12 Ordinary Annuities:

Periodic Payment, Number of

Payments, and Interest Rate

**Point of Interest (Section 12.1)**

***Retirement Dreams Then and Now***

1. The annual income at age 65 that will have the same purchasing power as

$50,000 at age 20 is

 = $50,000 = $189,080

**Concept Questions (Section 12.1)**

1. The payments will be (ii) *more* than half as large because you will pay more total interest if you pay off the loan over 10 years instead of 5 years. (The total of the principal components of the payments will be the same in both cases.)

**Exercise 12.1**

1. Given: *FV* = $500,000; *j* = 9%; *m* = 1. Then *i* = 9%

*a*. For a term of 25 years, *n* = 25. Solve for *PMT* in

9 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

0  **PV**

500000  **FV**

**CPT** **PMT**

*Ans*: –5903.13

$500,000 = *PMT* 

*PMT* = $5903.13

Earnings portion = $500,000 − 25($5903.13)

= $352,421.75

*b*. For a term of 30 years, *n* = 30. Solve for *PMT* in

$500,000 = *PMT* 

Same *I/Y, P/Y, C/Y*

Same *PV, FV*

30 **N**

**CPT** **PMT**

*Ans*: –3668.18

*PMT* = $3668.18

Earnings portion = $500,000 − 30($3668.18)

= $389,954.60

3. Given: *PV* = $400,000, Term = 20 years

*a.* With *j* = 4%, *m* = 4, we have *n* = 4(20) = 80, *i* = 1%

4 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

80 **N**

400000 **+ /** **–**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 7287.54

Solve for *PMT* in

$400,000 = *PMT* 

*PMT* = $7287.54

Total earnings = 80($7287.54) – $400,000

= $183,003.20

*b.* With *j* = 5%, *m* = 4, we have *n* = 80, *i* = 1.25%

Same *P/Y, C/Y*

Same *N,* *PV, FV*

5 **I/Y**

**CPT** **PMT**

*Ans*: 7938.61

Solve for *PMT* in

$400,000 = *PMT* 

*PMT* = $7938.61

Total earnings = 80($7938.61) – $400,000

= $235,088.80

Similarly.

*c.* With *j* = 6%, *m* = 4, *n* = 80, *i* = 1.5%, we obtain *PMT* = $8619.33

Total earnings = 80($8619.33) – $400,000

= $289,546.40

*d.* With *j* = 7%, *m* = 4, *n* = 80, *i* = 1.75%, we obtain *PMT* = $9328.37

Total earnings = 80($9328.37) – $400,000

= $346,269.60

5. Given: *FV* = $750,000; *j* = 5%; *m* = 1, *P/Y* = 12.

5 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

180 **N**

0  **PV**

750000  **FV**

**CPT** **PMT**

*Ans*: –2832.06

Then *i* = 5%, *c* = , and

 = – 1 = 0.004074124

*a*. For a term of 15 years, *n* = 12(15) = 180, and

$750,000 = *PMT* 

*PMT* = $2832.06

Total earnings = $750,000 − 180($2832.06)

= $240,229.20

*b*. For a term of 20 years, *n* = 12(20) = 240, and

Same *I/Y, P/Y, C/Y*

Same *PV, FV*

240 **N**

**CPT** **PMT**

*Ans*: –1848.18

$750,000 = *PMT* 

*PMT* = $1848.18

Total earnings = $750,000 − 240($1848.18)

= $306,436.80

*c.* For a term of 25 years, *n* = 12(25) = 300, we obtain *PMT* = $1280.44

Total earnings = $750,000 – 300($1280.44)

= $365,868.00

*d.* For a term of 30 years, *n* = 12(30) = 360, we obtain *PMT* = $919.82

Total earnings = $750,000 – 360($919.82)

