11 Ordinary Annuities:

Future Value and Present Value

**Concept Questions (Section 11.1)**

1. The two types of annuities are distinguished by comparing the payment interval to the compounding interval. If the payment interval *equals* the compounding interval, the annuity is a *simple* annuity. Otherwise, it is a general annuity.

3. No. Insurance premiums are paid at the beginning of the period of coverage. In the present case, the monthly payments will be made at the beginning of each month of coverage. To qualify as an ordinary annuity, the monthly payments would have to occur at the end of each month of coverage.

**Point of Interest** **(Section 11.2)**

***Your Potential to Become a Millionaire!***

1. *a.* The accumulated amount after 20 years will be

 =  = $280,512

*b.* The amount after 30 years will be = $768,912

*c.* The amount after 40 years will be = $1,966,155

3. If the inflation rate is 2.4% compounded monthly, you will need

 = $1,000,000(1.002)480 = $2,609,194

40 years from now to have the same purchasing power as $1,000,000 has today.

Therefore, the amount in 1(c) will have only a little more than half of the purchasing power of $1,000,000 today. You would/should not “feel” as wealthy as a person holding $1,000,000 today.

**Exercise 11.2**

1. Given: *j* = 4%; *m* = 1; term = 25 years

*i* =  = 4%; *n* = 1(25) = 25

 = *PMT* = 41.645908*PMT*

(*a*) *PMT* = $1000 (*b*) *PMT* = $2000 (*c*) *PMT* = $3000

*FV* = $41,645.91 *FV* = $83,291.82 *FV* = $124,937.72

Same *I/Y, P/Y, C/Y*

Same *N, PV*

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 83,291.82

Same *I/Y, P/Y, C/Y*

Same *N, PV*

3000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 124,937.72

4 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 41,645.91

3. *a.* Given: *n* = 20, *PMT* = $1000, *i* = 9%

9 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 51,160.12



= $1000

= $51,160.12

Similarly,

(b) *i* = 10% (c) *i* = 11% (d) *i* = 12%

Same PV, *P/Y, C/Y, N, PMT*

12 **I/Y**

**CPT** **FV**

*Ans*: 72,052.44

Same PV, *P/Y, C/Y, N, PMT*

11 **I/Y**

**CPT** **FV**

*Ans*: 64,202.83

Same PV, *P/Y, C/Y, N, PMT*

10 **I/Y**

**CPT** **FV**

*Ans*: 57,275.00

5. Given: *PMT* = $6000, *n* = 1(20) = 20, *i* =  = 4.5%

4.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

0 **PV**

6000 **+ / –** **PMT**

**CPT** **FV**

*Ans*:188,228.54



= $6000

= $188,228.54

7. Given: *PMT* = $2000, *n* = 2(12.5) = 25, *i* =  = 1.75%

3.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

25 **N**

0 **PV**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 62,054.92



= $2000

= $62,054.92

9. Given: *PMT* = $700, *n* = 4(7.75) = 31, *i* =  = 2.25%



9 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

31 **N**

0 **PV**

700 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 30,901.26

= $700

= $30,901.26

11. Given: *PMT* = $1000, *n* = 1(3) = 3, *i* =  = 4%

4 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

3 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 3121.60

 = $1000 = $3121.60

The future value after 3 years is $3121.60.

13. Given: *PMT* = $1000 every half year for 17 years.

6.4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

34 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 59,942.63

*n* = 2(17) = 34, *i* =  = 3.2%



= $1000

= $59,942.63

The client’s investments are worth $59,942.63 today.

15. *a.* Given: *PMT* = $500, *n* = 4(20) = 80, *i* = = 1.25%

 = $500 = $68,059.40

*b*. If, instead, *i* = = 2.5%

*FV* = $500 = $124,191.36

17. The amount 5 years from now will be the combined future value of the

$20,000 already saved and the annual $5000 contributions. That is,

 + 

= $20,000 + $5000

= $22,353.55 + $26,150.60

= $48,504.15

19. If contributions start this year, the amount

in the RRSP 30 years from now will be

8 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

30 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 113,283



= $1000

= $113,283

If contributions start in the sixth year and continue for 25 years, the RRSP will be worth

Same *I/Y,* *P/Y, C/Y, PV, PMT*

25 **N**

**CPT** **FV**

*Ans*: 73,106

*FV* = $1000 = $73,106

The RRSP will be worth $113,283 – $73,106 = $40,177 more

as a result of the first 5 years’ contributions.

