $0.05)7000– $300 = $50 per month

The additional 1000 copies add $50 per month profit.

9. Given: Capacity = 250 units per week

*S* = $20, *VC* = $12, *FC* = $1200 per week

*a.* *TR = (S)X* = $20*X*

*TC* = (*VC*)*X* + *FC* = $12*X* + $1200

To break even, *TR = TC*

Therefore, $20*X* = $12*X* + $1200

*X* = 150 units/week

*b.* *NI = TR – TC* = $20*X* – $12*X* – $1200 = $8*X* – $1200

(i) If *X* = 120 units/week,

then  *NI* = $8(120) – $1200 = –$240

There will be a loss of $240/week.

(ii) If *X* = 250 units/week,

then *NI* = $8(250) – $1200 = $800/week profit.

c. *NI = TR – TC* = $20*X* – $12*X* – $1200 = $8*X* – $1200

(i) *X* = = 245 units per week

*NI* = $8(245) - $1200 = $760 profit per week

(ii) *X* = = 98 units per week

*NI* = $8(98) - $1200 = $416 loss per week

*d.* If *NI* = $400/week,

then $400 = $8*X* – $1200

$1600 = $8*X*

*X* = 200 units/week

11. Given: Total variable costs are of total revenue,

*FC* = $10,000,000 per year

At full capacity, Sales revenue =  = $20,000,000

*a.* At the break-even point, *TR – TC* = 0

That is, *TR* – *TR* – $10,000,000 = 0

*TR* = $10,000,000

*TR* = $15,000,000

which is  $100% = 75% of capacity

*b.* At 70% of capacity, *TR* = 0.70($20,000,000) = $14,000,000

*NI = TR – TC* = $14,000,000 – ($14,000,000) – $10,000,000 = –$666,667

That is, Beta would have a $666,667 loss.

13. Given: *VC* = $43, *S* = $70, *FC* = $648,000/year = $54,000/month

Production capacity = 3200 composters per month

*a.* *TR* = (*S*)*X* = $70*X*

*TC* = (*VC*)*X* + *FC* = $43*X* + $54,000

At break even, *TR* = *TC*

$70*X* = $43*X* + $54,000

$27*X* = $54,000

*X* = 2000 composters /month

*b.* *NI = TR – TC* = $70*X* – $43*X* – $54,000 = $27*X* – $54,000

If *X* = 2500 composters/month,

*NI* = $27(2500) – $54,000 = $13,500/month

*c.* At 50% capacity, *X* = 0.5(3200 composters/month) = 1600 composters/month and

*NI* = $27(1600) – $54,000 = –$10,800/month.

Reflex will lose $10,800/month in the recession.

*d.* For *NI* = $226,800/year = $18,900/month,

$18,900 = $27*X* – $54,000

$72,900 = $27*X*

*X* = 2700 composters/month

which is 100% = 84.4% of capacity.

*e.* If *TR* = $71*X* while costs do not change, then

$71*X* = $43*X* + $54,000 at the break-even point

*X* =  = 1929 composters/month

The break‑even volume decreases by 71 composters/month

for a $1 increase in the selling price.

15. Given: In the first quarter (Q1), *TR* = $4,500,000; *NI* = $900,000; *X* = 60,000 tires.

In the second quarter (Q2), *NI* = $700,000; *X* = 50,000 tires.

Since *TR = (S)X*, then = $75 per tire

Also, *TR* = $75 × 50,000 = $3,750,000 in Q2.

Since *NI =* (*S – VC*)*X – FC*

then for Q1, $900,000 = (*S – VC*)60,000 − *FC* ➀

and for Q2, $700,000 = (*S – VC*)50,000 − *FC* ➁

Subtract: $200,000 = (*S – VC*)10,000

*VC* = *S –* $20 = $75 *–* $20 = $55 per tire

Substitute *S – VC* = $20 into ➁:

$700,000 = ($20)50,000 − *FC*

Therefore, *FC* = $1,000,000 − $700,000 = $300,000 per quarter

17. Given: Annual sales= $1,200,000; *FC* = $400,000 per year

Total variable costs = $600,000

*a.* Variable costs were one-half of sales revenue. At the break-even point,

*NI* = 0 = *TR – TC*

0 = *TR –* 0.5(*TR*) – $400,000

$800,000 at breakeven

*b.* Sales increase by 0.15($1,200,000) = $180,000

Since variable costs are 50% of total revenue,

half of increased revenue will be consumed by variable costs and

net income will increase by 0.50($180,000) = $90,000.