= $418,864.80

7. Given: *PV* = $50,000; *j* = 7.5%; *P/Y* = 12, *n* = 12(7) = 84.

7.5 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

84 **N**

50000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –760.86

*a*. If *m* = 1, then *i* = 7.5%, *c* = , and

 = – 1 = 0.006044919

Solve for *PMT* in

$50,000 = *PMT* 

*PMT* = $760.86

*b*. If *m* = 2, then *i* = 3.75%, *c* = , and

 = – 1 = 0.006154524

Same *I/Y, P/Y*

Same *N*, *PV, FV*

**C/Y** 2 **ENTER**

**CPT** **PMT**

*Ans*: –764.09

$50,000 = *PMT* 

*PMT* = $764.09

Similarly,

*c.* If *m* = 4, then *i* = 1.875%%, *c* = , and we obtain *PMT* = $765.77

*d.* If *m* = 12, then *i* = 0.625%%, *c* = 1, and we obtain *PMT* = $766.91

9. Given: *n* = 12(3) = 36, *FV* = $15,000; *j* = 4%; *m* = 12, *i* = = %

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

36 **N**

15000 **+ /** **–**  **FV**

0 **PV**

**CPT**  **PMT**

*Ans*: 392.86

The monthly annuity payment is the value of *PMT* in

$15,000 = *PMT* 

*PMT* = $392.86

11. Original loan = Present value of all payments

4 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

90 **N**

20000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –257.28

The payments in this case form an ordinary general annuity having

*PV* = $20,000, *n* = 12(7.5) = 90, *i* =  = 2%, *c* = 

 = – 1 = 0.00330589

$20,000 = *PMT* 

*PMT* = $257.28

The monthly payment is $257.28.

Total interest Karen will pay

90($257.28) − $20,000 = $3155.20.

13. The $600,000 balance is the present value of the

payments which form an ordinary general annuity.

8 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

80 **N**

600000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –14,839.78

*PV* = $600,000, *n* = 4(20) = 80, *i* = 8%, *c* =  = 0.25

 =  – 1 = 0.019426547

$600,000 = *PMT* 

*PMT* = $14,839.78

Henry’s quarterly payments will be $14,839.78.

Total interest Henry will pay is

80($14,839.78) − $600,000 = $587,182.40.

15. The future value of the replacement payments is $25,000.

*FV* = $25,000, *n* = 11, *i* =  = 0.45%

5.4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

11 **N**

0  **PV**

25000  **FV**

**CPT** **PMT**

*Ans*: –2222.05

$25,000 = *PMT* 

*PMT* = $2222.05

Eleven monthly payments of $2222.05 are economically

equivalent to the scheduled payment of $25,000.

17. *a*. Original loan = Present value of all payments

5.5 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 12 **ENTER**

12 **N**

7500  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –682.53

*PV* = $7500, *n* = 4(3) = 12, *i* =  = %, *c* =  = 3,

 = – 1 = 0.013813117

$7500 = *PMT* 

*PMT* = $682.53

Same *I/Y,* *P/Y, C/Y, FV*

4 **N**

682.53 **+ / –** **PMT**

**CPT** **PV**

*Ans*: 2638.38

*b.* Balance = Present value of the 4 remaining payments

= $682.53

= $2638.38

19. With a focal date at age 65,

*FV* of amount in RRSP at age 56 = *PV* of the annuity

5.4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

108 **N**

195000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 316,685.55

For the *FV* calculation, *PV* = $195,000,

*n* = 12(9) = 108, and *i* =  = 0.45%

For the *PV* calculation, *n* = 12(20) = 240,

and *i* =  = 0.35%.

$195,000 = *PMT*

Same *P/Y, C/Y*

4.2 **I/Y**

240 **N**

316685.55 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 1952.59

$316,685.55 = *PMT*

*PMT* = $1952.59

21. At a focal date 10 years from now,

Future value of the retiring allowance

= Present value of the 15‑year annuity

Viewed from the focal date,

the annuity is an ordinary simple annuity.

Putting the word equation into mathematics,

For the *FV* calculation, *PV* = $25,000.

*n* = 1(10) = 10, and *i* =  = 5%

For the *PV* calculation, *n* = 4(15) = 60,

and *i* =  = 1.3%.

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

25000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 40722.37

$25,000 = *PMT*

5.2 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

60 **N**

40722.37 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 981.65

*PMT* = $981.65

Elizabeth can expect payments of $981.65

at the end of each quarter.

23. At a focal date 4 years from now,

Future value of quarterly savings

Step 1

4.2 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

12 **N**

3000 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –35,194.20

= Present value of the monthly withdrawals

*PMT* = $3000

Step 2

6 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

16 **N**

0  **PV**

35194.20  **FV**

**CPT** **PMT**

*Ans*: –1962.61

The solution is *PMT* = $1962.61.

Tim and Justine must contribute $1962.61 to the fund at

the end of every calendar quarter for the next 4 years.

25. *a*. The nominal amount in the RRSP should be

= $500,000 = $905,680.79

*b.* The contributions form an ordinary general annuity having

8.5 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 2 **ENTER**

120 **N**

0  **PV**

905680.79  **FV**

**CPT** **PMT**

*Ans*: –1708.17

*FV* = $905,680.79, *n* = 4(30) = 120,

*i* =  = 4.25%, *c* =  = 0.5,

 = – 1 = 0.021028893

$905,680.79 = *PMT* 

*PMT* = $1708.17

Dr. Collins should make quarterly contributions of $1708.17.

27. At a focal date on Brice’s 65th birthday,

*FV* of $154,000 = *PV* of ordinary simple annuity payments

For the *FV* calculation, *PV* = $154,000, *n* = 12(11) = 132, and *i* =  = 0.6875%.

