9 Compound Interest: Future

Value and Present Value

**Concept Questions (Section 9.1)**

1. We compound interest when we convert it to principal and calculate subsequent interest on both the principal and the converted interest.

3. The “periodic rate of interest” is the percent interest earned in a single compounding period. The “nominal rate of interest” is the *annual* interest rate you obtain if you *extend* the periodic interest rate to a full year. This extension is done by multiplying the periodic rate of interest by the number of compounding periods in a year.

**Exercise 9.1**

1. *a.* *i* =  =  = 0.5% per month

*b.* *i* =  =  = 1.5% per quarter

*c.* *i* =  =  = 3.0% per half-year

3. *a.* *i* =  =  = 1.35% per quarter

*b.* *i* =  =  = 0.45% per month

5. *a.* *j = mi =* 2(3.6%) = 7.2% compounded semiannually

*b.* *j = mi =* 4(1.8%) = 7.2% compounded quarterly

*c.* *j = mi =* 12(0.6%) = 7.2% compounded monthly

7. *a.* *j = mi =* 4(1.25%) = 5.00% compounded quarterly

*b.* *j = mi =* 12(0.49167%) = 5.90% compounded monthly

9. *a.* *m* =  =  = 4 (that is, quarterly compounding)

*b.* *m* =  =  = 2 (that is, semiannual compounding)

*c.* *m* =  =  = 12 (that is, monthly compounding)

11. *a.* *m* =  =  = 4 per year

*b.* *m* =  =  = 12 per year

**Point of Interest (Section 9.2)**

***The “Magic” of Compound Interest***

1. Growth ratio = = 3.16

That is, the growth over 20 years will be 3.16 times the growth over 10 years.

3. A higher rate of return causes growth to accelerate disproportionately as time passes. Therefore, we expect that the growth ratio will be larger at a 10% rate of return than at an 8% rate of return. Checking our intuition,

Growth ratio = = 3.59

**Concept Questions (Section 9.2)**

1. The more frequent the compounding of the 6% nominal rate, the more interest will be earned by the investment. Therefore, 6% compounded quarterly is the preferred rate.   
(The other two rates both earn 3% interest in the 6-month term.)

(#1) 6% compounded quarterly;

(#2) 6% compounded semiannually and 6% simple interest (tied)

3. The fundamental reason is that money can be invested to earn interest. $100 received today can earn interest for a longer period than $100 received at a future date.

**Exercise 9.2**

1. *i* =  =  = 1.5% (per half year)

*n* = *m*(Term in years) = 2(7) = 14 compounding periods

Maturity value,  = $5000 = $6158.78

3. *i* =  =  = 0.208% (per month)

*n* = *m*(Term in years) = 12(3.25) = 39 compounding periods

Future value,  = $12,100 = $13,123.06

5. Given: *PV* = $10,000; *j* = 3% compounded quarterly; *i* =  = 0.75%

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Term | *n* | *Maturity amount* |
|  | *a.* | 15 years | 4(15) = 60 | $10,000 = $15,656.81 |
|  | *b.* | 20 years | 4(20) = 80 | $10,000 = $18,180.44 |
|  | *c.* | 25 years | 4(25) = 100 | $10,000 = $21,110.84 |
|  | *d.* | 30 years | 4(30) = 120 | $10,000 = $24,513.57 |

7. Given: *PV* = $10,000; Term = 25 years; *j* = 4%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *m* | *i* | *n* | *Maturity amount* |
|  | *a.* | 1 | = 4% | 25 | $10,000 = $26,658.36 |
|  | *b.* | 2 | = 2% | 2(25) = 50 | $10,000 = $26,915.88 |
|  | *c.* | 4 | = 1% | 4(25) = 100 | $10,000 = $27,048.14 |
|  | *d.* | 12 | = 0.% | 12(25) = 300 | $10,000 = $27,137.65 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 9. |  | *i* | *n* | *Maturity amount* |
|  | *a.* |  | 12 | $100 = $103.04 |
|  | *b.* |  | 4 | $100 = $103.14 |
|  | *c.* |  | 2 | $100 = $103.23 |
|  | *d.* | 3.3% | 1 | $100 = $103.30 |

The investor would prefer 3.3% compounded annually

since it produces the highest maturity value.

11. *PV* = $12,000; *i* =  = 1.8%; *n* = 4(1.5) = 6

Maturity value,  = $12,000 = $13,355.74

Interest charged = *FV* – *PV* = $13,355.74 – $12,000 = $1355.74

13. Maturity value at 5% =  = $10,000 = $33,863.55

Maturity value at 4% =  = $10,000 = $26,658.36

Difference: $7205.19

The difference is ×100% = 27.03% of the

maturity value at 4% compounded annually.

15. Maturity value after 25 years =  = $10,000 = $33,863.55

Maturity value after 20 years =  = $10,000 = $26,532.98

Difference: $7330.57

The difference is ×100% = 27.63% of the amount after 20 years.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 17. | *Interest rate* | *20 years* | *25 years* | *30 years* |
|  | 8% | $4660.96 | $6848.48 | $10,062.66 |
|  | 10% | $6727.50 | $10,834.71 | $17,449.40 |

19. Equivalent payment *=*  = $5000 = $5682.38

21. Equivalent amount *=* = $10,000 = $12,454.51

23. Calculate equivalent values at a focal date 4 years from now.

For the first payment, *PV* = $1300, *i* =  = 0.5%; *n* = 4(4) = 16

For the second payment, *PV* = $1800, *i* =  = 0.5%; *n* = 4(2.25) = 9

The combined equivalent value 4 years from now is

 = $1407.99 + $1882.64 = $3290.63

The single replacement payment is $3290.63.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 25. | Principal | *i* | *n* | *Maturity amount* |
|  | $3000 | 2.5% | 6 | $3000 = $ 3479.08 |
|  | $3500 | 2.5% | 4 | $3500 = $ 3863.35 |
|  | $4000 | 2.5% | 2 | $4000 = $ 4202.50 |
|  |  |  | Total amount owed = $11,544.93 | |

27. *PV* = $5000 and *i* =  = % in each case.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Grand-*  *child* | *Age* | *Time until*  *19th birthday* | *n* |
|  | Donna | 12 yr, 7 mo | 6 yr, 5 mo | 77 |
|  | Tim | 10 yr, 3 mo | 8 yr, 9 mo | 105 |
|  | Gary | 7 yr, 11 mo | 11 yr, 1 mo | 133 |

Donna will receive $5000= $8340.04

Tim will receive $5000 = $10,045.40

Gary will receive $5000 = $12,099.48

29. For the current GIC, *i* =  = 3.00%; *n* = 5(2) = 10

Maturity value *=* *=* $60,000  = $80,634.98

Maturity value of the second GIC will be:

