10 Compound Interest: Further

Topics and Applications

**Concept Questions (Section 10.1)**

1. If *FV* < *PV*, the quantity is decreasing in size as time passes. Therefore, the rate of growth is negative. That is, the value for *i* is negative.

3. Since the time interval is the same for both cases, the relative size of the periodic rates of return is indicated by the overall *percent* increase rather than the overall *dollar* increase. In the case “$1 grew to $2”, the final value is twice the initial value (100% increase). In the case “$3 grew to $5”, the final value is 1.667 times the initial value (66.7% increase). Therefore, the periodic rate of return was higher in the “$1 grew to $2” scenario.

**Exercise 10.1**

1. Given: *PV* = $3400, *FV* = $4297.91, *n* = 3

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

3 **N**

3400  **PV**

0 **PMT**

4297.91 **+ / –**  **FV**

**CPT** **I/Y**

*Ans*: 8.125



= – 1

= 0.08125 = 8.125% per year

Nominal rate,= 1(8.125%)

= 8.13% compounded annually.

3. Given: *PV* = $1800, *FV* = $2299.16, *n* = 4(2.75) = 11



**P/Y** 4 **ENTER**

(making *C/Y* = *P/Y* = 4)

11 **N**

1800 **+ / –**  **PV**

0 **PMT**

2299.16  **FV**

**CPT** **I/Y**

*Ans*: 9.00

= – 1

= 0.0225

= 2.25% per quarter

Nominal rate of return = *mi* = 4(2.25%)

= 9.00% compounded quarterly.

5. Given: *PV* = $950, *FV* = $1165.79, *n* = 12 = 29

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

29 **N**

950 **+ / –**  **PV**

0 **PMT**

1165.79  **FV**

**CPT** **I/Y**

*Ans*: 8.50



= – 1

= 0.0070833 = 0.70833% per month

Nominal int. rate = *mi* = 12(0.70833%)

= 8.50% compounded monthly.

7. Given: *PV* = $4600, *FV* = $332,000, *n* = 100

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

100 **N**

4600 **+ / –**  **PV**

0 **PMT**

332000  **FV**

**CPT** **I/Y**

*Ans*: 4.37



=  – 1

= 0.0437

= 4.37%

The bequest earned= 4.37% compounded annually.

9. Given: *PV* = $2550/month, *FV* = $4475/month, *n* = 11

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

11 **N**

2550 **+ / –**  **PV**

0 **PMT**

4475  **FV**

**CPT** **I/Y**

*Ans*: 5.25



=  – 1

= 0.0525 = 5.25%

Ander’s salary has grown at the rate of

= 5.25% compounded annually.

11. Given: *PV* = $198,000, *FV* = $430,000, *n* = 1(15) = 15

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

15 **N**

198,000 **+ / –**  **PV**

0 **PMT**

430,000  **FV**

**CPT** **I/Y**

*Ans*: 5.306



=  – 1

= 0.053061

= 5.306%

The value of the home increased at the rate of = 5.31% compounded annually.

13. Given: *PV* = $7000, *FV* = $7867.34, *n* = 12(3) = 36

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

36 **N**

7000 **+ / –**  **PV**

0 **PMT**

7867.34  **FV**

**CPT** **I/Y**

*Ans*: 3.900



=  – 1

= 0.003250

= 0.3250%

The GIC earned = 12(0.3250%) = 3.90% compounded monthly.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 15. | Province | *PV* | *FV* | *n* | *i* | *j* |
|  | Alberta | 2,237,700 | 4,330,206 | 37 | 1.80% ➀ | 1.80% ca |
|  | British Columbia | 2,744,500 | 5,016,322 | 37 | 1.64% | 1.64% ca |
|  | Newfoundland and Labrador | 567,700 | 525,073 | 37 | –0.21% | –0.21% ca |
|  | Nova Scotia | 847,400 | 964,693 | 37 | 0.35% | 0.35% ca |
|  | Ontario | 8,625,100 | 14,411,424 | 37 | 1.40% | 1.40% ca |