For the *PV* calculation, *n* = 12(20) = 240 and *i* = 0.6875%.

Hence, $154,000 = *PMT* 

8.25 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

132 **N**

154000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 380,447.16

Same *I/Y,* *P/Y, C/Y*

240 **N**

380447.16 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 3241.66

*PMT* =  = $3241.66

The annuity payments will be $3241.66 per month.

29. *a*. The original loan equals the present value of the loan

payments. Shifting the focal date to the date

6 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

5 **N**

3000000  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –3,477,822.22

2.5 years from now,

*FV* of $3,000,000 = *PV* of ordinary annuity

For the *FV* calculation, *PV* = $3,000,000,

*n* = 2(2.5) = 5, and *i* =  = 3%.

For the *PV* calculation, *n* = 2(15) = 30, and *i* = 3%.

Hence,

$3,000,000 = *PMT* 

*PMT* =  = $177,435.91

Same *I/Y,* *P/Y, C/Y*

30 **N**

3477822.22  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –177,435.91

The semiannual loan payments will be $177,435.91.

*b.* Total interest = Total of payments – Principal

= 30($177,435.91) – $3,000,000

= $2,323,077.30

31 At a focal date 28 years from now,

*FV* of RRSP contributions = *PV* of the annuity payments.

7.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

0  **PV**

7000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 99,029.61

The amount in the RRSP 10 years from now will be



= $7000

= $99,029.61

The left side of the word equation is the future value,

Same *I/Y,* *P/Y, C/Y*

18 **N**

99029.61 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 364,013.45

28 years from now, of the preceding amount and the

contributions for years 11 to 28 inclusive. That is,

$99,029.61 + *PMT* 

= $364,013.45 + *PMT*(35.677388)

The present value, 28 years from now,

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y =* 12)

300 **N**

6000 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –811,917.68

of the $6000 per month annuity is

$6000 = $811,917.68

Substituting these amounts into the original

word equation gives

$364,013.45 + *PMT*(35.677388) = $811,917.68

The solution is *PMT* = $12,554.29. Mr. Groman must

contribute $12,554.29 at each year-end for years 11 to 28 inclusive.

33 The amount, 20 years from now, that will have the

purchasing power of $6000 (in today’s dollars) is

2.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y =* 1)

20 **N**

6000  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –9831.70

 = $6000= $9831.70

At a focal date 20 years from now,

*FV* of the RRSP = *PV* of the 25‑year annuity

For the *FV* calculation, *PV* = $54,000,

*i* =  = 4%, and *n* = 40.

For the *PV* calculation, *PMT* = $9831.70,

*i* =  = 1.375%, and *n* = 4(25) = 100.

$54,000 + *PMT*  = $9831.70

8 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y =* 2)

40 **N**

54000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 259,255.11

5.5 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y =* 4)

100 **N**

9831.70 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –532,542.67

$259,255.11 + *PMT*(95.0255157) = $532,542.67

*PMT* = $2875.94

Semiannual RRSP contributions of $2875.94 are needed to reach the retirement goal.

**Concept Questions (Section 12.2)**

1. You will pay off the loan in (ii) less than half the time. If payments are doubled, you will pay less interest over the life of the loan. Therefore, the total of the nominal payments (principal + interest) will be reduced and you will pay off the loan in less than half the time.

**Exercise 12.2**

1. Given: *PV* = $50,000, *PMT* = $3874.48, *i* =  = 3.25%

6.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

50000  **PV**

3874.48 **+ / –** **PMT**

0  **FV**

**CPT** **N**

*Ans*: 17.00

 = – 

= – 

= 17.00

To pay off the loan requires 17 semiannual payments.

This will take  years = 8.5 years = 8 years and 6 months.

3. Given: *PV* = $200,000, *PMT* = $3341.74, *i* =  = 1.125%



4.5 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y =* 4)

200000 **+ / –**  **PV**

3341.74 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 100.00

= – 

= 100.00

The annuity consists of 100 payments requiring

100 calendar quarters. The term of the annuity is

 years = 25 years.

5. Given: *PV* = $100,000, *PMT* = $1000, *i* =  = 0.%

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

100000  **PV**

1000 **+ / –** **PMT**

0  **FV**

**CPT** **N**

*Ans*: 121.84



= 

= 121.84

The endowment can make 122 monthly payments with the last payment

smaller than $1000. The endowment can sustain the payments for

122 months =  years = 10.1 years = 10 years and 2 months.

7. Given: *FV* = $74,385, *PMT* = $1200, *i* = 4.75%, *c* =  = 0.25

4.75 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

0  **PV**

1200 **+ / –** **PMT**

74385  **FV**

**CPT** **N**

*Ans*: 46.91

 =  – 1 = 0.011669153



= 

= 46.91

William has made 47 quarterly contributions over the past 11 years and 9 months.