21. For the first 2 years (2017 to 2018 inclusive), *PMT* = $5500,

*n* = 2(1) = 2, *i* =  = 8.0%.

8 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

2 **N**

0 **PV**

5500 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 11,440.00

The future value after 2 years is



= $5500

= $11,440.00

For the next 15 years, *n* = 15(1) = 15,

*PV* = $11,440.00, *PMT* = $6000, *i* =  = 8.0%

The combined future value of these amounts

at the end of the 15 years is

Same *P/Y, C/Y,I/Y*

15 **N**

11440 **+ / –** **PV**

6000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 199,202.30

 + 

= $6000 + $11,440.00

= $$162,912.683 + $36,289.615

= $199,202.30

23. After 10 years, Marika's RRSP will be worth

4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

20 **N**

18000 **+ / –** **PV**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 75,341.79

 + 

= $18,000 + $2000

= $26,747.05 + $48,594.74

= $75,341.79

After a further 5 years, the RRSP will be worth

Same *I/Y*

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

60 **N**

75341.79 **+ / –** **PV**

300 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 111,881.76

*FV* = $75,341.79 + $300

= $91,992.07 + $19,889.69

= $111,881.76

25. For the first 4 ½ years, PMT = $1200, n= 4(4.5) =18,



Future value of quarterly payments is

 = $1200 = $23,033.66

Future value of quarterly payments after 9 months is

FV = $23,033.66(1.0075)3=$23,555.82

Future value after another 10 years is

FV= $23,555.82(1.01125)40 + $1500

= $36,850.18 + $75,250.25

= $112,100.43

**Point of Interest** **(Section 11.3)**

***“Got a Million Dollar Talent?”***

1. The economic value is the present value, on the date of winning the prize, of all future payments. For PMT = $25,000, n = 40 and i = 5%

*PV* =

= $428,977.16

For i = 7%,

*PV* =  = $333,292.72

For i= 9%,

*PV* = 

= $268,934.00

3. The equivalent value today is the present value of $25,000 discounted at 3% for 40 years.

PV = $25,000(1.03)-40 = $7,663.92

**Concept Questions (Section 11.3)**

1. The annuity’s present value will be (ii) smaller than before. The present value represents the amount which, invested at the discount rate, can generate the future payments. When money can earn a higher rate of return, *less* money needs to be invested *today* in order to generate the specified stream of future payments.

**Exercise 11.3**

1. Given: *j* = 5%; *m* = 1; term = 25 years

*i* =  = 5%; *n* = 1(25) = 25

 = *PMT* = 14.0939446 *PMT*

(*a*) *PMT* = $1000 (*b*) *PMT* = $2000 (*c*) *PMT* = $3000

*PV* = $14,093.94 *PV* = $28,187.89 *PV* = $42,281.83

Same *I/Y, P/Y, C/Y*

Same *N, FV*

3000 **PMT**

**CPT** **PV**

*Ans*: –42,281.83

Same *I/Y, P/Y, C/Y*

Same *N, FV*

2000 **PMT**

**CPT** **PV**

*Ans*: –28,187.89

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –14,093.94

3. *PMT* = $1000 and *n* = 20 in all parts.

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –12,462.21

*a*. For *j* = 5% compounded annually,



= $1000

= $12,462.21

Similarly,

*b.* For *i* = 10%, *PV* = $8513.56

*c.* For *i* = 11%, *PV* = $7963.33

*d.* For *i* = 15%, *PV* = $6259.33

5. Given: *i* =  = %; *PMT* = $75,

8 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

30 **N**

75 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –2033.16

*n* = 12(2.5) = 30



= $75

= $2033.16

7. Given: *i* =  = 0.25%; *PMT* = $175,

3 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

99 **N**

175 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –15,330.60

*n* = 12(8.25) = 99



= $175

= $15,330.60

The investor would pay $15,330.60 to

purchase the contract.