➀  =  – 1 = 0.01800 = 1.80%

17. Use *PV* = $1, *FV* = $3, term = 15 years

*a.*  =  – 1 = 0.07599 = 7.60%

*j = mi* = 1(7.60%) = 7.60% compounded annually

*b.* *i* =  – 1 = 0.01848 = 1.848%

*j* = 4(1.848%) = 7.39% compounded quarterly

*c.* *i* =  – 1 = 0.006122 = 0.6122%

*j* = 12(0.6122%) = 7.35% compounded monthly

19. Given: *PV* = $3800, *FV* = $5000, *n* = 2(3) = 6

 =  – 1 = 0.04680 = 4.680%

Monty realized a rate of return of= 2(4.680%) = 9.36% compounded semiannually.

21. Given: *PV* = 97.2, *FV* = 210.6, *n* = 10

 = – 1 = 0.0804 = 8.04%

The average rate of inflation during the 1970’s was 8.04% compounded annually.

23. Given: *PV* = 93.3, *FV* = 113.5, *n* = 10

 = – 1 = 0.0198 = 1.98%

The annual rate of inflation during the 10 years (decade of the 1990s) was 1.98% compounded annually.

25. *a.* If tuition fees had risen at the rate of inflation,





The 2018/2019 tuition fees would be x $1464 = $2550.07

*b.* Given: *PV* = $1464, *FV* = $6838, *n* = 28

 =– 1 = 0.0566 = 5.66%

The annual rate of increase of tuition fees was

*j* = 1*i* = 1(5.66%) = 5.66% compounded annually.

*c.* Given: *PV* = 76.7, *FV* = 133.6, *n* = 28

*i* =– 1 = 0.02002 = 2.00%

The compound annual rate of inflation was

*j* = 1*i* = 1(2.00%) = 2.00% compounded annually.

27. Maturity value of the note =  = $5700 = $7943.83

Based on proceeds of $6620 paid 18 months (*n* = 18 monthly periods) before maturity,

 =– 1 = 0.010179 = 1.0179%

The buyer will realize a rate of return of *mi* = 12(1.0179%)

= 12.22% compounded monthly on the investment.

29. Maturity value of the note =  = $6000

The purchase price ($6854.12) is the present value, 21 months (*n* = (36 –15) = 21) before

maturity, of the maturity value.

 = – 1 = 0.0090 = 0.90%

The nominal discount rate was *mi* = 12(0.9%) = 10.80% compounded monthly.

31. To maintain purchasing power after

2.3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

5 **N**

5630 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 6307.93

5 years, $5630 had to grow to

*FV* = $5630 = $6307.93

The $5630 actually grew to $8485.

Same *P/Y, C/Y*

Same *N, PMT*

6307.93 **+ / –**  **PV**

8485  **FV**

**CPT** **I/Y**

*Ans*: 6.11

Real rate of return

= 

= 6.11% compounded

annually

33. Suppose the initial investment in the portfolio was $100.

Its value after 5 years would have been

*FV* = $100 = $100(1.20)(0.80)(1.0)(1.20)(0.80) = $92.16.

The annually compounded rate of return that would have produced the

same final value is

 =  – 1 = –0.0162 = –1.62% (per year)

The equivalent rate of return was –1.62% compounded annually.

35. Suppose the initial investment for each period was $100.

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

3 **N**

100 **+ / –**  **PV**

0 **PMT**

117.43  **FV**

**CPT** **I/Y**

*Ans*: 5.50

For the last 3 years,

*FV* = $100(1.1942)(1.0759)(0.914) = $117.43

 =  – 1 = 0.055= 5.50%

The 3-year equivalent rate of return was

*j = mi* = 5.50% compounded annually.

Same *P/Y, C/Y*

Same *PV,* *PMT*,

5 **N**

122.17  **FV**

**CPT** **I/Y**

*Ans*: 9.73

For the last 5 years,

*FV* = $117.43(0.9277)(1.1214) = $122.17

*i* =  – 1 = 0.0409 = 4.09%

Same *P/Y, C/Y*

Same *PV,* *PMT*,

10 **N**

220.51  **FV**

**CPT** **I/Y**

*Ans*: 8.23

The 5-year equivalent rate of return was

*j = mi* = 4.09% compounded annually.