*c.* Net income will increase by the full reduction in *FC.*

That is, *NI* will increase by 0.10($400,000) = $40,000.

*d.* Net income will decrease by the full amount of the increase

in total variable costs, That is, *NI* will decrease by

0.10($600,000) = $60,000.

19. Given: *S* = $270, *VC* = $220, *FC* = $1400 for 15 to 36 participants

*a.* To break even,

*X* =  =  = 28 participants

*b.* If *X* = 36, then

*NI* =(*S – VC*)*X – FC* = ($270 – $220)36 – $1400 = $400

*c.* If a loss of $400 is incurred,

–$400 = ($270 – $220) *X* – $1400

*X* = 20

Hence, the minimum number of participants is 20.

21. Given: If *S* = $46, *FC* = $5400, then *VC* = $24 + 0.1($46) = $28.60

If *S* = $56, *FC* = $5400, then *VC* = $24 + 0.1($56) = $29.60

*a.* To break even,

*X* = =  *X* = 

= 310.34 tickets = 204.55 tickets

Rounding up (to avoid any loss), there must be 311 tickets sold at $46

or 205 tickets sold at $56.

*b.* *NI* = *(S – VC)X – FC* *NI* = *(S – VC)X – FC*

= ($46 – $28.60)400 – $5400 = ($56 – $29.60)300 – $5400

= $1560 = $2520

23. Given: *S* = $37.50, *VC* = $13.25, *FC* = $5600/month

*a.* Last year’s break-even volume =  =  = 231 units/month

This year, *VC* = $15, *FC* = $6000/month.

For the break-even volume to remain 231 griddles/month,

*S – VC*  =

*S –* $15 =  = $25.97

*S* = $15 + $25.97 = $40.97

*b.* Last year’s *NI* = (*S – VC*)*X – FC* = ($37.50 – $13.25)300 – $5600 = $1675.00.

In order that this year's profit also be $1675/month on sales of 300 griddles/month,

$1675 = 300(*S* – $15) – $6000

*S* = + $15 = $40.58

25. Given: *S* = $180/tonne, *VC* = $42 + $24 = $66 per tonne,

*FC* = $400 + $600 + $450 = $1450 per hectare

*a.* To break even,

*X* =  =  = 12.72 tonnes per hectare

*b.* If *S* = $190 per tonne instead of $180 per tonne,

*X* =  = 11.69 tonnes per hectare

The break-even tonnage will be reduced by 1.03 tonnes per hectare.

*c.* (i) *NI = (S – VC)X – FC* = ($180 – $66)15 – $1450 = $260 per hectare.

(ii) *NI* = ($180 – $66)10 – $1450 = –$310.00 per hectare.

That is, a loss of $310 per hectare.

**Concept Questions (Section 6.6)**

1. *a.* Since *CM = S – VC, CM* will increase if *S* is increased.

*b.* Raw materials are part of *VC. VC* will decrease if the cost of raw materials decreases. Therefore, *CM* will increase.

*c.* The property tax is part of *FC*. There will be no change in *CM*.

*d.* The salaries of executives are part of *FC*. There will be no change in *CM*.

*e.* Wages of production workers are part of *VC. VC* will increase and *CM* will decrease.

3. *a.* The break-even volume will be lowered since fewer units need to be sold to cover the reduced fixed costs.

*b.* If unit variable costs (*VC*) increase, the contribution margin (*CM* = *S – VC*) decreases and more units must be sold to cover the unchanged fixed costs. Therefore, the break-even point is higher.

*c.* No change—the actual sales volume does not affect the break-even point.

*d.* If *S* decreases, the contribution margin (*CM* = *S – VC*) decreases and more units must be sold to cover the unchanged fixed costs. Therefore, the break-even point is higher.

*e.* If the contribution rate (or ratio) *CR* increases, a larger portion of each unit’s selling price is available to pay fixed costs. Therefore, the break-even point is lower.