9. Given: *i* =  = 2.95%;

5.9 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

19 **N**

1240 **+ / –** **PMT**

0 **FV**

**CPT** **PV**

*Ans*: 17,840.38

*PMT* = $1240, *n* = 2(9.5) = 19



= $1240

= $17,840.38

The current balance on the loan is $17,840.38.

11. Given: *PMT* = $2000 per year; term = 10 years;

*j* = 7% compounded annually

7 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

2000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –14,047.16



= $2000

= $14,047.16

13. Given: *i* =  = 1.5%; *PMT* = $2500, *n* = 4(7) = 28



6 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

28 **N**

2500 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –56,816.79

= $2500

= $56,816.79

15. Given: Term = 20 years; *j* = 4.8% compounded monthly;

*PMT* = $1000 per month

4.8 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

240 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –154,093.30

Therefore, *i* =  = 0.4% and *n* = 12(20) = 240

The amount required to purchase the annuity

is its present value.



= $1000 = $154,093.30

17. The amount required to fund each annuity is the present value of the annuity

discounted at the interest rate the money can earn. For both annuities,

*PMT* = $200 monthly and *i* =  = 0.5%

For the 30-year annuity,

 = $200 = $33,358.32

For the 20-year annuity, *PV* = $27,916.15. It requires

$33,358.32 – $27,916.15 = $5442.17 more

to fund the extra 10 years. (This is only about 20% more money.)

19. Original loan = Present value of all payments

Given: *PMT* = $2000, *n* = 5, and *i* =  = 5%

Original loan = $2000= $8658.95

6.8 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

40 **N**

1037.33 **+ / –** **PMT**

0 **FV**

**CPT** **PV**

*Ans*: 22,500.00

21. *a*. Original loan = Present value of all payments

Original loan = 

= $1037.33

= $22,500.00

*b.* Balance = Present value of the remaining

Same *I/Y,* *P/Y, C/Y PMT*, *FV*

23 **N**

**CPT** **PV**

*Ans*: 16,369.18

40 – 2(8.5) = 23 payments

= $1037.33

= $16,369.18

23. Selling price = Down payment

+ Present value of the monthly payments

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

42 **N**

295.88 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –10,899.99

*PMT* = $295.88, *n* = 12(3.5) = 42, *i* =  = 0.625%

*PV* of monthly payments = 

= $295.88

= $10,899.99

Thus, the selling price was $2000 + $10,899.99 = $12,899.99

25. Household Finance paid the present value of the

scheduled payments discounted at the required rate

of return (19.5% compounded monthly).

Given: *PMT* = $250, *n* = 15, *i* =  = 1.625%

19.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

15 **N**

250 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –3304.30

Price = 

= $250

= $3304.30

27. The economic value of the contract at the date of termination is the

present value of the remaining payments.

Given: *PMT* = $90,000, *n* = 12(3) = 36, *i* =  = 0.625%



7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

36 **N**

90000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –2,893,312.18

= $90,000

= $2,893,312.18

The settlement amount is $2,893,312.18.

29. Equivalent lump payment = PV of annual payments

Given: *PMT* = $2000/year; *n* = 5; *j* = 4.5% compounded annually.

*PV* = $2000 = $8779.95

The equivalent lump payment is $8779.95.

31. The offer having the higher current economic value is

preferred. The present value of the five payments

of $2000 is

6 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

4 **N**

2000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –7434.20

*PV =* $2000 + $2000 = $9434.20

Therefore, Mr. Lindberg’s offer is worth

$9500 − $9434.20 = $65.80 more

in current dollars.

33. The amount Pierre and Pat must pay is the present value of all of the payments.

The desired payment stream is equivalent to a succession of four 5-year annuities of:

(i) $2000 per quarter for the first 5 years, (ii) $2500 per quarter for the next 5 years,

(iii) $3000 per quarter for the next 5 years, and (iv) $3500 per quarter for the final 5 years.