For the entire 10 years,

*FV* = $122.17(1.2726)(1.1190)(0.9752)(1.1071)(1.1740) = $220.51

*i* =– 1 = 0.0823 = 8.23%

The 10-year equivalent rate of return was

*j = mi* = 8.23% compounded annually.

37. Suppose the initial investment for each period was $100.

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

3 **N**

100 **+ / –**  **PV**

0 **PMT**

115.57  **FV**

**CPT** **I/Y**

*Ans*: 4.94

For the last 3 years,

*FV* = $100(1.1176)(1.1093)(0.9322) = $115.57

 =  – 1 = 0.0494= 4.94%

The 3-year equivalent rate of return was

Same *P/Y, C/Y*

Same *PV,* *PMT*,

5 **N**

129.34  **FV**

**CPT** **I/Y**

*Ans*: 5.28

*j = mi* = 4.94% compounded annually.

For the last 5 years,

*FV* = $115.57(1.1354)(0.9857) = $129.34

*i* =– 1 = 0.0528 = 5.28%

The 5-year equivalent rate of return was

*j = mi* = 5.28% compounded annually.

Same *P/Y, C/Y*

Same *PV,* *PMT*,

10 **N**

208.79  **FV**

**CPT** **I/Y**

*Ans*: 7.64

For the entire 10 years,

*FV* = $129.34(1.1983)(1.0971)(0.9738)(1.0693)(1.1792)

= $208.79

*i* =– 1 = 0.0764 = 7.64%

The 10-year equivalent rate of return was

*j = mi* = 7.64% compounded annually.

**Concept Questions (Section 10.2)**

1. In the case of annual compounding, the value calculated for *n* will equal the number of years in the term of the loan or investment.

**Exercise 10.2**

6.3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

1100 **+ / –**  **PV**

0 **PMT**

4483.92  **FV**

**CPT** **N**

*Ans*: 23.00

1. Given: *PV* = $1100, *FV* = $4483.92, *j* = 6.3%, *m* = 1

 = 

= 

= 23.00 periods

Since the compounding period is 1 year,

the term of the investment was 23 years.

3. Given: *PV* = $5000, *FV* = $5789.73, *j* = 3.7%, *m* = 2

3.7 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

5000 **+ / –**  **PV**

0 **PMT**

5789.73  **FV**

**CPT** **N**

*Ans*: 8.00

 = 

= 

= 8.0 periods

Term of the GIC = 8(0.5 year)

= 4 years

5. Given: *PV* = $1450; *FV* = $1528.01; *j* = 4.5%, *m* = 12

4.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

1450 **+ / –**  **PV**

0 **PMT**

1528.01  **FV**

**CPT** **N**

*Ans*: 14.00

 =  = 14.00 periods

Marilyn made the equivalent payment 14 months after the scheduled date, that is, on May 1 of the following year.

7. Given: *PV* = $6000; *FV* = $10,968.25;

9 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

6000 **+ / –**  **PV**

0 **PMT**

10,968.25  **FV**

**CPT** **N**

*Ans*: 7.00

*j* = 9% compounded annually

 =  =  = 7.00

The client held the investment for 7.00 years.

9. *a.* Given: *PV* = 32,500; *FV* = 40,000; *j* = 3%; *m* = 1

3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

32500 **+ / –**  **PV**

0 **PMT**

40000  **FV**

**CPT** **N**

*Ans*: 7.0246

 =  = 7.0246 periods

Each compounding period is 1 year. The partial

period is 0.0246 x 12 months = 0.295 months.

Rounded to the nearest month, it will take 7 years

for the population to grow from 32,500 to 40,000.