**Exercise 6.6**

1. Given: *S* = $30; *VC* = $10; *FC* = $100,000 per year

*a.* Each toy sold contributes

*CM* = *S – VC* = $30 – $10 = $20

to the payment of fixed costs. To cover $100,000 of fixed costs

(and break even), Toys-4-U needs to sell

= 5000 toys per year

*b.* Break-even revenue = 5000($30) = $150,000 per year

c. Break-even volume = = 62.5% of capacity

d. *NI* = ($30 - $10)(6200) - $100,000

= ($20)(6200) - $100,000

*NI* = $24,000 profit

e. $10,000 = $20*X* - $100,000

$20*X* = $110,000

*X* = 5500 toys per year

3. Given: *S* = $3.20; *VC* = $1.20; *FC* = $250 per month

*a.* *FC* = 12($250) = $3000 per year

Each jar sold contributes

*CM* = *S – VC* = $3.20 – $1.20 = $2.00

to the payment of fixed costs. To cover $3000 of fixed costs

(and break even), Ingrid must sell

= 1500 jars per year

*b.* If Ingrid sells 3000 jars per year, this sales level is 1500 jars above break-even. Each of these jars contributes *CM* = $2.00 to profit.

Total profit = 1500($2.00) = $3000

c. A loss of $1200 would be a net income of -$1200

-$1200 = ($2.00)*X* - $3000

$1800 = $2.00*X*

*X* = 900 jars per year

5. Given: *S* = $2.50; *VC* = $1.00; *FC* = $60,000 per month

*a.* *CM* = *S – VC* = $2.50 – $1.00 = $1.50 per CD

Break-even volume =  =  = 40,000 CDs per month

*b.* For *NI* = $7500 per month, sales must be

units above breakeven.

Total sales must be 40,000 + 5000 = 45,000 CDs per month

c. If total revenue = $130,000 then CDs per month = = 52,000 CDs

*NI* = ($1.50)(52,000) - $60,000

= $18,000 profit per month

d. *TR* = *S*(*X*) = $3.50*X*

To break even, *NI = $0 = CM(X) - FC*

$0 = $2.50*X* - $60,000

$2.50*X* = $60,000

*X* = 24,000 CDs per month to break even

7. Given: *S* = 10¢ per copy; *FC* = $300 per month;

*VC* = 1.5¢ + + 0.5¢ = 5¢ per copy

*a.* *CM* = *S – VC* = 10¢ – 5¢ = 5¢ per copy

Break‑even volume =  =  = 6000 copies per month

*b.* Each additional 1000 copies per month will add profit of

1000*CM* = 1000($0.05) = $50 per month

9. Given: Capacity = 250 units per week

*a.* *S* = $20, *VC* = $12, *FC* = $1200 per week

*CM = S – VC* = $20 – $12 = $8.00

Break-even volume =  =  = 150 units per week

*b.* (i) At 30 units per week short of breakeven,

there will be a loss of *X*(*CM*) = 30($8) = $240/week.

(ii) At 100 units per week above breakeven,

there will be a profit of 100($8) = $800/week.

c. *NI =* CM(X) - FC = $8*X* – $1200

(i) *X* = = 245 units per week

*NI* = $8(245) - $1200 = $760 profit per week

(ii) *X* = = 98 units per week

*NI* = $8(98) - $1200 = $416 loss per week

*d*. For a net income of $400/week, sales must be

 =  = 50 units above breakeven

Hence, sales must be 150 + 50 = 200 units per week.





If revenue of $18,000,000 represents 90% of capacity, then

Revenue at full capacity = 

and break-even revenue represents = 75% of capacity

*b.* At 70% of capacity,

Revenue = 0.70($20,000,000) = $14,000,000

and Total variable costs = = $4,666,667

Net income = $14,000,000 – $10,000,000 – $4,666,667 = –$666,667

That is, Beta would lose $666,667.

13. Given: *VC* = $43, *S* = $70, *FC* = $648,000/year = $54,000/month

Production capacity = 3200 composters per month

*a*. *CM* = *S – VC* = $70 – $43 = $27

Break-even volume =  =  = 2000 composters/month

*b.* At 500 composters/month in excess of break even,

*NI* = 500(*CM*) = 500($27) = $13,500/month

*c.* At 50% of capacity,

*X* = 0.5(3200 composters/month) = 1600 composters/month

This is 400 composters/month below breakeven.