*a.* *FV* = $80,634.98 = $108,366.67

*b.* *FV* = $80,634.98 = $113,743.60

*c.* *FV* = $80,634.98 = $103,219.59

31. For the first 15 months, *PV* = $7000; *i* =  = 2.375%; *n* =  = 5

 *=* $7000 = $7871.68

For the next 6 months, *PV* = $7871.68; *i* =  = 4.25%; *n* = 1

Total amount owed = $7871.68 = $8206.23

33. Amount owed after 2years =  = $3000= $3394.22

Balance owed after $1000 payment = $2394.22

Amount owed after another 6 months = $2394.22= $2454.08

Amount owing today (after another 2 years) = $2454.08 = $2766.14

35. Amount owed after the $2500 payment =  – $2500

= $10,000– $2500

= $8006.03

Amount owed 3 months later = $8006.03= $8138.86

Amount owed after the $3000 payment = $8138.86– $3000

= $5426.21

Amount owed 6 months later = $5426.21

= $5617.79

37. a. At 5% compounded annually, *FV* = $100 (1.0525) = $338.64

At 4% compounded annually, *FV* = $100 (1.0425) = $266.58

The investment will be worth  x 100% = 27.03% more

b. For 20 years, *FV =* $100 (1.0520) = $265.33

For 25 years, *FV =* $100 (1.0525) = $338.64

The investment will be worth  x 100% = 27.63% more

**Exercise 9.3**

1. Given: *j* = 2.5%; *m* = 1; *FV* = $10,000; *i* =  = = 2.5% (per year)

*a.* For a term of 10 years, *n* = 1(10) = 10, and

Investment today, *=* = $7811.98

*b.* For a term of 20 years, *n* = 1(20) = 20, and

Investment today, *=* = $6102.71

*c.* For a term of 30 years, *n* = 1(30) = 30, and

Investment today, *=* = $4767.43

3. Given: *FV* = $10,000; Term = 25 years; *j* = 4%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *m* | *i* | *n* | *Initial investment* |
|  | *a.* | 1 | = 4% | 25 | $10,000 = $3751.17 |
|  | *b.* | 2 | = 2% | 2(25) = 50 | $10,000 = $3715.28 |
|  | *c.* | 4 | = 1% | 4(25) = 100 | $10,000 = $3697.11 |
|  | *d.* | 12 | = | 12(25) = 300 | $10,000 = $3684.92 |

5. Given: *FV* = $10,000; *i* =  = 4.5% (per year); *n* = 1(10) = 10 periods

Present value, *=* = $6439.28

7. Given: *FV* = $9704.61; *i* = = 1.75% (per half year); *n* =  = 7 periods

Original investment, *=* = $8594.83

9. Given: *i* =  = 5.5%; *n* = 1(4) = 4; *FV* = $10,000

Required investment *=**=*  = $8072.17

11. Equivalent payment today = *PV* of future payment

= $5000

= $4494.57

13. Given: *i* =  = 3.1%; *FV* = $7000

With the focal date 1.5 years from now, *n* = 2(6.5) = 13

Equivalent value *=**=* $7000 = $4706.90

15. Equivalent amount *=**=* $2600 = $2241.95

17. Equivalent value 3 years from now of the $1400 payment is

 *=* $1400 = $1531.33

Equivalent value 3 years from now of the $1800 payment is

*=* $1800= $1695.56

Combined equivalent value = $1531.33 + $1695.56 = $3226.89

19. Sum of the equivalent values of the payments, 1 year from now,

= $1000+ $2000

= $934.00+ $1768.69

= $2702.69

The payee should be willing to accept a payment of $2702.69, one year from now.

21. Equivalent payment 6 months from now = Sum of the equivalent values

= $500+ $800

= $510.515+ $790.073

= $1300.59

23. Sum of the equivalent values of the payments, 6 months from now,

= $2100 + $1300+ $800

= $2297.379 + $1329.526 + $747.877

= $4374.78

The equivalent single payment, 6 months from now, is $4374.78.

25. The economic value of an offer is its present value.

*PV* of second offer = $49,000 + $49,000 + $49,000

= $49,000 + $47,939.53 + $46,902.01

= $143,841.54

The $145,000-cash offer is worth $1158.46 more.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time: | 0 yr | 0.5 yr | 1.5 yr | 2.5 yr | 3.5 yr |
| Salary: | $4.8 M | $10 M | $17.2 M | $17.5 M | $18.5 M |

27.

Economic value of Pete’s contract

= Present value of payments

= $4.8M +  +  +  + 

= $4,800,000 + $9,756,098 + $15,971,910 + $15,467,450 + $15,563,407

= $61,559,000 rounded to the nearest $1000

29. The data for the three certificates are presented in the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *Maturity*  *Value* | *Term* | *j* | *m* | *i* | *n* |
|  | $4000 | 3.5 years | 5% | 2 | 2.5% | 7 |
|  | $5000 | 4.5 years | 5% | 2 | 2.5% | 9 |
|  | $6000 | 5.5 years | 5.6% | 4 | 1.4% | 22 |

Michelle should invest:

*=* $4000 = $3365.06 in a 3-year certificate

*=* $5000 = $4003.64 in a 4-year certificate

*=* $6000= $4418.92 in a 5-year certificate

31. Total price = Sum of the present values

= $950+ $780+ $1270

= $899.498 + $718.637 + $1186.177

= $2804.31

33. Let *x* represent the final loan payment.

Loan = Present value of all payments

$15,000 = $4000+ $4000+ *x*

$15,000 = $3788.75+ $3399.14+ 0.762397906*x*

*x* = $10,246.76

The third loan payment will be $10,246.76.

35. Let *x* represent the size of each loan payment.

$10,000 = *x*+ *x*+ *x*

= 0.90573081*x* + 0.85349037*x* + 0.75787503*x*

= 2.51709621*x*

*x* = $3972.83

The amount of each loan payment is $3972.83.

37. Let *x* represent the size of the first payment.

$7500 = *x*+ 2*x*+ 4*x*

= 0.98349871*x* + 1.93453943*x* + 3.80523409*x*

= 6.72327223*x*

*x* = $1115.53

The second payment is 2*x* = $2231.06.

39. The scheduled payments are:

*=* $950 = $988.57 due in 4 months,

*=* $780 = $827.99 due in 6 months, and

*=* $1270 = $1334.78 due in 5 months.

The total price to be paid for these scheduled payments is the sum of their present values.

Total price = $988.57+$827.99+$1334.78

= $2945.55

41. Interest = *FV* – *PV* =  – *PV*

$1175.98 = *PV*– *PV* = *PV*(1.9799873) – *PV* = 0.9799873*PV*

Original investment, *PV* =  = $1200.00

**Exercise 9.4**

**Financial Calculator Solutions to Odd-Numbered Problems in Exercise 9.2**

1. Given: *PV* = $5000; *j* = 3% ; *m* = 2

3 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

14 **N**

5000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 6158.78

*n* = *m*(Term in years)

= 2(7)

= 14 compounding periods

Maturity value, *FV* = $6158.78

2.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

39 **N**

12100 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 13,123.06

3. Given: *PV* = $12,100; *j* = 2.5%; *m* = 12

*n* = *m*(Term in years)

= 12(3.25)