*b.* Given: *PV* = 40,000; *FV* = 32,500; *j* = –3%; *m* = 1

 =  = 6.8170 periods

Same *P/Y, C/Y, PMT*

3 **+ / –** **I/Y**

40000 **+ / –**  **PV**

32500  **FV**

**CPT** **N**

*Ans*: 6.8170

The partial period is 0.8170(12 months) = 9.8 months.

Rounded to the nearest month, it will take

6 years and 10 months for the population to

decline from 40,000 to 32,500.

9 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

1 **+ / –**  **PV**

0 **PMT**

3  **FV**

**CPT** **N**

*Ans*: 12.748

11. *a.* Given: *PV* = 1; *FV* = 3; *j* = 9%; *m* = 1

 =  = 12.748 periods

Since each period is 1 year, the investment

will triple in 12 years and 9 months.

*b.* Given: *PV* = 1; *FV* = 3; *j* = 8%; *m* = 4

Same *PV,* *PMT*, *FV*

8 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

**CPT** **N**

*Ans*: 55.478

*n* =  = 55.478 periods

Each period is 3 months. The partial period is

0.478(3 months) = 1.43 months.

Rounded to the nearest month, the investment will triple in 13 years and 10 months.

13. Given: *PV* = $5000; *FV* = $10,000; *j* = 5%; *m* = 1

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

5000 **+ / –**  **PV**

0 **PMT**

10,000  **FV**

**CPT** **N**

*Ans*: 14.207

 =  =  = 14.207

Since each compounding period is 1 year,

14.21 periods = 14 years and 0.207(12 months)

= 14 years and 2.48 months

Rounded to the nearest month, $10,000 will be equivalent to $5000 paid 14 years and 2 months earlier.

15. Given: *PV* = $250,000; *FV* = $325,000;

7 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

250000 **+ / –**  **PV**

0 **PMT**

325000  **FV**

**CPT** **N**

*Ans*: 3.87776

*j* = 7% compounded annually

 =  =  = 3.87776

Each compounding period is one year.

0.87776 year = 0.87776 × 12 = 10.53 months

Therefore, the term is 3 years and 11 months.

17. If money loses half of its purchasing power, $100 (*FV*) at the end of

the period will buy as much as $50 (*PV*) at the beginning of the period.

*a.*  =  = 28.071 periods

Each period is 1 year. The partial period is 0.071(12 months) = 0.85 months. Rounded

to the nearest month, money will lose half its purchasing power in 28 years & 1 month.

*b.*  =  = 20.149 periods

Each period is 1 year. The partial period is 0.149(12 months) = 1.79 months. To the

nearest month, money will lose half its purchasing power in 20 years and 2 months.

19. Maturity value, *FV* = = $2600= $3677.33

12.25 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

3 **N**

2600 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –3677.33

Discounted price = $3283.57

The number of compounding periods between the

discount date and maturity is

Same *PMT*, *FV*

10.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

3283.57  **PV**

**CPT** **N**

*Ans*: 13.00



= 

= 13.0 periods (months)

Hence, the discounting took place 36 – 13 = 23 months after the issue date.

21. The number of months required for $4000 borrowed

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

4000  **PV**

0 **PMT**

5000 **+ / –**  **FV**

**CPT** **N**

*Ans*: 35.8144

at 7.5% compounded monthly to grow to $5000 is

=  = 35.8144 periods (mo.)

The partial period (month) is 0.8144(30 days) = 24.43 days

To the nearest day, the $5000 payment was made

35 months and 24 days, or 2 years, 11 months and 24 days after the initial loan.

23. $557.05 represents the present value of $1000

5.22 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

557.05 **+ / –**  **PV**

0 **PMT**

1000  **FV**

**CPT** **N**

*Ans*: 22.7089

on the purchase date. The number of half-years

between the purchase date and maturity date was

 =  = 22.7089

The partial period was

0.7089(182 days) = 129 days

Therefore, Wilf purchased the strip bond

11 years and 129 days before its maturity date.

25. The $9380.24 payout figure represents the present

9 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

16 **N**

7500 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –10,707.16

value on the prepayment date of the loan's maturity

value.