Hence, Reflex will lose 400($27) = $10,800/month

*d.* For *NI* = $226,800/year = $18,900/month,

*X* = 2000 + = 2700 units/month

which is 100% = 84.4% of capacity

*e.* If *S* is $1 higher while *VC* and *FC* are unchanged,

then *CM* will increase by $1 to $28 and

Break-even volume =  = 1929

That is, the break-even volume decreases by 71 composters /month for

a $1 increase in the selling price.

15. Given: In the first quarter (Q1), *TR* = $4,500,000; *NI* = $900,000; *X* = 60,000 tires.

In the second quarter (Q2), *NI* = $700,000; *X* = 50,000 tires.

Since *TR = (S)X*, then = $75 per tire

Also, *TR* = $75 × 50,000 = $3,750,000 in Q2.

Since *NI =* (*CM*)*X – FC*

then for Q1, $900,000 = (*CM*)60,000 − *FC* ➀

and for Q2, $700,000 = (*CM*)50,000 − *FC* ➁

Subtract: $200,000 = (*CM*)10,000

 = $20 per tire

Substitute *CM* = $20 into ➁:

$700,000 = ($20)50,000 − *FC*

Therefore, *FC* = $1,000,000 − $700,000 = $300,000 per quarter

Since *CM = S − VC*

then *VC = S − CM* = $75 − $20 = $55 per tire

17. Given: Annual sales= $1,200,000; *FC* = $400,000 per year

Total variable costs = $600,000

*a.* Variable costs were one-half of sales revenue. At the break-even point,

*NI* = $0 = *CM(TR) - FC*

$0 = 0.5(*TR*) – $400,000

$800,000 at breakeven

*b.* Sales increase by 0.15($1,200,000) = $180,000

Since variable costs are 50% of total revenue,

half of increased revenue will be consumed by variable costs and

net income will increase by 0.50($180,000) = $90,000.

*c.* Net income will increase by the full reduction in *FC.*

That is, *NI* will increase by 0.10($400,000) = $40,000.

*d.* Net income will decrease by the full amount of the increase

in total variable costs, That is, *NI* will decrease by

0.10($600,000) = $60,000.

19. Given: *S* = $270, *VC* = $220, *FC* = $1400 for 15 to 36 participants

*a.* *CM = S – VC* = $270 – $220 = $50

To break even, *X* = =  = 28 participants

*b.* If *X* = 36 (which is 8 participants *above* breakeven),

then *NI* = 8(*CM*) = 8($50) = $400

*c.* A loss of $400 will be incurred if there are

=  = 8 *fewer* participants than breakeven

Hence, the minimum number of participants is 28 – 8 = 20.

21. Given: If *S* = $46, *FC* = $5400, then *VC* = $24 + 0.1($46) = $28.60

If *S* = $56, *FC* = $5400, then *VC* = $24 + 0.1($56) = $29.60

If *S* = $46 \_\_ If *S* = $56

*a.* *CM* = $46 – $28.60 = $17.40 *CM* = $56 – $29.60 = $26.40

To break even, To break even,

*X* = == 310.34 = 311 *X* = == 204.55 = 205

We have rounded up to get the *smallest* number of tickets

that will make *NI* = 0 or a small profit.

*b.* If 400 tickets are sold, If 300 tickets are sold,

*NI* = $17.40(400 – 311) *NI* = $26.40(300 – 205)

= $1548.60 = $2508.00

23. Given: *S* = $37.50, *VC* = $13.25, *FC* = $5600/month

*a.* Last year's *CM* *= S – VC* = $37.50 – $13.25 = $24.25

Last year’s break-even volume =  =  = 231 units/month

This year, *VC* = $15, *FC* = $6000/month. For the break-even

volume to still be 231 griddles/month,

*CM* = =  = $25.97 and

*S = VC + CM* = $15 + $25.97 = $40.97

*b.* Last year’s *NI = (CM)X – FC* = ($24.25)300 – $5600 = $1675.00. In order that