= 39 compounding periods

Maturity value, *FV* = $13,123.06

5. Given: *PV* = $10,000; *j* = 3% compounded quarterly for all parts

*a*. *n* = 4(15) = 60 *b.* *n* = 4(20) = 80

3 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

60 **N**

10000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 15,656.81

Same *I/Y,* *C/Y,* *P/Y*

Same *PV, PMT*

80 **N**

**CPT** **FV**

*Ans*: 18,180.44

Future value, *FV* = $18,180.44

Future value, *FV* = $15,656.81

*c.* *n* = 4(25) = 100 *d.* *n* = 4(30) = 120

Same *I/Y,* *C/Y,* *P/Y*

Same *PV, PMT*

120 **N**

**CPT** **FV**

*Ans*: 24,513.57

Same *I/Y,* *C/Y,* *P/Y*

Same *PV, PMT*

100 **N**

**CPT** **FV**

*Ans*: 21,110.84

Future value, *FV* = $21,110.84

Future value, *FV* = $24,513.57

7. Given: *PV* = $10,000; Term = 25 years; *j* = 4.0% for all parts

*a*. *m* = 1; *n* = 1(25) = 25 *b.* *m* = 2; *n* = 2(25) = 50

Same *I/Y,* *PV,* *PMT*

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

50 **N**

**CPT** **FV**

*Ans*: 26,915.88

4 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

10000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 26,658.36

Future value, FV = $26,915.88

Future value, FV = $26,658.36

*c.* *m* = 4; *n* = 4(25) = 100 *d.* *m* = 12; *n* = 12(25) = 300

Same *I/Y,* *PV,* *PMT*

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

300 **N**

**CPT** **FV**

*Ans*: 27,137.65

Same *I/Y,* *PV,* *PMT*

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

100 **N**

**CPT** **FV**

*Ans*: 27,048.14

Future value, FV = $27,048.14 Future value, FV = $27,137.65

9. Given: *PV* = $100; Term = 1 year for all parts

*a*. *j* = 3.0%; *m* = 12; *n* = 12(1) = 12 *b.* *j* = 3.1%; *m* = 4; *n* = 4(1) = 4

Same *PV,* *PMT*

3.1 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

4 **N**

**CPT** **FV**

*Ans*: 103.14

3 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

12 **N**

100 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 103.04

*c.* *j* = 3.2%; *m* = 2; *n* = 2(1) = 2 *d.* *j* = 3.3%; *m* = 1; *n* = 1(1) = 1

Same *PV,* *PMT*

3.2 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

2 **N**

**CPT** **FV**

*Ans*: 103.23

Same *PV,* *PMT*

3.3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

1 **N**

**CPT** **FV**

*Ans*: 103.30

The investor would prefer 3.3% compounded annually because it

produces the highest maturity value.

11. Given: *PV* = $12,000; Term = 18 months = 1.5 years;

7.2 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

6 **N**

12000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: -13,355.74

*j* = 7.2%; *m* = 4

*n* = *m*(Term) = 4(1.5) = 6 compounding periods

Maturity value, *FV* = $13,355.74

Interest charged = *FV* – *PV*

= $13,355.74 – $12,000

= $1355.74

13. Given: *PV* = $10,000; Term = 25 years

Maturity value at *j* = 5%, *m* = 1: Maturity value at j = 4%, *m* = 1:

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

10000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 33,863.55

Same *P/Y, C/Y*

Same *N,* *PV, PMT*,

4 **I/Y**

**CPT** **FV**

*Ans*: 26,658.36

The difference is $33,863.55 − $26,658.36 = $7205.19

which is ×100% = 27.03% of the maturity

value at 4% compounded annually.

15. Given: *PV* = $10,000; *j* = 5%, *m* = 1

Maturity value after 25 years: Maturity value after 20 years:

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

10000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 33,863.55

Same *I/Y,* *P/Y, C/Y*

Same *PV*, *PMT*

20 **N**

**CPT** **FV**

*Ans*: 26,532.98

The difference is $33,863.55 − $26,532.98 = $7330.57

which is ×100% = 27.63% of the amount

after 20 years.

8 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

1000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 4660.96

17. The solution for the case *PV* = $1000, *j* = 8%, *m* = 1,

and *n* = 1(20) = 20 is presented in the box to the right.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Interest rate* | *20 years* | *25 years* | *30 years* |
|  | 8% | $4660.96 | $6848.48 | $10,062.66 |
|  | 10% | $6727.50 | $10,834.71 | $17,449.40 |

3.25 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

4 **N**

5000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 5682.38

19. Given: *PV* = $5000; *j* = 3.25%; *m* = 1

Term = 1.5 + 2.5 = 4 years

*n* = 1(4) = 4 compounding periods

Equivalent payment, *FV* *=*  $5682.38

21. Given: *PV* = $10,000; *j* = 5.5%; *m* = 12

5.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

48 **N**

10000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 12,454.51

Term = 4 years

*n* = 12(4) = 48 compounding periods

Equivalent amount, *FV* *=* $12,454.51

23. For both payments, *j* = 2%; *m* = 4

For the first payment: For the second payment:

*PV* = $1300, Term = 4 years *PV* = $1800, Term = 4 − 1.75 =2.25 years

*n* = 4(4) = 16 *n* = 4(2.25) = 9

Same *I/Y,* *P/Y, C/Y, PMT*

9 **N**

1800 **+ / –** **PV**

**CPT** **FV**

*Ans*: 1882.64

2 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

16 **N**

1300 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 1407.99

The single replacement payment 4 years from now is

$1407.99 + $1882.64 = $3290.63

25. The amount owed will be the combined future value.

For each amount borrowed, *j* = 5%, *m* = 2

*PV* = $3000; *n* = 3(2) = 6: *PV* = $3500; *n* = 4: *PV* = $4000; *n* = 2:

5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

6 **N**

3000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –3479.08

Same *I/Y,* *P/Y, C/Y, PMT*

2 **N**

4000 **PV**

**CPT** **FV**

*Ans*: –4202.50

Same *I/Y,* *P/Y, C/Y, PMT*

4 **N**

3500 **PV**

**CPT** **FV**

*Ans*: –3863.35

Total amount owed = $3479.08 + $3863.35 + $4202.50

= $11,544.93

27. Given: *PV* = $5000, *j* = 8%, and *m* = 12 in each case.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Grand-*  *child* | *Age* | *Time until*  *19th birthday* | *n* |
|  | Donna | 12 yr, 7 mo | 6 yr, 5 mo | 6(12) + 5 = 77 |
|  | Tim | 10 yr, 3 mo | 8 yr, 9 mo | 8(12) + 9 = 105 |
|  | Gary | 7 yr, 11 mo | 11 yr, 1 mo | 11(12) + 1 = 133 |

Donna: Tim: Gary:

Same *I/Y,* *P/Y, C/Y*

Same *PV, PMT*

133 **N**

**CPT** **FV**

*Ans*: 12,099.48

Same *I/Y,* *P/Y, C/Y*

Same *PV, PMT*

105 **N**

**CPT** **FV**

*Ans*: 10,045.40

8 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

77 **N**

5000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 8340.04

Donna will receive $8340.04, Tim will receive $10,045.40, and Gary will receive $12,099.48.