The present values of these annuities (at their respective beginnings) are:

(i) *PV* = $2000 = $35,198.632; (ii) $43,998.290

(iii) *PV* = $3000 = $52,797.948; (iv) $61,597.606

(i) (ii) (iii) (iv)

Same *I/Y,* *P/Y, C/Y*

Same *N, FV*

2500 **PMT**

**CPT** **PV**

*Ans*: –43,998.290

5 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

20 **N**

2000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –35,198.632

Same *I/Y,* *P/Y, C/Y*

Same *N, FV*

3000 **PMT**

**CPT** **PV**

*Ans*: –52,797.948

Same *I/Y,* *P/Y, C/Y*

Same *N, FV*

3500 **PMT**

**CPT** **PV**

*Ans*: –61,597.606

The present value today of these four amounts is

$35,198.632 +  + +

Same *I/Y,* *P/Y, C/Y*

20 **N**

0 **PMT**

43998.290 **FV**

**CPT** **PV**

*Ans*: –34,319.042

Same *I/Y,* *P/Y, C/Y*

60 **N**

0 **PMT**

61597.606 **FV**

**CPT** **PV**

*Ans*: –29,232.228

Same *I/Y,* *P/Y, C/Y*

40 **N**

0 **PMT**

52797.948 **FV**

**CPT** **PV**

*Ans*: –32,122.976

= $35,198.632 + $34,319.042 + 32,122.976 + 29,232.254

= $130,872.90

Pierre and Pat must pay $130,872.90.

35. The highest price is the price that provides the lowest acceptable rate of return (7.5% compounded monthly). This price is the present value of all mortgage payments

discounted at 7.5% compounded monthly.

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

42 **N**

800 **PMT**

45572 **FV**

**CPT** **PV**

*Ans*: –64,550.64

Given: *PMT* = $800, *i* =  = 0.625%,

*n* = 12(3.5) = 42, and *FV* = $45,572

Price =  + 

= $800 + $45,572

= $64,550.64

37. Consider a $1000 purchase. It requires a $100 down

payment plus 12 monthly payments of $900/12 = $75.

Flemmings should be willing to accept a cash amount

equal to the present value of the payments discounted

at the rate of return that Flemmings can earn on this

money. The present value of the payments is

8.4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

12 **N**

75 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –860.35

*PV* = 

= $75

= $860.35

Cash price = $100 + $860.35 = $960.35

This is a $39.65 or 3.97% discount from the $1000 list price.

39. Let represent the normal monthly pension payment

at age 60. The choice at age 55 is between:

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

336 **N**

0.85 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –119.24

(i) receiving 0.85*PMT* per month for 28 years, or

(ii) waiting 5 years and then receiving *PMT*

per month for 28 − 5 = 23 years.

The economic values of the two alternatives are their

present values at age 55. For option (i),

*PV* = 0.85*PMT* = 119.24*PMT*

For option (ii), the present value at

age 60 of the 12(23) = 276 monthly

Same *I/Y,* *P/Y, C/Y, FV*

276 **N**

1 **PMT**

**CPT** **PV**

*Ans*: –131.34

payments of *PMT* is



= *PMT* 

= 131.34*PMT*

The *PV* at age 55 of this amount is

Same *I/Y,* *P/Y, C/Y*

60 **N**

0 **PMT**

131.34 **FV**

**CPT** **PV**

*Ans*: –90.37



= 131.34*PMT*

= 90.37*PMT*

Hence, the pension-at-age-55 option has a

× 100% = 31.9%

higher economic value

**Exercise 11.4**

1. Given: *j*1 *=* 3%; *m*1 *=* 1; *i*1 *=*  = 3%

*a.* For *m*2 *=* 2, *b*. For *m*2 *=* 4, *c*. For *m*2 *=* 12,

  

= – 1 = – 1 = – 1

= 0.01489 = 0.00742 = 0.00247

=1.489% =0.742% = 0.247%

**2nd** **ICONV**

3 **ENTER**

**↑**  1 **ENTER**

**↑**  **CPT**

**↓**  12 **ENTER**

**↓**  **CPT**

**÷**  12 =

*Ans*: 0.247

**2nd** **ICONV**

3 **ENTER**

**↑**  1 **ENTER**

**↑**  **CPT**

**↓**  4 **ENTER**

**↓**  **CPT**

**÷**  4 =

*Ans*:0.742

**2nd** **ICONV**

3 **ENTER**

**↑**  1 **ENTER**

**↑**  **CPT**

**↓**  2 **ENTER**

**↓**  **CPT**

**÷**  2 =

*Ans*: 1.489

The periodic interest rates are (*a*) 1.489% per half year; (*b*) 0.742% per quarter;

and (*c*) 0.247% per month .