9. Given: *FV* = $700,000, *PMT* = $1000

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

0  **PV**

1000 **+ / –** **PMT**

700000  **FV**

**CPT** **N**

*Ans*: 361.79

*a.* *j* = 4%, *m* = 12, *i* = %

*n* = 

= 361.79

It will take 362 contributions requiring 362 months, or

30 years, 2 months for the RRSP to grow to $700,000.

*b.* *j* = 6%, *m* = 12, *i* =  = 0.5%



Same *P/Y, C/Y*

Same *PV, PMT, FV*

6 **I/Y**

**CPT** **N**

*Ans*: 301.57

= 

= 301.57

It will take 302 contributions requiring 302 months, or

25 years, 2 months for the RRSP to grow to $700,000.

Similarly.

*c.* For *j* = 8%, *m* = 12, *i* = %, we obtain *n* = 261.06

requiring 262 months, or 21 years, 10 months

*d.* For *j* = 9%, *m* = 12, *i* =  = 0.75%, we obtain *n* = 245.26

requiring 246 months, or 20 years, 6 months

11. Given: *PV* = $100,000, *PMT* = $740,

*a.* *j* = 7.5%, *m* = 1, *i* = 7.5%, *c* = 

 = – 1 = 0.006044919

7.5 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

100000  **PV**

740 **+ / –** **PMT**

0  **FV**

**CPT** **N**

*Ans*: 281.68



= 

= 281.68

The loan requires 282 monthly payments,

taking 23 years, 6 months.

*b.* *j* = 7.5%, *m* = 2, *i* = 3.75%, *c* = 

Same *I/Y, P/Y*

Same *PV, PMT, FV*

**C/Y** 2 **ENTER**

**CPT** **N**

*Ans*: 290.43

 = – 1 = 0.006154524

*n* = 

= 290.43

The loan requires 291 monthly payments,

taking 24 years, 3 months.

Similarly,

*c.* For *j* = 7.5%, *m* = 4, then *i* = 1.875%, *c* = , and we obtain *n* = 295.32

The loan requires 296 monthly payments, taking 24 years, 8 months.

*d.* For *j* = 7.5%, *m* = 12, then *i* = 0.625%, *c* = 1, and we obtain *n* = 298.80

The loan requires 299 monthly payments, taking 24 years, 11 months.

13. The deposits form an ordinary simple annuity having

*FV* = $100,000, *PMT* = $500 per month,

5.25 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

0  **PV**

500 **+ / –** **PMT**

100000  **FV**

**CPT** **N**

*Ans*: 143.996

and *i* =  = 0.4375%



= 

= 143.996

Rounded to the next higher month it will require a total of 144 months

or  = 12 years to accumulate $100,000.

15. The withdrawals constitute an ordinary simple annuity having

*PV* = $500,000, *PMT* = $3000/month, and *i* =  = 0.2708%

3.25 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

500000 **+ / –**  **PV**

3000 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 221.97

Substitute into formula (11-2n), giving



= 

= 221.97

The $500,000 will sustain monthly withdrawals of $3000 for 222 months

or  = 18 years and 6 months. (The last withdrawal will be less than $3000.)

17. The purchase price represents the present value of the annuity. We have

*PV* = $300,000 with *PMT* = $2500 and *i* =  = 0.625%.

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

300000 **+ / –**  **PV**

2500 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 222.50



= 

= 222.50

The annuity will have 223 monthly payments. Its term will

be 223 months or 18 years and 7 months.

19. In each case, the contributions form an ordinary simple annuity.

With *PMT* = $500, *FV* = $500,000, and *i* =  = 0.625%



7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

0  **PV**

500 **+ / –** **PMT**

500000  **FV**

**CPT** **N**

*Ans*: 317.95

= 

= 317.95

Rounded to the nearest month, it will take 318 months

for monthly contributions of $500 to reach $500,000.

With *PMT* = $550 and *i* = 0.625%,



Same *I/Y,* *P/Y, C/Y*

Same *PV, FV*

550 **+ / –** **PMT**

**CPT** **N**

*Ans*: 304.85

= 

= 304.85

Rounded to the nearest month, it will take 305 months for monthly contributions

of $550 to reach $500,000. Therefore, it will take 318 – 305 = 13 months longer

for the smaller payments to grow to $500,000.

21. The original loan equals the present value of the loan payments.

With *PV* = $100,000, *PMT* = $1000, and *i* =  = 0.875%,



10.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

100000  **PV**

1000 **+ / –** **PMT**

0  **FV**

**CPT** **N**

*Ans*: 238.69

= 

= 238.69

That is, it will take 239 months to pay off the loan

(with the last payment smaller than the others).