29. Maturity value of current GIC *a.* Maturity value of second GIC:

having *j* = 6%, *m* = 2, *n* = 2(5) = 10 having *j* = 6%, *m* = 2, *n* = 2(5) = 10

Same *I/Y,* *P/Y, C/Y*

Same *N, PMT*

80634.98 **+ / –** **PV**

**CPT** **FV**

*Ans*: 108,366.67

6 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

10 **N**

60000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 80,634.98

*b.* Maturity value of second GIC: *c.* Maturity value of second GIC:

having *j* = 7%, *m* = 2, *n* = 2(5) = 10 having *j* = 5%, *m* = 2, *n* = 2(5) = 10

Same *P/Y, C/Y*

Same *N, PV, PMT*

5 **I/Y**

**CPT** **FV**

*Ans*: 103,219.59

Same *P/Y, C/Y*

Same *N, PV, PMT*

7 **I/Y**

**CPT** **FV**

*Ans*: 113,743.60

31. For the first 15 months, *j* = 9.5% For the next 6 months, *j* = 8.5%

*m* = 4, *PV* = $7000; *n* = = 5 *m* = 2, *PV* = $7871.68, *n* = 1

8.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 1)

1 **N**

7871.68 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –8206.23

9.5 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

5 **N**

7000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –7871.68

Total amount owed = $8206.23

33. The solution requires three steps:

*Step 1*: Calculate the amount owed after 2.5 years;

*Step 2*: Calculate the amount owed after an additional 6 months;

*Step 3*: Calculate the amount owed after an additional 2 years.

*Step 1*: *Step 2*: *Step 3*:

*PV* = $3000, *j* = 5%, *PV* = $2394.22, *j* = 5%, *PV* = $2454.08, *j* = 6%,

*m* = 2, *n* = 2(2.5) = 5 *m* = 2, *n* = 2(0.5) = 1 *m* = 12, *n* = 12(2) = 24

Same *I/Y,* *P/Y, C/Y, PMT*

1 **N**

2394.22 **PV**

**CPT** **FV**

*Ans*: –2454.08

Same *PMT*

6 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

24 **N**

2454.08 **PV**

**CPT** **FV**

*Ans*: –2766.14

5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

5 **N**

3000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –3394.22

Total amount now owed = $2766.14

35. The solution requires four steps:

*Step 1*: Calculate the amount owed after 9 months;

*Step 2*: Calculate the amount owed after an additional 3 months;

*Step 3*: Calculate the amount owed after an additional 6 months;

*Step 4*: Calculate the amount owed after an additional 6 months.

*Step 1*: *Step 2*:

*PV* = $10,000, *j* = 6.6%, *PV* = $10,506.03 − $2500 = $8006.03

*m* = 12, *n* = 9 *n* = 3

6.6 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

9 **N**

10000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –10,506.03

Same *I/Y,* *P/Y, C/Y, PMT*,

3 **N**

8006.03 **PV**

**CPT** **FV**

*Ans*: –8138.86

*Step 3*: *Step 4*:

*PV* = $8138.86, *j* = 7%, *m*=4, *n*=2 *PV* = $8426.21 − $3000 = $5426.21

Same *I/Y,* *P/Y, C/Y*

Same *N,* *PMT*

5426.21 **PV**

**CPT** **FV**

*Ans*: –5617.79

Same *PMT*

7 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

2 **N**

8138.86 **PV**

**CPT** **FV**

*Ans*: –8426.21

The amount owed after two years was $5617.79.

**Exercise 9.4**

**Financial Calculator Solutions to Odd-Numbered Problems in Exercise 9.3**

1. Given: *j* = 2.5%; *m* = 1; *FV* = $10,000; *i* =  =  = 2.5% (per year)

*a.* *For a 10-year term*: *b. For a 20-year term*: *c. For a 30-year term*:

*n* = 1(10) = 10 *n* = 1(20) = 20 *n* = 1(30) = 30

Same *I/Y,* *P/Y, C/Y*

Same *PMT, FV*

20 **N**

**CPT** **PV**

*Ans*: -6102.71

2.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

10000 **FV**

0 **PMT**

**CPT** **PV**

*Ans*: -7811.98

Same *I/Y,* *P/Y, C/Y*

Same *PMT, FV*

30 **N**

**CPT** **PV**

*Ans*: -4767.43

The initial investments must be:

(*a*) $7811.98 (*b*) $6102.71 (*c*) $4767.43

3. Given: *FV* = $10,000; Term = 25 years; *j* = 4%

*a*. *m* = 1; *n* = 1(25) = 25 *b.* *m* = 2; *n* = 2(25) = 50

Same *I/Y,* *FV,* *PMT*

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

50 **N**

**CPT** **PV**

*Ans*: -3715.28

4 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

10000 **FV**

0 **PMT**

**CPT** **PV**

*Ans*: -3751.17

*c.* *m* = 4; *n* = 4(25) = 100 *d.* *m* = 12; *n* = 12(25) = 300

Same *I/Y,* *FV,* *PMT*

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

300 **N**

**CPT** **PV**

*Ans*: -3684.92

Same *I/Y,* *FV,* *PMT*

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

100 **N**

**CPT** **PV**

*Ans*: -3697.11

4.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

0 **PMT**

10000 **FV**

**CPT** **PV**

*Ans*: –6439.28

5. Given: *FV* = $10,000; *j* = 4.5%; *m* = 1

*n* = *m*(Term in years)

= 1(10)

= 10 compounding periods

Present value = *PV* = $6439.28

7. Given: *FV* = $9704.61; *j* = 3.5%; *m* = 2

3.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

7 **N**

0 **PMT**

9704.61 **FV**

**CPT** **PV**

*Ans*: –8594.83

*n* = *m*(Term in years)

= 2

= 7 compounding periods

Original investment = *PV* = $8594.83

5.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

4 **N**

0 **PMT**

10000 **FV**

**CPT** **PV**

*Ans*: –8072.17

9. Given: *FV* = $10,000; *j* = 5.5%; *m* = 1

*n* = *m*(Term in years)

= 1(4)

= 4 compounding periods

Investment today = *PV* = $8072.17

11. Given: *FV* = $5000; *j* = 5.4%; *m* = 2

5.4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

4 **N**

0 **PMT**

5000 **FV**

**CPT** **PV**

*Ans*: –4494.57

*n* = *m*(Term in years)

= 2(2)

= 4 compounding periods

Equivalent amount today = *PV* = $4494.57

13. Given: *FV* = $7000; *j* = 6.2%; *m* = 2

6.2 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

13 **N**

0 **PMT**

7000 **FV**

**CPT** **PV**

*Ans*: –4706.90

There is a 6.5 year interval between the focal date

1.5 years from now and the scheduled payment date.

*n* = *m*(Interval in years)

= 2(6.5)

= 13 compounding periods

Equivalent amount 1.5 years from now

= *PV* = $4706.90

15. The economically equivalent amount is the present value of $2600 at a time that is

5.4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

33 **N**

0 **PMT**

2600 **FV**

**CPT** **PV**

*Ans*: –2241.95

1.5 years + 15 months = 33 months earlier

Given: *FV* = $2600; *j* = 5.4%; *m* = 12

Then *n* *=* 33 compounding periods

The equivalent amount 15 months ago is $2241.95.

17. The equivalent amount is the sum of the equivalent values of

the scheduled payments at a date three years from now.

*First payment*: *Second payment*:

*PV* = $1400, *j* = 3%, *FV* = $1800, *j* = 3%,

*m* = 4, *n* = 4(3) = 12 *m* = 4, *n* = 4(2) = 8

3 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

12 **N**

1400 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –1531.330

Same *I/Y,* *P/Y, C/Y, PMT*

8 **N**

1800 **FV**

**CPT** **PV**

*Ans*: –1695.556

Sum of the equivalent values of the payments, 3 years from now,

= $1531.330+ $1695.556

= $3226.89

19. The payee should be willing to accept the sum of the equivalent values

of the scheduled payments at a date one year from now.