3. Given: *j*1 *=* 4%; *m*1 *=* 4; *i*1 *=*  = 1%

*a.* For *m*2 *=* 1, *b*. For *m*2 *=* 2, *c*. For *m*2 *=* 12,

  

= – 1 = – 1 = – 1

= 0.04060 = 0.0201 = 0.003322

**2nd** **ICONV**

4 **ENTER**

**↑**  4 **ENTER**

**↑**  **CPT**

**↓**  1 **ENTER**

**↓**  **CPT**

**÷**  1 =

*Ans*: 4.060

= 4.060% = 2.010% = 0.332%

**2nd** **ICONV**

4 **ENTER**

**↑**  4 **ENTER**

**↑**  **CPT**

**↓**  12 **ENTER**

**↓**  **CPT**

**÷**  12 =

*Ans*: 0.3322

**2nd** **ICONV**

4 **ENTER**

**↑**  4 **ENTER**

**↑**  **CPT**

**↓**  2 **ENTER**

**↓**  **CPT**

**÷**  2 =

*Ans*: 2.0100

The periodic interest rates are (*a*) 4.060% per year; (*b*) 2.010% per half year;

and (*c*) 0.332% per month .

5. Given: *PMT* = $1000, Term = 25 years, *i* = 6%

6 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 54,864.51

*a.* *m* = 1, *n* = 1(25) = 25



= $1000

= $54,864.51

*b*. *m*1 = 2, *i*1 *=*  = 3%, *m*2 *=* 1

; *n* = 1(25) = 25

 =  = 0.0609

Same *I/Y, N, PV, PMT*

**P/Y** 1 **ENTER**

**C/Y** 2 **ENTER**

**CPT** **FV**

*Ans*: 55,564.96

Substitute in

giving *FV* = $1000

= $55,564.96

*c*. *m*1 = 4, *i*1 *=*  = 1.5%, *m*2 *=* 1

; *n* = 1(25) = 25

Same *I/Y, N, PV, PMT*

**P/Y** 1 **ENTER**

**C/Y** 4 **ENTER**

**CPT** **FV**

*Ans*: 55,929.71

 =  = 0.061363551

Then *FV* = $1000

= $55,929.71

*d*. *m*1 = 12, *i*1 *=*  = 0.5%, *m*2 *=* 1

; *n* = 1(25) = 25

Same *I/Y, N, PV, PMT*

**P/Y** 1 **ENTER**

**C/Y** 12 **ENTER**

**CPT** **FV**

*Ans*: 56,178.55

 =  = 0.061677812

Then *FV* = $1000

= $56,178.55

7. Given: *PMT* = $400; Term = 11 years; *j* = 6.5%; *m* = 1

6.5 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

44 **N**

400 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –12,598.41

Then *i* =  = 6.5%; *n* = 4(11) = 44;

=  = 0.25

 = – 1 = 0.015868285

*a.* 

= $400

= $12,598.41

Same *I/Y*, *P/Y, C/Y*

Same *PMT, N,*

0 **PV**

**CPT** **FV**

*Ans*: –25,186.12

*b.* 

= $400

= $25,186.12

9. Given: *i* =  = ; *n* = 2(3.5) = 7;

4 **I/Y**

**P/Y** 2 **ENTER**

**C/Y** 12 **ENTER**

7 **N**

2750 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −17,786.53

=  = 6

 = – 1 = 0.020167409

*a.* 

= $2750

Same *I/Y*, *P/Y, C/Y*

Same *PMT, N,*

0 **PV**

**CPT** **FV**

*Ans*: –20,454.61

= $17,786.53

*b.*  = $2750

= $20,454.61

11. *i* =  = 0.4375%; *n* = 17;