With *PMT* increased to $1100 per month,

Same *I/Y,* *P/Y, C/Y*

Same *PV*, *FV*

1100 **+ / –** **PMT**

**CPT** **N**

*Ans*: 182.16

*n* =  = 182.16

The loan will be paid off in only 183 months.

Therefore, the $1000 payments will take

(239 – 183) months = 56 months = 4 years and 8 months

longer to pay off the loan.

23. The purchase price of the annuity is the present value of

the annuity payments. In this case, the payments form

an ordinary general annuity having

*PV* = $200,000, *PMT* = $5000, *i* =  = 2.75%, *c* =  = 0.5

 = – 1 = 0.013656747

The number of payments that the annuity will provide is

5.5 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 2 **ENTER**

200000 **+ / –**  **PV**

5000 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 58.26



= 

= 58.26

The 59 quarterly payments will last for

 years = 14.75 years = 14 years and 9 months

25. In effect, the customer borrows $1395 – $50 = $1345.

The loan equals the present value of the payments. Thus,

*PV* = $1345, *PMT* = $50, *i* =  = 1.125%

13.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

1345 **+ / –**  **PV**

50 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 32.22

The number of payments will be



= 

= 32.22

The final payment (smaller than $50) will be the 33rd payment occurring

33 months or 2 years and 9 months after the date of purchase.

27. The economic value at age 65 of each annuity is the present value of the

payments discounted at the time value of money.

For the 25-year-term annuity, *PMT* = $307,

6 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y*= 12)

300 **N**

307 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –47,648.51

*n* = 12(25) = 300, and *i* =  = 0.5%



= $307

= $47,648.51

For payments of $408 per month to have the

same economic value,



Same *I/Y,* *P/Y, C/Y*

Same *PV,* *FV*

408 **PMT**

**CPT** **N**

*Ans*: 175.82

= 

= 175.82

Therefore, if the man receives 176 payments, the life annuity will have the higher

economic value. He must live at least 176 months (14 years and 8 months)

beyond age 65. That is, he must live at least to an age of 79 years and 8 months.

29. At focal date 41 months from now,

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

41 **N**

10000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 12,910.46

*FV* of $10,000 = *PV* of ordinary simple annuity payments

For the *FV* calculation, *PV* = $10,000,

*i* =  = 0.625%, *n* = 12(3) + 5 = 41

 = $10,000 = $12,910.46

Now substitute *PV* = $12,910.46, *PMT* = $300,

and *i* = 0.625% into formula (11-2n) to obtain *n*.



Same *I/Y,* *P/Y, C/Y*

12910.46 **+ / –**  **PV**

300 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 50.28

= 

= 50.28

The fund will provide 51 monthly withdrawals

(with the last one less than $300).

31. At a focal date 3 years ago,

*FV* of $17,000 = *PV* of the RRIF withdrawals

The amount on the left side is

8 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

40912.53 **+ / –**  **PV**

1000 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 86.11

 = $17,000 = $40,912.53

Now substitute *PV* = $40,912.53, *PMT* = $1000,

and *i* =  = 2% into formula (11-2n) to obtain *n*.

*n* =  = 86.11

There will be 87 withdrawals in all.

4(3) = 12 withdrawals have already been made

and 87 – 12 = 75 withdrawals remain. These will continue for another

 = 18.75 years = 18 years and 9 months

33. The original loan equals the present value of the payments. Equivalently,

At a focal date 23 months from now,

8 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

23 **N**

20000  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: –23,178.87

*FV* of $20,000 = *PV* of ordinary general annuity payments

For the *FV* calculation, *PV* = $20,000, *i* = 8%,

*c* =  = , *n* = 12 + 11 = 23, and

 = – 1 = 0.006434030



= $20,000

= $23,178.87

Now substitute *PV* = $23,178.87, *PMT* = $300, and

= 0.006434030 into formula (11-2n) to obtain *n*.



Same *I/Y,* *P/Y, C/Y*

23178.87  **PV**

300 **+ / –** **PMT**

0  **FV**

**CPT** **N**

*Ans*: 107.18

= 

= 107.18

The 108th payment will extinguish the debt. It will

occur 23 + 108 = 131 months or 10 years and 11 months

after the date the $20,000 was originally borrowed.

35. At a focal date 6 years from now,

8.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

6 **N**

139000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 226,773.98

*FV* of $139,000

= *PV* of the ordinary general annuity payments

The left side is

 = $139,000 = $226,773.98

For the right side of the equation,

*PV* = $226,773.98, *PMT* = $5000,

*i* =  = 0.625%, *c* =  = 3, and

 = – 1 = 0.018867432

7.5 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 12 **ENTER**

226773.98 **+ / –**  **PV**

5000 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 103.58

Using formula (11-2n) to obtain *n*, we get



= 

= 103.58

There will be 104 payments lasting 104 quarters or

26 years after the purchase of the annuity.