this year's profit also be $1675/month on sales of 300 griddles/month,

$1675 = 300*CM* – $6000

*CM* =  = $25.58

and *S = CM + VC* = $25.58 + $15 = $40.58

25. Given: *S* = $180/tonne, *VC* = $42 + $24 = $66 per tonne,

*FC* = $400 + $600 + $450 = $1450 per hectare

*a.* *CM = S – VC* = $180 – $66 = $114 per tonne

Break-even yield =  =  = 12.72 tonnes/hectare

*b.* If *S* = $190 per tonne, then *CM* = $124 per tonne and

Break-even yield =  = 11.69 tonnes/hectare

The break-even yield is lowered by

12.72 – 11.69 = 1.03 tonnes/hectare

*c.* (i) If *X* = 15 tonnes/hectare (which is 2.28 tonnes/hectare *above* breakeven),

*NI* = 2.28(*CM*) = 2.28($114) = $259.92 per hectare

(ii) If *X* = 10 tonnes/hectare (which is 2.72 tonnes/hectare *below* breakeven),

the loss will be 2.72($114) = $310.08 per hectare.

**Review Problems**

1. Given: *S* = $180; *VC* = $110; *FC* = $1,260,000 per year

*a.* *TR* = (*S*)*X* = $180*X*

*TC* = (*VC*)*X* + *FC* = $110*X* + $1,260,000

To break even, *TR = TC*

Then, $180*X* = $110*X* + $1,260,000

$70*X* = $1,260,000

*X* = 18,000 units

*b.* Break-even sales revenue = (*S*)*X* = $180(18,000) = $3,240,000

*c.* (i) If *X* = 20,000 units are sold,

*NI* =(*S – VC*)*X* – *FC*

= ($180 – $110)20,000 – $1,260,000

= $140,000

That is, the company will have a profit of $140,000.

(ii) If *X* = 17,500 units are sold,

*NI* =(*S – VC*)*X* – *FC*

= ($180 – $110)17,500 – $1,260,000

= –$35,000

That is, the company will lose $35,000.

d. *NI* =(*S – VC*)*X* – *FC*

$315,000 = ($180 – $110)*X* – $1,260,000

$1,575,000 = $70*X*

*X* = 22,500 units

e. *NI* =(*S – VC*)*X* – *FC*

-$124,250 = ($180 – $110)*X* – $1,260,000

$1,135,750 = $70*X*

*X* = 16,225 units

f. New *FC* = $1,260,000(1 – 10%) = $1,134,000

To break even *TR* = *TC*

$180*X* = $110*X* + $1,134,000

$70*X* = $1,134,000

*X* = 16,200 units

3. Given: *FC* = $45,000 + $7000 + $8000 = $60,000;

*VC* = $8.00 + 0.08($35) = $10.80; *S* = $35.00

*a.* Break-even volume =  =  = 2480 books

*b.* *NI =* (*S – VC*)*X – FC* = ($35.00 – $10.80)4800 – $60,000 = $56,160

*c.* If the price is reduced 10%,

*S* = $35(1 – 0.10) = $31.50

and if sales are 15% higher,

*X* = 4800(1 + 0.15) = 5520 units

New VC = $8.00 +0.08($31.50) = $10.52

Then *NI* = ($31.50 – $10.52)5520 – $60,000 = $55,809.60

Select the $35 price because (forecast) net incomewill be larger (by approximately $350).

*d.* If *FC* = $65,000 and *VC* = $11.80, then

Break-even volume =  =  = 2802 books

That is, the break-even volume would be increased by 2802 – 2480 = 322 books

5. Given: *NI* = loss of $12 million on revenue of $270 million from *X* = 9000 cars;

Break-even volume = 10,000 cars

*a.* *NI =* (*S – VC*)*X – FC*,

If *X* = 9000 cars,

–$12,000,000 = (*S – VC*)9,000 *– FC* ➀

At the break-even volume of *X* = 10,000 cars,

0 = (*S – VC*)10,000 *– FC* ➁

➁ – ➀ :gives:

$12,000,000 = (*S – VC*)1000

Hence, *S – VC*  = $12,000

Substitute this value into ➁ to obtain *FC* = $120 million per year

*b.* If sales = 12,000 units (which are 2000 more than breakeven),

*NI* = (*S – VC*)*X – FC* = $12,000(12,000) – $120 million = $24 million

7. Given: *S* = $100, *FC* = $200,000, *VC* = $60

Forecast *X* = 8000 units

*a.* *CM = S – VC* = $100 – $60 = $40

Break-even volume =  =  = 5000 units

*b.* Volume = Break-even volume + = 5000 +  = 7500 units

*c.* If sales are 8000 – 5000 = 3000 units above breakeven, then

*NI* = 3000(*CM*) = 3000($40) = $120,000

*d.* (i) *NI* = (*CM*)*X* – *FC* = 8000(*CM*) – *FC*

For each $1 increase in *FC*, *NI* will drop $1.