*First payment*: *Second payment*:

*FV* = $1000, *j* = 2.75%, *FV* = $2000, *j* = 2.75%,

*m* = 2, *n* = 2(2.5) = 5 *m* = 2, *n* = 2(4.5) = 9

2.75 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

5 **N**

1000 **FV**

0 **PMT**

**CPT** **PV**

*Ans*: –933.997

Same *I/Y,* *P/Y, C/Y, PMT*

9 **N**

2000 **FV**

**CPT** **PV**

*Ans*: –1768.692

One year from now, the payee should be willing to accept

= $933.997 + $1768.692

= $2702.69

21. The single equivalent payment is the sum of the equivalent values of

the scheduled payments at a date six months from now.

*First payment*: *Second payment*:

*PV* = $500, *j* = 2.5%, *FV* = $800, *j* = 2.5%,

*m* = 12, *n* = 4 + 6 = 10 *m* = 12, *n* = 12 – 6 = 6

2.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

10 **N**

500 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –510.515

Same *I/Y,* *P/Y, C/Y, PMT*

6 **N**

800 **FV**

**CPT** **PV**

*Ans*: –790.073

Sum of the equivalent values of the payments, 6 months from now,

= $510.515 + $790.073

= $1300.59

23. The single economically equivalent payment is the sum of the equivalent

values of the three scheduled payments at a date six months from now.

*First payment*: *Second payment*: *Third payment*:

*PV* = $2100, *j* = 4.5%, *PV* = $1300, *j* = 4.5%, *FV* = $800, *j* = 4.5%,

*m* = 12, *n* = 12(2) = 24 *m* = 12, *n* = 12(0.5) = 6 *m* = 12, *n* = 12(1.5) = 18

4.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

24 **N**

2100 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –2297.379

Same *I/Y,* *P/Y, C/Y, PMT*

6 **N**

1300 **PV**

**CPT** **FV**

*Ans*: –1329.526

Same *I/Y,* *P/Y, C/Y, PMT*

18 **N**

800 **FV**

**CPT** **PV**

*Ans*: –747.877

The single equivalent payment is

$2297.379 + $1329.526 + $747.877 = $4374.78

25. The economic value today of an offer is the present value of the payments.

Given: *j* = 4.4%; *m* = 4

*PV* of payment due in 6 months *PV* of payment due in 12 months

Same *I/Y,* *P/Y, C/Y*

Same *PMT,* *FV*

4 **N**

**CPT** **PV**

*Ans*: –46,902.01

4.4 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

2 **N**

0 **PMT**

49000 **FV**

**CPT** **PV**

*Ans*: –47,939.53

The economic value of the second offer is

$49,000 + $47,939.53 + $46,902.01 = $143,841.54

The $145,000-cash offer is worth $1158.46 more

27. The economic value of Pete’s contract is the sum of the present values of the payments.

Given: *j* = 5%; *m* = 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time: | 0 yr | 0.5 yr | 1.5 yr | 2.5 yr | 3.5 yr |
| Salary: | $4.8 M | $10 M | $17.2 M | $17.5 M | $18.5 M |
| *n* = | 0 | 1 | 3 | 5 | 7 |
| *PV* = | $4,800,000 | $9,756,098 | $15,971,910 | $15,467,450 | $15,563,407 |

5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

3 **N**

0 **PMT**

17200000 **FV**

**CPT** **PV**

*Ans*: –15,971,910

Sample calculation: *PV* of third payment is calculated in the box at the right.

Rounded to the nearest $1000, the sum of the present values is $61,559,000.

29. The data for the three certificates are presented in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Maturity*  *Value* | *Term* | *j* | *m* | *n* |
|  | $4000 | 3.5 years | 5% | 2 | 2(3.5) = 7 |
|  | $5000 | 4.5 years | 5% | 2 | 2(4.5) = 9 |
|  | $6000 | 5.5 years | 5.6% | 4 | 4(5.5) = 22 |

In each type of certificate, Michelle should invest a principal amount equal to the present value of the maturity value.

*First certificate*: *Second certificate*: *Third certificate*:

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

9 **N**

5000 **FV**

**CPT** **PV**

*Ans*: –4003.64

Same *PMT*

5.6 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

22 **N**

6000 **FV**

**CPT** **PV**

*Ans*: –4418.92

5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

7 **N**

0 **PMT**

4000 **FV**

**CPT** **PV**

*Ans*: –3365.06

Michelle should invest $3365.06 in a 3.5-year certificate, $4003.64 in a 4.5-year certificate, and $4418.92 in a 5.5-year certificate.

31. Total price = Sum of the present values of the three contracts.

*First contract*: *Second contract*: *Third contract*:

Same *I/Y,* *P/Y, C/Y, PMT*

5 **N**

1270 **FV**

**CPT** **PV**

*Ans*: –1186.177

16.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

4 **N**

0 **PMT**

950 **FV**

**CPT** **PV**

*Ans*: –899.498

Same *I/Y,* *P/Y, C/Y, PMT*

6 **N**

780 **FV**

**CPT** **PV**

*Ans*: –718.637

Total price = $899.498 + $718.637 + $1186.177

= $2804.31

33. Let *x* represent the final loan payment.

Loan = Present value of all payments

*PV* of f*irst payment*: *PV* of *second payment*: *PV* of *third payment*:

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

10 **N**

1 **+ / –** **FV**

**CPT** **PV**

*Ans*:

0.76239791

Same *I/Y,* *P/Y, C/Y*

Same *PMT*, *FV*

6 **N**

**CPT** **PV**

*Ans*: 3399.14

5.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

2 **N**

0 **PMT**

4000 **+ / –** **FV**

**CPT** **PV**

*Ans*: 3788.75

Hence,

$15,000 = $3788.75 + $3399.14 + 0.76239791*x*

*x* = $10,246.76

The third loan payment will be $10,246.76.

35. Let *x* represent the size of each loan payment.

*PV* of f*irst payment*: *PV* of *second payment*: *PV* of *third payment*:

Same *I/Y,* *P/Y, C/Y*

Same *PMT*, *FV*

14 **N**

**CPT** **PV**

*Ans*: 0.757875025

4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

5 **N**

0 **PMT**

1 **+ / –** **FV**

**CPT** **PV**

*Ans*: 0.90573081

Same *I/Y,* *P/Y, C/Y*

Same *PMT*, *FV*

8 **N**

**CPT** **PV**

*Ans*: 0.853490371

Hence,

$10,000 = 0.90573081*x* + 0.853490371*x* + 0.757875025*x*

= 2.517096206*x*

*x* = $3972.83

The amount of each loan payment is $3972.83.

37. Let *x* represent the size of the first payment.

$7500 = Present value of all three payments

*PV* of f*irst payment*: *PV* of *second payment*: *PV* of *third payment*:

Same *I/Y,* *P/Y, C/Y, PMT*

10 **N**

2 **+ / –** **FV**

**CPT** **PV**

*Ans*: 1.934539434

Same *I/Y,* *P/Y, C/Y, PMT*

15 **N**

4 **+ / –** **FV**

**CPT** **PV**

*Ans*: 3.805234085

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

5 **N**

0 **PMT**

1 **+ / –** **FV**

**CPT** **PV**

*Ans*: 0.983498712

Hence,

$7500 = 0.983498712*x* + 1.934539434*x* + 3.805234085*x*

= 6.723272231*x*

*x* = $1115.53

The second payment is 2*x* = $2231.06.