The maturity value was

*FV* == $7500= $10,707.16

The number of compounding periods (calendar quarters)

between prepayment and maturity was

 = 

Same *P/Y, C/Y, PMT*, *FV*

7 **I/Y**

9380.24 **PV**

**CPT** **N**

*Ans*: 7.62638

= 7.62638

The partial period (quarter) was

0.62638(91 days) = 57.0 days long

Hence, the loan was prepaid

1 year, 9 months and 57 days before maturity.

**Point of Interest (Section 10.3)**

***Not in Your Best Interest***

1. *i* =  = 0.016438356% per day; *m* = 365

 *=* 1.00016438356365 – 1 = 1.05127 – 1 = 0.06183 = 6.183%

**Concept Questions (Section 10.3)**

1. The effective rate of interest is the *equivalent annually* *compounded* rate.

3. Yes. The effective interest rate equals the nominal rate for annual compounding.

**Exercise 10.3**

1. For a financial calculator approach, we will use the calculator’s

financial functions to obtain the future value of $100 after 1 year.

*a.* *j* = 6%, *m* = 2 *b.* *j* = 6%, *m* = 4 *c.* *j* = 6%, *m* = 12

  

= – 1 = – 1 = – 1

= 0.0609 = 0.06136 = 0.06168

= 6.09% = 6.14% = 6.17%

Same *I/Y, PV, PMT*

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12

12 **N**

**CPT** **FV**

*Ans*: 106.17

Same *I/Y, PV, PMT*

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4

4 **N**

**CPT** **FV**

*Ans*: 106.14

6 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2

2 **N**

100 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 106.09

3. *a.* *j* = 9% semiannually *b.* *j* = 9% quarterly *c.* *j* = 9% monthly

  

=  – 1 =  – 1 =  – 1

= 0.0920 = 0.0931 = 0.0938

= 9.20% = 9.31% = 9.38%

**2nd** **ICONV**

9 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 9.203

4 **ENTER**

**↑**  **CPT**

*Ans*: 9.308

12 **ENTER**

**↑**  **CPT**

*Ans*: 9.381

5. *a. f = j =* 5% compounded annually

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2

2 **N**

100 **+ / –**  **PV**

0 **PMT**

105  **FV**

**CPT** **I/Y**

*Ans*: 4.939

*b.*  Using the financial

=  -1 functions, find the

= 0.02470 semiannually com-

= 2.470% pounded interest

rate at which $100

*j* = *mi* will grow to

= 2(2.470%) $105 after 1 year.

= 4.94% compounded semiannually

*c.*  = – 1 = 0.01227 = 1.227%

*j* = *mi* = 4(1.227%) = 4.91% compounded quarterly

*d.*  Using the ICONV worksheet

**2nd** **ICONV**

5 **ENTER**

12 **ENTER**

**CPT**

*Ans*: 4.89

=  – 1 to convert an effective rate

= 0.004074 to a nominal interest rate.

= 0.4074%

*j* = *mi*

= 12(0.4074%)

= 4.89% compounded monthly

7. *f = j =* 12% compounded annually

*j* = 11.9% semiannually *j* = 11.8% quarterly *j* = 11.7% monthly

  

=  – 1 =  – 1 =  – 1

= 0.12254 = 0.1233 = 0.1235

= 12.25% = 12.33% = 12.35%

The nominal rate of 11.7% compounded monthly has the highest effective rate.

9. For a loan, choose the rate with the lowest effective rate.

*j* = 9% monthly *c.* *j* = 9.1% quarterly *d.* *j* = 9.2% semiannually

  

=  – 1 =  – 1 =  – 1

= 0.09381 = 0.09415 = 0.09412

= 9.38% = 9.42% = 9.41%

**↑**  9.2 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 9.412

**↑**  9.1 **ENTER**

**↑**  4 **ENTER**

**↑**  **CPT**

*Ans*: 9.415

**2nd** **ICONV**

9 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 9.381

If *j* = 9.3% compounded annually, *f* = 9.3%

Choose the lowest effective rate--9.3% compounded annually

11. Since the interest is charged each month, we have monthly compounding.

The nominal annual rate charged is

*j* = *mi* = 12(0.62%) = 7.44% compounded monthly.