5.25 **I/Y**

**P/Y** 1 **ENTER**

**C/Y** 12 **ENTER**

17 **N**

3500 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −38,367.94

=  = 12

 = – 1 = 0.053781887

*a.* 

= $3500

= $38,367.94

Same *I/Y*, *P/Y, C/Y*

Same *PMT, N,*

0 **PV**

**CPT** **FV**

*Ans*: -93,482.61

*b.* 

= $3500

= $93,482.61

13.Given: *PMT* = $352; Term = 4 years; *j* = 7.55%; *m* = 2

Then *i* =  = 3.775%; *n* = 12(4) = 48;

=  = 

7.55 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

48 **N**

352 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –14576.73

 = – 1 = 0.006194928



= $352

= $14,576.73

The purchase price of the car including the $1200 down payment is $15,776.73

15. Given*:* = 2.0%

4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y*)

16 **N**

5000 **PV**

0 **PMT**

**CPT** **FV**

*Ans*: −6863.93

*a*. *FV* = *PV* = $5000 = $6863.93

*b*. *PMT* = $900; *i* =  = 2.0%; *n* = 8; = 2



Same *I/Y*

**P/Y** 1 **ENTER**

**C/Y** 2 **ENTER**

8 **N**

0 **PV**

900 **PMT**

**CPT** **FV**

*Ans*: −8304.63

= – 1

= 0.0404



= $900

= $8304.63

Same *I/Y, PV*

**P/Y** 4 **ENTER**

**C/Y** 2 **ENTER**

20 **N**

400 **PMT**

**CPT** **FV**

*Ans*: −8803.36

*c.* *PMT* = $400; *i* = 2.0%; *n* = 20;

 =  = 0.5

 = – 1 = 0. 009950494

*FV* = $400

= $8803.36

17. The amount required to purchase the annuity is the present value of

the payments discounted at the rate of return on the annuity.

*PMT* = $2500, *n* = 12(20) = 240, *i* = 4.75%,

4.75 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

240 **N**

2500 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −390,165.12

 =  = 

= – 1 = 0.003874685



= $2500

= $390,165.12

19. Selling price = Down payment + Present value of the monthly payments

Given: *PMT* = $259.50; *n* = 12(3.5) = 42.

7.5 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

42 **N**

259.50 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –9600.00

Then *i* =  = 7.5%, *c* =  = , and

 = – 1 = 0.006044919



= $259.50

= $9600.00

Thus, the selling price was $2000 + $9600.00 = $11,600.00.

21. For the case of month-end contributions of $100,

*i* =  = 4.25%; *PMT* = $100; *n* = 240;  = 

 = – 1 = 0.003474495

The amount in the TFSA after 20 years will be

 = $100 = $37,384.03

For quarter-end contributions of $300,

*PMT* = $300; *n* = 80; *i* = 4.25%; *c* =  = 0.25, and

 = – 1 = 0.010459743

The amount in the TFSA after 20 years will be

 = $300 = $37,254.44

The TFSA with monthly contribution is worth $129.59 more.

23. For the first 10 years, *i* =  = 0.5%;

*PMT* = $1500; *n* = 2(10) = 20; *c* =  = 6

6 **I/Y**

**P/Y** 2 **ENTER**

**C/Y** 12 **ENTER**

20 **N**

0 **PV**

1500 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 40,460.694

 = – 1 = 0.030377509

and the future value at the end of the 10 years is



= $1500

= $40,460.694

For the next 15 years, *PMT* = $1500;