39. First calculate the scheduled payments:

*First contract*: *Second contract*: *Third contract*:

12 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

4 **N**

950 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 988.57

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

5 **N**

1270 **+ / –** **PV**

**CPT** **FV**

*Ans*: 1334.78

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

6 **N**

780 **+ / –** **PV**

**CPT** **FV**

*Ans*: 827.99

The total price to be paid for these scheduled payments is the sum of their present values.

*First contract*: *Second contract*: *Third contract*:

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

5 **N**

1334.78 **FV**

**CPT** **PV**

*Ans*: −1246.681

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

6 **N**

827.99 **FV**

**CPT** **PV**

*Ans*: −762.851

16.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y*= 12)

4 **N**

0 **PMT**

988.57 **FV**

**CPT** **PV**

*Ans*: −936.017

Total price = $936.017 + $762.851 + $1246.681

= $2945.55

41. First calculate how much interest $1 would earn if

8.2 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

17 **N**

1 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 1.9799873

invested for 8.5 years at 8.2% compounded semiannually.

The future value of $1 is $1.9799873. Therefore, each $1

earns $0.9799873 interest. The number of dollars that had

to be invested to earn total interest of $1175.98 was

 = $1200.00

# Exercise 9.5

1. Semiannual interest payment = *i*(*PV*) = × $18,000 = $378.00

3. *i* =  = 1.125%; *n* = 2(7) = 14; *PV* = $30,000

Maturity value = = $30,000= $35,086.56

5. Suppose that $1000 is invested for 3 years at each rate.

4.8 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

36 **N**

1000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 1154.55

The maturity value at 4.8% compounded monthly is



= $1000

= $1154.55

The maturity value at 4.9% compounded semiannually is

Same *PV, PMT*

4.9 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

6 **N**

**CPT** **FV**

*Ans*: 1156.30



= $1000

= $1156.30

An investor should choose 4.9% compounded semiannually since it will produce the larger maturity value.

7. The future value will not differ since

FV = $1000(1 +0.02)(1+0.03)(1+0.04) = $1000(1+0.04)(1+0.03)(1+0.02) = $1092.62

Difference = $0.00

9. Value of the $1000 investment in the 2%, 3% 4% GIC is

*FV* = *PV*(1+) = $1000(1.02) = $1020

Interest earned in Year 2 = 0.03 x $1020 = $30.60

Value of the $1000 investment in the 4%, 3%, 2% GIC is

*FV* = *PV*(1+) = $1000(1.04) = $1040

Interest earned in Year 2 = 0.03 x $1040 = $31.20

11. Given: *PV* = $2000; *j* = 2.1%; *m* = 1; term = 5 years

Then *i* =  = 2.1% and *n* = 1(5) = 5

and  *=* $2000 = $2219.01

13. Given: *PV* = $8000; term = 5 years

*i*1 = 2%; *i*2 = 2.5%; *i*3 = 3%; *i*4 = 3.5%; *i*5 = 5%;

Maturity value = *PV* 

= $8000(1.02)(1.025)(1.03)(1.035)(1.05)

= $9362.26

15. Maturity value of the compound-interest RateOptimizer GIC

= *PV* 

= $5000(1.018)(1.0225)(1.026)(1.03)(1.0325)

= $5678.79

Interest earned on RateOptimizer GIC = $5678.79 – $5000 = $678.79

Maturity value of fixed-rate compound-interest GIC

= *PV*= $5000= $5726.37

Interest earned on fixed-rate GIC = $5726.37 – $5000.00 = $726.37

17. Interest earned in Year 4 = (Value at end of Year 3)

= ()*PV*(1+)(1+)(1+)

Year 4 interest on Escalating Rate GIC = 0.0325($10,000)(1.02)(1.025)(1.03) = $349.98

Year 4 interest on fixed‑rate GIC = 0.0275($10,000) = $298.32

19. After 20 years, the cost of goods that cost $100 today will be the future value

of $100 (compounded at the rate of inflation.)

*a. j* = 2%, *m* = 1: *b. j* = 3%, *m* = 1: *c. j* = 4%, *m* = 1:

  

*=* $100 = $100 = $100

= $148.59 = $180.61 = $219.11

2 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

100 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 148.59

Same *P/Y, C/Y*

Same *N,* *PV, PMT*

4 **I/Y**

**CPT** **FV**

*Ans*: 219.11

Same *P/Y, C/Y*

Same *N,* *PV, PMT*

3 **I/Y**

**CPT** **FV**

*Ans*: 180.61

21. To keep pace with inflation, the hourly rate should also

3.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

15 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 21.16

grow at *j* = 3.5% compounded annually.

Ten years from now, the hourly rate would then be



= $15

= $21.16 per hour

23. The retirement income goal is $35,000 adjusted for 15 years’

nominal growth at the projected annual rate of inflation.

*a. j* = 2%, *m* = 1: *b. j* = 3%, *m* = 1: *c. j* = 5%, *m* = 1:

  

*=* $35,000 = $35,000 = $35,000

= $47,105.39 = $54,528.86 = $72,762.49

2 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

15 **N**

35000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 47,105.39

Same *P/Y, C/Y*

Same *N,* *PV, PMT*

5 **I/Y**

**CPT** **FV**

*Ans*: 72,762.49

Same *P/Y, C/Y*

Same *N,* *PV, PMT*

3 **I/Y**

**CPT** **FV**

*Ans*: 54,528.86

25. Price = Present value of the face value discounted at the market rate of return

2.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

44 **N**

0 **PMT**

1000 **FV**

**CPT** **PV**

*Ans*: −578.92

= 

= $1000

= $578.92

27. *a*. With 11½ years until maturity, *b*. With 7½ years until maturity,

*n* = 2(11½) = 23; 1.086% *n* = 2(7½) = 15

*PV* =  *PV* = 

=$10,000 = $10,000

= $7800.22 = $8504.23

Same *I/Y,* *P/Y, C/Y*

Same *PV,* *PMT*

15 **N**

**CPT** **PV**

*Ans*: –8504.23

2.172 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

23 **N**

0 **PMT**

10000 **FV**

**CPT** **PV**

*Ans*: –7800.22

Same *I/Y,* *P/Y, C/Y, PMT*

8 **N**

7800.22 **+ / –** **PV**

**CPT** **FV**

*Ans*: 8504.23

*c.* With *PV* = $7800.22,

*n* = 2(4) = 8, *i* = 1.086%,



= $7800.22

= $8504.23

*d.* The strip bond’s market value on any date impounds a rate of return equal to 2.172% compounded semiannually (csa) to the investor. The investor can earn this either in the bond or in another investment earning 2.172% csa.

29. Market price of one strip bond = *FV*

5.25 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

26 **N**

0 **PMT**

1000 **FV**

**CPT** **PV**

*Ans*: –509.82

= $1000

= $509.82

The number of strip bonds that may be purchased

with $12,830 is the integer portion of

 = 25.17

That is, 25 $1000 face value strip bonds can be purchased.

31. The maturity value at 5.00% compounded annually will be

 = $10,000= $12,762.82

The maturity value at 4.875% compounded semiannually will be

$10,000= $12,723.01

The maturity value at 4.75% compounded monthly will be

$10,000= $12,674.81

An investor will earn

$12,762.82 – $12,674.81 = $88.01

more at 5.00% compounded annually than at 4.75% compounded monthly.