The effective interest rate charged is

 =  – 1 = 0.07699 = 7.70%

13. Because the 3% compounds every quarter, the annual growth

rate will exceed 4(3%) = 12%. The actual annual growth is

 =  – 1 = 0.1255 = 12.55%

15. The essential question is: “What annually compounded rate of interest would

produce the same maturity value after 2.25 years (27 months)?” This rate is

*f* *=*  = – 1 = 0.1121 = 11.21%

17. *f* (semiannual compounding) =  =  – 1 = 0.0465 = 4.65%

*f* (monthly compounding) =  =  – 1 = 0.0459 = 4.59%

Craig should choose the semiannually compounded GIC since it has

a slightly higher (by 0.06%) effective rate.

19. *f* (ABC) =  =  – 1 = 0.1677 = 16.77%

*f* (DEF) =  =  – 1 = 0.1699 = 16.99%

21. For monthly compounding,

 =  – 1 = 0.005654 = 0.5654%

*j* = 12*i* = 6.78% compounded monthly

Similarly, for semiannual compounding,

*i* =  – 1 = 3.441% and *j* = 2*i* = 6.88% compounded semiannually

and for annual compounding, *j* = *i* = *f* = 7.00% compounded annually

23. *f* (old) =  =  – 1 = 0.2242

*f* (new) = 0.2242 – 0.03 = 0.1942

*i* (new) = =  – 1 = 0.0149

The monthly periodic rate should be dropped to 1.49% per month.

**Exercise 10.4**

**Note:** We will use the abbreviations:

ca = compounded annually cq = compounded quarterly

csa = compounded semiannually cm = compounded monthly

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *Problem* |  |  |  |  |  |
| 1. *a*. | 10% | 1 | 2 | = 0.04881 | 2(0.04881) = 9.76% csa |
| *b*. | 10% | 1 | 4 | = 0.02411 | 4(0.02411) = 9.64% cq |
| *c*. | 10% | 1 | 12 | = 0.007974 | 12(0.007974) = 9.57% cm |
| 3. *a*. | 10% | 4 | 1 | = 0.1038 | 1(0.1038) = 10.38% ca |
| *b*. | 10% | 4 | 2 | = 0.050625 | 2(0.050625) = 10.13% csa |
| *c*. | 10% | 4 | 12 | = 0.008264 | 12(0.008264) = 9.92% cm |
| 5. *a*. | 6% | 2 | 1 | = 0.0609 | 1(0.0609) = 6.09% ca |
| *b.* | 6% | 4 | 1 | = 0.06136 | 1(0.06136) = 6.14% ca |
| *c.* | 6% | 12 | 1 | = 0.06168 | 1(0.06168) = 6.17% ca |
| 7. *a*. | 6% | 1 | 4 | = 0.01467 | 4(0.01467) = 5.87% cq |
| *b.* | 6% | 2 | 4 | = 0.01489 | 4(0.01489) = 5.96% cq |
| *c.* | 6% | 12 | 4 | = 0.015075 | 4(0.015075) = 6.03% cq |
| 9. | 4% | 12 | 2 | = 0.02017 | 2(0.02017) = 4.03% csa |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *Problem* |  |  |  |  |  |
| 11. | 6% | 4 | 12 | = 0.004975 | 12(0.004975) = 5.97% cm |
| 13. | 10.5% | 12 | 4 | = 0.02648 | 4(0.02648) = 10.59% cq |
| 15. | 2.5% | 2 | 1 | = 0.02516 | 1(0.02516) = 2.52% ca |
| 17. | 3.45% | 2 | 12 | = 0.002855 | 12(0.002855) = 3.43% cm |
| 19. | 2.0% | 2 | 12 | = 0.001660 | 12(0.001660) = 1.99% cm |
| 21. *a*. | 7.5% | 12 | 1 | = 0.07763 | 1(0.07763) = 7.76% ca |
| *b*. | 7.5% | 12 | 2 | = 0.03809 | 2(0.03809) = 7.62% csa |
| *c*. | 7.5% | 12 | 4 | = 0.01887 | 4(0.01887) = 7.55% cq |
| 23. | 9% | 52 | 4 | = 0.02273 | 4(0.02273) = 9.09% cq |