Same *PMT, P/Y*

7 **I/Y**

**C/Y** 1 **ENTER**

30 **N**

40460.694 **+ / –** **PV**

**CPT** **FV**

*Ans*: 188,316.36

*n* = 2(15) = 30; *i* =  = 7%; *c* =  = 0.5

 = – 1 = 0.034408043

The combined future value at the end of the 25 years is

$40,460.694+ $1500

= $111,632.331 + $76,684.027

= $188,316.36

25. For the first 10 years, *i* =  = 0.625%;

7.5 **I/Y**

**P/Y** 1 **ENTER**

**C/Y** 12 **ENTER**

10 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 14,324.71

*PMT* = $1000; *n* = 10; *c* =  = 12

and  =  – 1

= 0.077632599



= $1000

= $14,324.71

Same *PMT, P/Y*

8 **I/Y**

**C/Y** 2 **ENTER**

15 **N**

14324.71 **+ /** **–** **PV**

**CPT** **FV**

*Ans*: 73,953.35

For the next 15 years, *PV* = $14,324.71; *PMT* = $1000;

*i* =  = 4%; = 2; *n* = 15

and  =  – 1 = 0.0816

The combined future value after 25 years will be

$14,324.71+ $1000 = $73,953.35

27. For the past 9 years,

*i* =  = 2%; *PMT* = $2000; *n* = 9; = 4; and

 = – 1 = 0.08243216

8 **I/Y**

**P/Y** 1 **ENTER**

**C/Y** 4 **ENTER**

9 **N**

0 **PV**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 25,230.137

The current value of Gloria’s RRSP is



= $2000

= $25,230.137

For the next 15 years, *PMT* = $2000; *n* = 30;

*i* = 2%; *c* =  = 2; *PV* = $25,230.137;

Same *I/Y, PMT*

**P/Y** 2 **ENTER**

**C/Y** 4 **ENTER**

30 **N**

25230.137 **+ /** **–** **PV**

**CPT** **FV**

*Ans*: 195,703.17

and  = – 1 = 0.0404

The total amount 15 years from now will be

$25,230.137+ $2000

= $195,703.17

29. Cost to purchase the annuity = *PV* of all payments

5 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

120 **N**

1500 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –142,148.386

First calculate the *PV,* 15 years from now, of the last

10 years’ payments of *PMT* = $1500/month with

*j* = 5%, *m* = 1, *c* =  = , *n* = 12(10) = 120, and

 = – 1 = 0.004074124

*PV* = $1500 = $142,148.386

Next determine the combined *PV* of this amount and the

Same *I/Y,* *P/Y, C/Y*

180 **N**

1000 **PMT**

142148.386 **FV**

**CPT** **PV**

*Ans*: –195,760.96

first 15 years’ payments of *PMT* = $1000/month

with the same *i*2 and *n* = 12(15) = 180.

*PV* =  + $1000

= $68,375.804 + $127,385.158

= $195,760.96

31. For the first 7 years, *i* =  = 2%, *PMT* = $3000,

*n* = 1(7) = 7, *c* =  = 4, and

8 **I/Y**

**P/Y** 1 **ENTER**

**C/Y** 4 **ENTER**

7 **N**

0 **PV**

3000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 26,968.51

 = – 1 = 0.08243216

Amount in the RRSP after 7 years will be

*FV* = $3000 = $26,968.51

For the next 5 years, *PMT* = $500, *i* =  = 2%,

*n* = 12(5) = 60, *c* =  = , and

 = – 1 = 0.00662271

8 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 4 **ENTER**

60 **N**

26968.51 **+ / –** **PV**

500 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 76,761.75

Amount in the RRSP after 12 years will be

*FV* =  + 

= $26,968.51 + $500

= $76,761.75

For the last 13 years, *PMT* = $500, *n* = 12(13) = 156,

*i* =  = 3.5%, *c* =  = , and

Same  *P/Y, PMT*

7 **I/Y**

**C/Y** 2 **ENTER**

156 **N**

76761.75 **+ /** **–** **PV**

**CPT** **FV**

*Ans*: 313,490.72

 = – 1 = 0.005750039

Amount in the RRSP after 25 years will be

*FV* =  + 

= $76,761.75 + $500

= $313,490.72

**Review Problems**

1. *a*. *PMT* = $1000, *n* = 2(20) = 40, *i* =  = 2.75%

5.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

40 **N**

0 **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 71,268.14



= $1000

Same *I/Y*, *PV*

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y*)

20 **N**

2000 **+ /** **–** **PMT**

**CPT** **FV**

*Ans*: 69,736.64

= $71,268.14

*b.* *PMT* = $2000, *n* = 20, *i* = 5.5%

*FV* = $2000

= $69,736.64

3. The finance company will pay an amount equal to the present value of the

15 payments discounted at its required rate of return.