33. Sales for Year 3 =  = $28,600,000 = $25,303,450

Sales for Year 7 = $25,303,450 = $34,425,064

35. Proceeds = Present value of the maturity value

For the maturity value calculation,

13.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

48 **N**

8000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –13,686.73

*PV* = $8000, *j* = 13.5%, *m* = 12, and

*i* =  = 1.125%, *n* = 12(4) = 48



= $8000

= $13,686.73

For the present value (discounting) calculation,

*FV* = $13,686.73, *j* = 12%, *m* = 4,

Same *PMT*, *FV*

12 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

9 **N**

**CPT** **PV**

*Ans*: 10,489.74

*i* =  = 3%, and *n* = 4 = 9

Proceeds = 

= $13,686.73

= $10,489.74

37. The appropriate price to pay is the present value, on the date of purchase,

of the scheduled payments.

Step 1: Calculate the balance after the $2000 payment.

Step 2: Calculate the second payment.

Step 3: Calculate the present value (on the date of purchase) of the two payments.

*Step 1*: *Step 2*:

*PV* = $4000, *j* = 9%, *m* = 4, *PV* = $2779.32;

= 2.25%, *n* = 4(2) = 8 same *j, m,* and *i* ; *n* = 4(1) = 4

*FV* = $4000 = $4779.32 *FV* = $2779.32= $3038.03

9 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

8 **N**

4000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –4779.32

Same *I/Y,* *P/Y, C/Y, PMT*

4 **N**

2779.32 **PV**

**CPT** **FV**

*Ans*: –3038.03

Balance after the $2000 payment

= $4779.32 − $2000 = $2779.32

*Step 3:* Calculate *PV*s discounting at *j* = 10%, *m* = 2, = 5%,

*PV of* *first payment*: *PV of* *second payment*:

*FV* = $2000.00; *n* = 2(1.5) = 3 *FV* = $3038.03; *n* = 2(2.5) = 5

*PV* = $2000= $1727.675 *PV* = $3038.03= $2380.376

Same *I/Y,* *P/Y, C/Y, PMT*

5 **N**

3038.03 **FV**

**CPT** **PV**

*Ans*: −2380.376

10 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

3 **N**

0 **PMT**

2000 **FV**

**CPT** **PV**

*Ans*: −1727.675

The appropriate price is the sum of the present values, that is,

$1727.675 + $2380.376 = $4108.05

# Exercise 9.6

1. Let *x* represent the replacement payment due in 24 months. The equivalent value

of the scheduled payments at a focal date 24 months from today is

$3000+ $2000 = $3184.797 + $2045.338 = $5230.135

Same *I/Y,* *P/Y, C/Y, PMT*

3 **N**

2000 **PV**

**CPT** **FV**

*Ans*: **−**2045.338

3 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

8 **N**

3000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –3184.797

Same *I/Y,* *P/Y, C/Y*

Same *N, PMT*

1500 **PV**

**CPT** **FV**

*Ans*: −1534.004

The equivalent value of the replacement payments on the same date is

$1500 + *x* = $1534.004 + *x*

For equivalence of the two payment streams,

$1534.004 + *x* = $5230.135

*x* = $3696.13

The second payment due in 24 months is $3696.13.

3. Let *x* represent the amount of the third replacement payment. Calculate *x* so that the

future values of the two payment streams at a focal date 7 months from now are equal.

FV of original stream

= $1000 + $1500 = $1068.637 + $1569.702 = $2638.339

Same *I/Y,* *P/Y, C/Y, PMT*

13 **N**

1500 **PV**

**CPT** **FV**

*Ans*: –1569.702

4.2 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

19 **N**

1000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –1068.637

FV of replacement stream

= $800 + $900 + *x* = $816.948 + $909.483 + *x* = $1726.431 + *x*

Same *I/Y,* *P/Y, C/Y, PMT*

3 **N**

900 **PV**

**CPT** **FV**

*Ans*: –909.483

Same *I/Y,* *P/Y, C/Y, PMT*

6 **N**

800 **PV**

**CPT** **FV**

*Ans*: –816.948

Solve for *x* in

$1726.431 + *x* = $2638.339

*x* = $911.91

The third payment should be $911.91.

5. Let *x* represent the size of each replacement payment. For equivalence at a focal date

coinciding with the first replacement payment (3 months from now),

$850+ $1760 = *x* + *x*

$963.30 + $1834.96 = *x* + 0.9725772*x*

$2798.26 = 1.9725772*x*

*x* = $1418.58

Each replacement payment should be $1418.58.

7. Let *x* represent the size of each replacement payment.

For equivalence at a focal date 1 year from now,

$5000 + $10,000 = *x* + *x*  + *x* 

$5250.00 + $8227.025 = *x* + 0.90702948*x* + 0.82270248*x*

*x* = $4937.12

Each replacement payment should be $4937.12.

9. Let *x* represent the size of the first replacement payment.

For equivalence at a focal date 1 year from now,

$2000 + $2000 = *x* + 0.5*x* 

$2038.00 + $1926.11 = *x* + 0.46373862*x*

*x* = $2708.21

The replacement payments are $2708.21 due in 1 year and $1354.11 due in 3 years.

11. Furniture City should be willing to accept a cash price that equals the down payment plus

the discounted balance that would otherwise be received from the finance company.

Cash price = 0.25($1595) + 0.75($1595) = $1492.77

13. The winner should choose the alternative having

3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

5 **N**

0 **PMT**

10000 **FV**

**CPT** **PV**

*Ans*: −8626.09

the larger current economic value.

*a.* Economic value of (1) = $10,000 + $10,000

= $18,626.09

The economic value of Alternative (2) is

$6700 + $6700 + $6700 + $6700 = $21,765.38

Same *I/Y,* *P/Y, C/Y*

Same *PMT, FV*

15 **N**

**CPT** **PV**

*Ans*: −4300.475

Same *I/Y,* *P/Y, C/Y*

Same *PMT, FV*

10 **N**

**CPT** **PV**

*Ans*: −4985.429

Same *I/Y,* *P/Y, C/Y, PMT*

5 **N**

6700 **FV**

**CPT** **PV**

*Ans*: −5779.479

Alternative (2) should be chosen since it is worth $3139.29 more.

*b.* Economic value of (1) = $10,000 + $10,000 = $18,024.51

The economic value of alternative (2) is

$6700 + $6700 + $6700+ $6700 = $19,852.76

Alternative (2) should be chosen since it is worth $1828.25 more.

15. Let *x* represent the amount of the second replacement payment due 4 years

from today. For equivalence at a focal date 4 years from today,

$5000 + $7000 = 0.5*x* + *x*

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

5 **N**

0.5 **PV**

**CPT** **FV**

*Ans*: −0.54530828

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

2 **N**

7000 **FV**

**CPT** **PV**

*Ans*: −6761.28

3.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

2 **N**

5000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: −5176.53

Hence, $5176.53 + $6761.28 = 0.54530828*x* + *x*

*x* = $7725.20

The replacement payments are $3862.60 due in 1 years and $7725.20 due in 4 years.