21. Use of ICONV worksheet to solve Problem 21.

Equivalent annually Equivalent semiannually Equivalent quarterly

compounded rate compounded rate compounded rate

**2nd** **ICONV**

7.5 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 7.763

1 **ENTER**

**CPT**

*Ans*: 7.763

**↑**  4 **ENTER**

**CPT**

*Ans*: 7.547

**↑**  2 **ENTER**

**CPT**

*Ans*: 7.618

# Review Problems

1. Given: *PV* = $85,000, *FV* = $215,000, *n* = 13

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

13 **N**

85000 **+ / –**  **PV**

0 **PMT**

215000  **FV**

**CPT** **I/Y**

*Ans*: 7.399

The compound annual rate of appreciation

of the home has been

*j =* 

= – 1

= 0.0740

= 7.40%

3. The preferred loan is the one with the lowest effective rate.

*j* = 7.6% cq *j* = 7.5% cm *j* = 7.7% csa

  

= – 1 = – 1 = – 1

= 0.07819 = 0.07763 = 0.07848

= 7.82% = 7.76% = 7.85%

**2nd** **ICONV**

7.6 **ENTER**

**↑**  4 **ENTER**

**↑**  **CPT**

*Ans*: 7.819

**↑**  7.7 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 7.848

**↑**  7.5 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 7.763

For a loan, we prefer the lowest effective rate, that is, 7.5% compounded monthly.

5. Given: *PV* = 24, *FV* = 400,000,000, *n* = 1926 – 1859 = 67 annual compounding periods

 = =0.28171 = 28.17%

7. *PV* = $165,000; *FV* = $485,000, *n* = 8, *m* = 1



**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1

8 **N**

165000 **+ / –**  **PV**

0 **PMT**

485000  **FV**

**CPT** **I/Y**

*Ans*: 14.43

= – 1

= 0.1443

= 14.43%

Sales grew at a compound annual rate of

*j = mi =* 1(14.43%) = 14.43% during the 8-year period.

9. The effective rate corresponding to 10.5% compounded monthly is



**2nd** **ICONV**

10.5 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 11.020

= – 1

= 0.1102

= 11.02%

The periodic 6-month rate having the same effective rate is the value of *i* in

0.1102 = 

2 **ENTER**

**CPT**

*Ans*: 10.732

 = 1.1102

*i* = – 1 = 0.05366 = 5.366%

The equivalent nominal rate is

*j* = 2*i* = 10.73% compounded semiannually

For any nominal interest rate below 10.73% compounded semiannually, you should choose semiannual compounding.

11. Given: *i* = 1.2%, *m* = 12, *j = mi* = 14.4%

= – 1

**2nd** **ICONV**

14.4 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 15.389

= 0.1539

= 15.39%

13. Given: *j* = 6.9%, *m* = 2

**2nd** **ICONV**

6.9 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 7.019



= – 1

= 0.0702

= 7.02%

15. *Bank* (*j* = 8.75% csa) *Broker* (*j* = 8.6% cm)

*f* (bank) =  – 1 *f* (broker) = – 1

= – 1 = 0.08947

= 0.08941 = 8.947%

= 8.941%

**↑**  8.6 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 8.947

**2nd** **ICONV**

8.75 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 8.941

Camille should take the bank mortgage since its effective interest rate is 0.006% lower.

17. Given: *j*= 4.625% compounded semiannually

Hence *i*1 =*j*/*m*1 = 4.625%/2 = 2.3125%

The equivalent periodic rate for 3 months (*m*2 = 4) is

 = – 1 = 0.01150

The corresponding nominal rate is

*j*2 = *m*2*i*2 = 4*i*2= 4(1.150%) = 4.60% compounded quarterly

4.60% compounded quarterly will produce the same maturity value as 4.625% compounded semiannually.