If *FC* is 5% (or $10,000) higher, *NI* will be $10,000 lower.

(ii) If *FC* is 10% (or $20,000) lower, *NI* will be $20,000 higher.

*e.* (i) If *VC* is 10% (or $6) higher, then

*CM* = *S – VC* = $100 – $66 = $34, and

*NI* = 8000(*CM*) – *FC* = 8000($34) – $200,000 = $72,000

That is, *NI* will be $120,000 – $72,000 = $48,000 lower.

(ii) If *VC* is 5% (or $3) lower, then

*CM* = $100 – $57 = $43 and

*NI* = 8000($43) – $200,000 = $144,000

That is, *NI* will be $144,000 – $120,000 = $24,000 higher.

Note the “leverage” effect here. That is, (% change in *NI*) = 4(% change in *VC*)

*f.* (i) If *S* is 5% (or $5) higher, then

*CM* = $105 – $60 = $45, and

*NI* = 8000($45) – $200,000 = $160,000

That is, *NI* will be $160,000 – $120,000 = $40,000 higher.

(ii) If *S* is 10% (or $10) lower, then

*CM* = $90 – $60 = $30 and

*NI* = 8000($30) – $200,000 = $40,000

That is, *NI* will be $120,000 – $40,000 = $80,000 lower.

Note the “leverage” effect here. That is,

% change in *NI* = ×(% change in *S*)

(assuming that the sales volume does not change).

*g.* If *X* = 8000, *VC* = 1.1($60) = $66. *S* = $100, and

*FC* = 0.9($200,000) = $180,000, then

*CM* = $100 – $66 = $34 and

*NI* = 8000($34) – $180,000 = $92,000

That is, *NI* will be $120,000 – $92,000 = $28,000 lower.

9. *a*. Contribution margin, *CM = S – VC* = $110 – $24 = $86

The number of room rentals per month needed to break even is

*X* = = = 325.6

Full occupancy would be 30 × 30 = 900 rentals per month.

Break-even occupancy rate =  × 100% = 36.2%

*b.* (i) At 40% occupancy, *X* = 0.4 × 900 = 360 rentals per month.

*NI* = (*CM*)*X – FC* = ($86)360 – $28,000 = $2960/month

(ii) At 30% occupancy, *X* = 0.3 × 900 = 270 rentals per month.

*NI* = ($86)270 – $28,000 = –$4780

That is, a loss of $4780 per month.

*c.* In part *b* (i), we obtained a net income of $2960/month for

a rental rate of $110 and a 40% occupancy.

At a rental rate of $94 per unit, *CM* = $94 – $24 = $70 per unit.

At 50% occupancy, *X* = 450 units/month and

*NI* = (*CM*)*X – FC* = ($70)450 – $28,000 = $3500/month

The owner should reduce the rental rate to $94 per unit per night since

the net income will increase by $3500 – $2960 = $540 per month.

**Case (Chapter 6)**

***Estimating the Contribution Rate in a Multi-Product Business***

1. Since *NI* = *CR*(Total revenue) – *FC*, then

For Year 1: $105,000 = $750,000*CR* – *FC* ➀

For Year 2: $127,500 = $825,000*CR* – *FC* ➁

Subtract: –$22,500 = –$75,000*CR*

*CR* = 0.30 = 30%

Substitute into ➀: $105,000 = $750,000(0.30) – *FC*

*FC* = $225,000 – $105,000 = $120,000

MML’s total fixed costs are $120,000 per year and its average contribution rate is 30%.

3. Forecast *NI* = *CR*(Forecast total revenue) – *FC*

= 0.3($875,000) – $120,000

= $142,500