17. Let *x* represent the size of the second replacement payment.

For equivalence at a focal date 9 months from today,

$8000 + $6000(1.016) = $4000 + *x* + 3*x*

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

3 **N**

3 **FV**

**CPT** **PV**

*Ans*: −2.8604880

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

3 **N**

4000 **PV**

**CPT** **FV**

*Ans*: −4195.09

6.4 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

8 **N**

8000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: −9083.22

Hence, $9083.22 + $6096.00 = $4195.09 + *x* + 2.8604880*x*

*x* = $2845.27

The last two replacement payments are $2845.27 due in 9 months and

$8535.81 due in 1 years.

19. The scheduled payments to Andrea are:

$2000 = $2844.20 in 1 year and $1000 = $1315.93 in 2 years.

9 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

8 **N**

2000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: −2844.20

Same *P/Y, C/Y, PMT*

8 **I/Y**

7 **N**

1000 **PV**

**CPT** **FV**

*Ans*: −1315.93

Let *x* represent the size of each replacement payment.

For equivalence at a focal date 1 year from now,

$2844.20 + $1315.93 = *x* + *x*

6 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

4 **N**

0 **PMT**

1315.93 **FV**

**CPT** **PV**

*Ans*: −1239.85

Same *I/Y,* *P/Y, C/Y, PMT*

8 **N**

1 **FV**

**CPT** **PV**

*Ans*: −0.88771112

Hence, $2844.20 + $1239.85 = *x* + 0. 88771112*x*

*x* = $2163.49

The two replacement payments should each be $2163.49.

**Review Problems**

1. Equivalent amount = *FV*

3 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

9 **N**

0 **PMT**

4800 **FV**

**CPT** **PV**

*Ans*: −4198.04

= $4800

= $4198.04

3. Suppose the current level of waste discharge is 100 units.

10 **+ / –** **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

5 **N**

100 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –59.05

The target level after 5 years is the future value of 100 units compounded at *j* = −10% compounded annually.

That is, *FV* = 100

= 100

= 100

= 59.05 units

The target level is 59.05% of the current level.

5.

The predicted population in the next 15 years is:



1.85 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

15 **N**

583,500 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: −768,168.26

= 583,500

= 768,168.26

The predicted population for 2031 is 768,000 people.

7. Original principal, *PV* = *FV* = $2297.78 = $2012.56

Interest portion = $2297.78 – $2012.56 = $285.22

9. *a.* Maturity value = *PV*

= $12,000

= $12,928.44

*b.* Interest earned in the second year

= Value after 2 years – Value after 1 year

= $12,000 – $12,000

= $12,549.14 – $12,241.20

= $307.94

11. *a*. Maturity value of Springboard GIC = *PV* 

= $10,000(1.0225)(1.03)(1.0375)(1.045)(1.065)

= $12,160.59

Maturity value of fixed-rate GIC = *PV* = $10,000= $11,876.86

*b.* Interest earned in the third year on the Springboard GIC

= ()*PV*(1+)(1+)

= 0.0375($10,000)(1.0225)(1.03)

= $394.94

Interest earned in the third year by the fixed-rate GIC

= *i*(*PV*) = 0.035($10,000) = $374.93

13. Today’s economic value of Offer 1 = $10,000 + $15,000+ $15,000

= $10,000 + $14,616.32 + $13,878.16

= $38,494.48

Today’s economic value of Offer 2 = $8000 + $17,500+ $17,500

= $8000 + $16,616.20 + $15,777.03

= $40,393.23

Donnelly should accept Offer 2. Its current economic value is almost $2000

more than that of Offer 1.

15. Amount owed after the $1200 payment 1 year ago

= $3000 – $1200

= $2687.34

Balance owed today = $2687.34 = $2980.82

17. The sum of the scheduled payments’ equivalent values on August 1 is

$1500 + $1500 = $3005.43

4.25 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

3 **N**

1500 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: −1515.994

Same *I/Y,* *P/Y, C/Y*

Same *PMT*

2 **N**

1500 **FV**

**CPT** **PV**

*Ans*: −1489.431

A single payment of $3005.43 six on August 1 is equivalent to the

two scheduled payments.

19. Amount owed on June 1 (following graduation)

= $3000 + $3500 + $4000(1.02)

= $3312.24 + $3714.23 + $4080.00

= $11,106.47

21. *a.* Portfolio value = *FV* of $1000

7.9 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

1000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 2139.02

= $1000

= $2139.02

*b.* Amount required on December 31, 2018 to have the same purchasing power as $1000 on December 31, 2008

= *PV* = $1000 = $1177.81

*c.* The percent increase in purchasing power of the

funds originally invested in the S&P/TSX portfolio is

×100% = 81.61%

23. *Payment due in 2 yr.: Payment due in 4 yr.:*

*FV* = $1500= $1757.49 *FV* = $2500= $3431.96

Same *I/Y,* *P/Y, C/Y, PMT*

16 **N**

2500 **+ / –** **PV**

**CPT** **FV**

*Ans*: 3431.96

8 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

8 **N**

1500 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 1757.49

The fair market value of the note, 18 months after the issue date, is the

present value on the date of sale of the scheduled payments. That is,

Price = $1757.49 + $3431.96 = $1669.82 + $2657.25 = $4327.07

Same *I/Y,* *P/Y, C/Y, PMT*

5 **N**

3431.96 **FV**

**CPT** **PV**

*Ans*: −2657.25

10.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

1 **N**

0 **PMT**

1757.49 **FV**

**CPT** **PV**

*Ans*: −1669.82

25. Let *x* represent the size of each loan payment.

$6500 = *x* + *x* + *x*

11.25 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

3 **N**

0 **PMT**

1 **FV**

**CPT** **PV**

*Ans*: −0.9723942

Same *I/Y,* *P/Y, C/Y*

Same *FV, PMT*

12 **N**

**CPT** **PV**

*Ans*: −0.8940658

Same *I/Y,* *P/Y, C/Y*

Same *FV, PMT*

6 **N**

**CPT** **PV**

*Ans*: −0.9455505

= 0.9723942*x* + 0.9455505*x* + 0.8940658*x*

$6500 = 2.8120105*x*

*x* = $2311.51

Each loan payment should be $2311.51.

27. *Current price: Price in 4 years: Price in 5 years:*

*PV* = *FV* *PV* = $1000 *PV* = $1000

= $1000 = $418.03 =$443.06

= $331.28

5.9 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

38 **N**

0 **PMT**

1000 **FV**

**CPT** **PV**

*Ans*: −331.28

Same *I/Y,* *P/Y, C/Y*

Same *PMT, FV*

28 **N**

**CPT** **PV**

*Ans*: −443.06

Same *I/Y,* *P/Y, C/Y*

Same *PMT, FV*

30 **N**

**CPT** **PV**

*Ans*: −418.03

Hence, the increase in value during the fifth year will be

$443.06 – $418.03 = $25.03

29. *a.* Accumulated value of the GIC after 4 years

= *PV*  = $1000 = $1169.86

Interest earned in fifth year = 0.04($1169.86) = $46.79

*b.* Amount required 5 years from now to have the same purchasing

power as $1000 today is

$1000 = $1114.95

Maturity value of GIC = $1169.86 + $46.79 = $1216.65

The percent increase in purchasing power of the funds invested in the GIC will be

×100% = 9.12%

31. Let *x* represent the original investment. Its maturity value was

*x* = *x* 

Hence, *x* – *x* = $618.55

1.09516119*x* – *x* = $618.55

*x* = $6500.02

The original investment was $6500.02.