**Point of Interest (Section 12.3)**

***Should You Choose a Cash-Discount Incentive or Low-Interest-Rate Financing?***

1. With *PV* = $30,000, *n* = 36, *j* = 0.99% compounded monthly, *i* = %

$30,000 = *PMT*

*PMT* = $846.11

The monthly payment will be $846.11.

3. If the bank charges less than 5.55% compounded monthly, the monthly payments will be less than $846.11. Choose the bank loan if the interest rate is less than 5.55% compounded monthly.

**Exercise 12.3**

1. Given: *PV* = $50,000, *PMT* = $1941.01, *n* = 4(7.75) = 31

**P/Y** 4 **ENTER**

(making *C/Y* = *P/Y* = 4)

31 **N**

50000  **PV**

1941.01 **+ / –** **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 4.800

Then *j* = 4.80% compounded quarterly

and *i* =  =  = 1.20% per quarter



**2nd** **ICONV**

4.8 **ENTER**

**↑**  4 **ENTER**

**↑**  **CPT**

*Ans*: 4.887

= – 1

= 4.89%

3. Given: *FV* = $500,000, *PMT* = $3030.02, *n* = 2(25) = 50

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

50 **N**

0  **PV**

3030.02 **+ / –** **PMT**

500000  **FV**

**CPT** **I/Y**

*Ans*: 8.500

Then *j* = 8.50% compounded semiannually

*i* = 

**2nd** **ICONV**

8.50 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 8.681

= 

= 4.25% per half year

and 

= – 1

= 8.68%

5. Given: *PMT* = $500 monthly, *n* = 12(25) = 300

*a.* For a *FV* of $400,000, *b.* For a *FV* of $500,000, *c.* For a *FV* of $600,000,

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

300 **N**

0  **PV**

500 **+ / –** **PMT**

400000  **FV**

**CPT** **I/Y**

*Ans*: 6.921

Same *I/Y,* *P/Y, C/Y*

Same *N, PV,* *PMT*

600000  **FV**

**CPT** **I/Y**

*Ans*: 9.406

Same *I/Y,* *P/Y, C/Y*

Same *N, PV,* *PMT*

500000  **FV**

**CPT** **I/Y**

*Ans*: 8.308

The RRSP must earn (*a*) 6.92% compounded monthly;

(*b*) 8.31% compounded monthly;

(*c*) 9.41% compounded monthly.

7. The purchase price represents the present value

of the annuity. That is,

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

240 **N**

100000  **PV**

830 **+ / –** **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 7.896

*PV* = $100,000 for *PMT* = $830 and *n* = 12(20) = 240.

Then *j* = 7.90% compounded monthly

*i* = 

**2nd** **ICONV**

7.896 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 8.188

= 

= 0.658% per month



= – 1

= 8.19%

9. The accumulated amount represents the

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

21 **N**

0  **PV**

2000 **+ / –** **PMT**

65727.82  **FV**

**CPT** **I/Y**

*Ans*: 8.501

future value of the contributions.

That is, *FV* = $65,727.82 with

*PMT* = $2000 and *n* = 2(10.5) = 21.

Then *j* = 8.50% compounded semiannually

*i* =  = 

**2nd** **ICONV**

8.501 **ENTER**

**↑**  2 **ENTER**

**↑**  **CPT**

*Ans*: 8.682

= 4.25%

and 

= – 1

= 8.68%

11. The projected future value of the plan equals the future value of the contributions. That is,

**P/Y** 1 **ENTER**

(making *C/Y* = *P/Y* = 1)

25 **N**

0  **PV**

10,000**PMT**

1,000,000**+ / –**  **FV**

**CPT** **I/Y**

*Ans*: 10.11

*FV* = $1,000,000, *PMT* = $10,000.00, and *n* = 1(25) = 25.

Then *j* = *f* = 10.11% compounded annually.

13. The initial purchase price equals the

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

240 **N**

150000 **+ / –**  **PV**

1200 **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 7.4084

present value of the annuity payments.

That is, *PV* = $150,000 with *PMT* = $1200

and *n* = 12(20) = 240.

Then *j* = 7.4084% compounded monthly

*i* =  = 

**2nd** **ICONV**

7.4084 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 7.6652

= 0.61737% per month

and 

=  – 1

= 7.67%

15. The interest rate being charged is the discount

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

12 **N**

260  **PV**

26 **+ / –** **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 35.074

rate that makes *PV* = $260 for

*n* = 12 payments of *PMT* = $26. This rate is

*j* = 35.074% compounded monthly

*i* = 

**2nd** **ICONV**

35.074 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 41.300

= 

= 2.9228% per month

and 

= – 1

= 41.30%

17. You can choose between:

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

48 **N**

23498  **PV**

545.79 **+ / –** **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 5.436

(i) borrowing $26,198 at 0% compounded monthly;

(ii) paying a cash price of $23,498.