19. If money loses one-third of its purchasing power, you will need $150 (*FV*) at that point to purchase the same amount as $100 (*PV*) buys now. With the rate of inflation at

3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

100 **+ / –**  **PV**

0 **PMT**

150  **FV**

**CPT** **N**

*Ans*: 13.717

*j* = 3% compounded annually

 =  = 13.717 periods

Each period is 1 year. The partial period is

0.717(12 months) = 8.6 months.

Rounded to the nearest month, money will lose one-third

of its purchasing power in 13 years and 9 months.

21. Each $100 invested grew to $153 after 5 years. The purchasing power

of the $153 in terms of beginning-of-Year 1 dollars was

$153 × = $153 × = $137.51

The annually compounded real rate of return on the portfolio was

 = – 1 = 0.0658 = 6.58%

23. Given: *PV* = $20,822.89, *FV* = $50,000, *j* = 5.38%, *m*=2

 =  = 33.00 periods

Since each compounding period is 6 months, 16 years and 6 months

remain until maturity of the strip bond.

25. The annual rate of population decline during the 5-year period is

 = – 1 = –0.03691 = –3.691%

If the population continues to decline at the same annual rate, it will take

 =  =  = 6.164 periods

for the population to drop another 3000 to 11,500.

Each period is 1 year. The partial year is 0.164(12 months) = 1.968 months.

To the nearest month, it will take 6 years and 2 months for the additional decline.

27. Suppose the company began the 5‑year period with sales

10 **+ / –** **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

5 **N**

1000 **+ / –** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 590.49

of 1000 units per year. (The answer to the problem will not

depend on the initial number chosen because the given

information is in terms of percentage changes.)

After the first 5 years, annual sales will be

*FV* = *PV* = 1000 = 590.49 units per year

*a.* To return to 1000 units per year over the subsequent

5 years, the compound annual growth of sales must be



Same *N,* *PMT*, *P/Y, C/Y*

590.49 **+ / –**  **PV**

1000  **FV**

**CPT** **I/Y**

*Ans*: 11.111

= – 1

= 0.11111

= 11.11%

*b.* If sales grow at 10% per year, it will take

Same *P/Y, C/Y*

Same *PV, PMT, FV*

10 **I/Y**

**CPT** **N**

*Ans*: 5.527

 =  = 5.527 years

or 5 years and 6 months (to the nearest month)

to return to the original level of sales.

**Exercise 10C**

1. The simple annualized rate of inflation was

×0.5% = 3.00%

The effective annualized rate of inflation was

 = – 1 = 0.0304 = 3.04%

3. Increase in 1 month = ×100% = 0.81239%

Simple annualized rate = × 0.81239% = 9.75%

Effective annualized rate = – 1 = 0.10196 = 10.20%

5. The compound or effective increase in house prices for the year will be

 = – 1 = 0.0872 = 8.72%

7. Holding-period return = × 4.6% = 1.5753%

The corresponding effective annualized rate of return is

 = – 1 = 0.04670 = 4.67%

9. Percent change during 3 months = ×100% = –2.3328%

Simple annualized rate = (–2.3328%) = – 9.33%

Effective annualized rate,  = – 1 = – 9.01%

The shares in the mutual fund declined at a simple annual rate

of 9.33% and an effective annual rate of 9.01%.

11. Current (simple) yield = × Holding-period return

= × 0.097%

= 5.06%

Effective annualized yield,  = – 1 = 0.05185 = 5.19%

13. The return for the most recent 7 days was

4.54% × = 0.08707%

The corresponding effective (annualized) yield is

 = – 1 = 0.04643 = 4.64%

15. The discounter pays out $170 now and gets back the full $200 when

the tax refund is received. The holding-period return is

×100% = 17.647%

*a.* If the holding period is 25 days, the discounter’s effective annualized rate of return is

 = – 1 = 9.727 = 972.7%

*b.* It the holding period is 50 days,

*f* *=* – 1 = 2.275 = 227.5%