*PMT* = $180.50, *n* = 15, *i* =  = 10.5%, *c* =  = 

21 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

15 **N**

180.50 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −2376.15

 = – 1 = 0.01678012

Price = 

= $180.50

= $2376.15

The finance company will pay $2376.15 for the contract.

5. The amount of the lost wages plus interest at the end of the 2‑year period was

 = $5500 = $135,865.50

This amount plus additional interest to the judgment date is

 = $135,865.50 = $138,953.22

7. The minimum amount is the present value of the payments, discounted

at 2.5% compounded semiannually.

*PMT* = $1000, *n* = 12(12) = 144, *i* =  = 1.25%,

=  = 

= – 1 = 0.002072565

 = $1000 = $124,388.36

9. For the first 2 years, *PMT* = $300, *n* = 2(12) = 24, *i* =  = 0.75%

For the subsequent 3 years, *PMT* = $300, *n* = 36, *i* =  = 0.625%

The future value, 2 years from now, of the first 24 payments is

 = $300 = $7856.54

The future value, 5 years from now, of this amount and the last 36 payments is

$7856.54 + $300 = $21,901.45

11. The amount (future value) in Dr. Krawchuk’s RRSP

when he left general practice was

10 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

24 **N**

0 **PV**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 64,698.076



= $2000

= $64,698.076

After an additional 2.5 years with no further contributions, this amount grew to

Same *I/Y*, *P/Y, C/Y*

10 **N**

64698.076 **+ /** **–** **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 82,819.01



= $64,698.08

= $82,819.01

13. The current economic value of the award is the present value of all the payments discounted at the time value of money.

6 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

120 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −90,073.45

The present value, 5 years from now,

of the 120 monthly payments of $1000 is



= $1000

= $90,073.45

The present value today of this amount and

Same *I/Y*, *P/Y, C/Y*

60 **N**

800 **PMT**

90073.45 **FV**

**CPT** **PV**

*Ans*: –108,158.40

the 60 monthly payments of $800 is

*PV =*  + 

= $90,073.45 + $800

= $108,158.40

The economic value of the award, 1 month before the first payment, is $108,158.40.

15. The amount she will have will be the future value of all contributions after 9 years.

For the first 4 years, *PMT* = $800, *n*=4(4) = 16, 

 = $800 = $13,675.30

For the subsequent 12(5) = 60 months,

*PMT* = $200, *n* = 60 *i* =  = 0.875%, *c* =  = 

 = – 1 = 0.002908201

The combined future value, 9 years from now,

of this amount and the last 60 payments is

*FV* =  + 

= $13,675.30 + $200

= $16,278.25 + $13,089.87

= $29,368.12

Norma will be $30,000 – $29,368.12 = $631.88 short of her goal.

17. If $3000 is contributed every 6 months for 20 years,

8 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

40 **N**

0 **PV**

3000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 285,076.55



= $3000

= $285,076.55

If $6000 is contributed every 6 months for the

final 10 years,

*FV* = $6000 = $178,668.47

Same *I/Y*, *P/Y, C/Y, PV*

20 **N**

6000 **+ /** **–** **PMT**

**CPT** **FV**

*Ans*: 178,668.47

The early start results in

×100% = 59.56% more

funds in the RRSP after 20 years.

19. The appropriate price to pay is the present value

of the payments discounted at the required rate

7 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

28 **N**

1500 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −32,980.43

of return. The present value, 5 years from now,

of the last 7 years’ payments is



= $1500

= $32,980.43

Today's present value of this amount

and the first 5 years’ payments is

Same *P/Y, C/Y, PMT*

20 **N**

6 **I/Y**

32980.43 **FV**

**CPT** **PV**

*Ans*: –50,239.95

$32,980.43 + $1500

= $24,486.99 + $25,752.96

= $50,239.95

The appropriate price to pay is $50,239.95