The interest rate under the option (i) loan is the

interest rate on a loan of $23,498 loan for which

the monthly payment is = $545.79

Therefore, determine *j* for the case

*PV* = $23,498, *n* = 48, *PMT* = $545.79 and *FV* = 0.

The solution is *j* = 5.436% compounded monthly.

Then *i* =  = 0.453% per month and

**2nd** **ICONV**

5.4355 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 5.574



=  – 1

= 5.57%

The effective interest rate for the

“0% financing” loan is 5.57%.

19. The customer can choose between:

(i) borrowing $20,000 at 1.9% compounded monthly;

(ii) taking the $1250 cash rebate and borrowing

$18,750 at market rates.

1.9 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

48 **N**

20000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –433.03

The effective interest rate under the option (i) loan

is the interest rate on an $18,750 loan

that has the same monthly payment.

*Step* 1: Determine the monthly payments under option (i).

*PV* = $20,000, *n* = 48, *i* =  = %

Formula (11-2) gives *PMT* = $433.03.

*Step* 2: Calculate *j* for an option (ii) loan where

*PV* = $18,750, *n* = 48, *PMT* = $433.03.

Same *P/Y, C/Y*

Same *N, PMT, FV*

18750  **PV**

**CPT** **I/Y**

*Ans*: 5.145

The solution is *j* = 5.145% compounded monthly.

*Step* 3: Convert to an effective rate, *f*.

*i* =  = 0.42873% per month and

**2nd** **ICONV**

5.145 **ENTER**

**↑**  12 **ENTER**

**↑**  **CPT**

*Ans*: 5.268

 = – 1 = 5.27%

The effective rate on the “1.9%” loan was 5.27%.

**Review Problems**

1. Given: *FV* = $500,000; *i* =  = 1.875%; *n* = 2(20) = 40

Substitute these values into formula (11-1) giving

$500,000 = *PMT* 

*PMT* = $8504.56

Semiannual investments of $8504.56 are required.

3. The selling price of the annuity represents the present value of the annuity payments.

Hence, *PV* = $100,000 where *PMT* = $802.76 and *n* = 12(20) = 240. Substitute into

formula (11-2) and solve for *i*.

*i* = 0.62114% per month

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

240 **N**

100000 **+ / –** **PV**

0  **FV**

802.76 **PMT**

**CPT** **I/Y**

*Ans*: 7.454

*j = mi* = 12(0.62114%) = 7.45% compounded monthly

 = – 1 = 7.71%

5. The accumulated amount is the future value of the contributions which form an ordinary

annuity. That is, *FV* = $205,064, *PMT* = $2000, *n* = 4(13.75) = 55

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

55 **N**

205064 **+ / –** **FV**

0  **PV**

2000 **PMT**

**CPT** **I/Y**

*Ans*: 8.500

Solving formula (11-1) for *i* gives *i* = 2.125% per quarter.

*j = mi* = 4(2.125%) = 8.50% compounded monthly

Then,

 = – 1 = 8.77%

7. The original loan equals the present value of all payments.

With *PV* = $100,000, *i* =  = 0.5%, and *PMT* = $1000,

 =  = 138.98

Hence, 139 monthly payments are required to pay off the loan. If the payments are

increased to $1050, we obtain *n* = 129.65. That is, only 130 payments will be required

to pay off the loan. Therefore, it takes 139 – 130 = 9 months longer to pay off the loan

with the smaller payments.

9. The purchase price equals the present value of the annuity payments

discounted at the rate of return earned by the invested funds. The payments

form an ordinary general annuity having

*PV* = $175,000, *PMT* = $4000, *i* =  = 2%, *c* =  = 0.5

 = – 1 = 0.009950494

 =  = 57.72

The payments will continue for 58 calendar quarters or 14 years and 6 months.

11. The original loan equals the present value of the loan payments. The payments

form an ordinary general annuity having

*PV* = $90,000, *n* = 2(10) = 20, *i* =  = 0.5625%, *c* =  = 6

 = – 1 = 0.034228184

Solve for *PMT* in formula (11-2),

$90,000 = *PMT* 

*PMT* = $6288.35

13. With *PV* = $65,000, *PMT* = $625, and *i* =  = 0.6%,

 =  = 163.52

Thus, 164 payments are required to satisfy the loan. With *PMT* reduced to $600,

we obtain *n* = 175.49. That is, 176 payments are required. The larger payments will

pay off the loan (176 – 164) months = 12 months = 1 year sooner.

The interest savings are approximately

175.49($600) – 163.52($625) ≈ $3094.

15. The payments form an ordinary general annuity having

*FV* = $15,000, *n* = 4(8.5) = 34, *i* = 7.5%, *c* =  = 0.25, and

 = – 1 = 0.018244601

Substitute into formula (11-1).

$15,000 = *PMT* 

*PMT* = $322.29

Quarterly payments of $322.29 are required.

17. With the focal date at the date of the last $500 contribution,

*FV* of the contributions = *PV* of $10,770.82

For the *PV* calculation, *FV* = $10,770.82, *i* =  = 0.875%, *n* = 4(3) = 12

Then  = $10,770.82 = $9701.66

For the *FV* calculation, *FV* = $9701.66, *PMT* = $500, and *i* = 0.875%

Then  =  = 18.00

Eighteen contributions of $500 were made.

19. Viewed from the date each trust is set up, the payments each child will receive form

a deferred general annuity whose present value is $20,000. That is,

*PV* = $20,000, *n* = 4(15) = 60, *i* =  = 4.625%, *c* =  = 0.5 and

 = – 1 = 0.022863627

Lena will receive her first payments in 5 years. At a focal date 4.75 years from now,

*FV* of $20,000 = *PV* of ordinary annuity

$20,000 = *PMT* 

*PMT* =  = $946.39

Axel will receive his first payment in 9.5 years. At a focal date 9.25 years from now,

*FV* of $20,000 = *PV* of ordinary annuity

$20,000 = *PMT* 

*PMT* = $46,162.17 ÷ 32.471047 = $1421.64

Lena will receive quarterly payments of $946.39 and Axel will receive

quarterly payments of $1421.64.

21. *a*. Use formula (9‑2) to adjust *PV* = $400,000 for *n* = 25 years of

inflation at *i* = 2.5% per year.

 = $400,000 = $741,577.64

Mr. Braun should have $741,577.64 in his RRSP 25 years from now to have

the same purchasing power as $400,000 current dollars.

*b*. The contributions form an ordinary general annuity having

*FV* = $741,577.64, *n* = 4(25) = 100, *i* =  = 3.75%, *c* =  = 0.5, and

 = – 1 = 0.018577439

Hence,

$741,578.64 = *PMT* 

*PMT* = $2598.90

Mr. Braun should make quarterly contributions of $2598.90.

23. The economic value of each annuity is the present value of the payments discounted

at the time value of money. For the 20‑year‑term annuity,

*PMT* = $394, *n* = 12(20) = 240, *i* =  = 0.6%

 = $394 = $50,041.32

Next calculate *n* in order for the life annuity (*PMT* = $440) to have the same present value.

 =  = 191.72

The man must live at least 192 months (16 years) beyond age 70. That is, he must live

to at least age 86 years.

25. With a focal date 2 years and 9 months from today

*FV* of $30,000 = *PV* of ordinary simple annuity

The left side of the equation is

 = $30,000 = $36,307.79

For the *PV* calculation, *PV* = $36,307.79, *i* = 1.75%, and *PMT* = $2000.

Then the number of payments is

 =  = 22.035

There can be 23 withdrawals (with the last one being only about 0.035 × $2000 = $70).

27. The nominal amount desired at age 60 is

 = $250,000 = $443,961.17

This amount represents the future value of his RRSP contributions. That is,

*FV* = $443,961.17 with *n* = 29 and *i* = 8%.

Substituting in formula (11-1),

$443,961.17 = *PMT* 

*PMT* = $4270.26

Justin should make annual contributions of $4270.26.

**Case (Chapter 12)**

***Should You Borrow to Make an RRSP Contribution?***

1. Substitute *PV* = $600, *n* = 12, and *i* = = 0.5% into formula (11-2).



*PMT* = $51.64 per month for 12 months

3. The after-tax cost is 60% of the RRSP contribution. That is,

$51.64 = 0.60(Monthly RRSP contribution)

Monthly RRSP contribution =  = $86.07

5. After another *n* = 12(19) = 228 compounding periods,

 = $1061.72 = $3310.34

The RRSP’s value after 20 years will be $3310.34.

7. If you borrow for a lump RRSP contribution,

 = $1000 = $6009.15 after 20 years

If you contribute the pretax equivalent of the monthly loan payments,

 = $86.07 = $1076.528 after 1 year

 = $1076.528 = $5914.22 after 20 years

In this scenario, you will end up with more in your RRSP if you borrow to make

an initial lump contribution. You should borrow for the RRSP contribution.

9. Borrow to make an RRSP contribution if the interest rate on the loan is lower than the rate of return you will earn on the RRSP investment.