13 Annuities Due

**Point of Interest (Section 13.1)**

***A Painful Truth and a Positive Spin***

1. The percent increase is the overall percent difference between 4 and 10 percent.



3. If 10% of this is invested then the payment made at the beginning of every month is:

10% x $2708.33 = $270.83

If contributions are monthly and interest compounds semi-annually then

PMT = $270.83, n = 20 x 12 = 240, ,  and per month

The future value of investing 10% of after-tax income is:



 $124,660.93

**Concept Questions (Section 13.1)**

1. Insurance premium payments, rent payments, lease payments, newspaper and magazine subscriptions, membership dues.

3. Each payment in an annuity due earns interest for *one more* payment interval than the corresponding payment in an ordinary annuity.

**Exercise 13.1**

1. Given: *PMT* = $1000, *j* = 7%, *m* = 1, *n* = 25, *i* = 7%

7 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

0  **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 63,249.04

*a.* 

= $1000

= $63,249.04

*b.* 

**BGN mode**

Same *I/Y, P/Y, C/Y*

Same *N, PV, PMT*

**CPT** **FV**

*Ans*: 67,676.47

= $1000(1.07)

= $67,676.47

*c*. Percent difference = 

= 7.0%

3. *a.* Given: *PMT* = $100, *n* = 300, *i* =  = %

**BGN mode**

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

300 **N**

0  **PV**

100 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 51,584.33



= $100(1.00)

= $51,584.33

*b.* If instead, *i* =  =%

**BGN mode**

Same *P/Y, C/Y*

Same *N, PV, PMT*

8 **I/Y**

**CPT** **FV**

*Ans*: 95,736.66

= $100

= $95,736.66

5. Given: *PMT* = $10,000/year, *n* = 14,

**BGN mode**

5.1 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

14 **N**

0  **PV**

10000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 207,416.86

*j* = 5.1% compounded annually, *i*  = 5.1%



= $10,0001.051

= $207,416.86

The total value of the portfolio after 14 years

will be $207,416.86.

7. Given: *PMT* = $2500, *n* = 16, *i* =  = 0.4375%,

**BGN mode**

5.25 **I/Y**

**P/Y** 1 **ENTER**

**C/Y** 12 **ENTER**

16 **N**

0  **PV**

2500 **PMT**

**CPT** **FV**

*Ans*: –64,273.29

*c* ==  = 12

 = – 1 = 0.0537818867

= $2500(1.0537818867)

= $64,273.29

Gus’s TFSA will be worth $64,273.29 after 16 years.

9. *a.* Given: *PMT* = $500, *n* = 4(6.5) = 26, *i* =  = 7.6%,

**BGN mode**

7.6 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

26 **N**

0  **PV**

500 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 16,803.44

*c* = =  = 0.25

 = – 1 = 0.01848132

= $500(1.01848132)

= $16,803.44

Amount in mutual fund will be $16,803.44.

*b.* Earnings =$16,803.44 − 26($500) = $3803.44

11. With August 1, 2025 as the focal date, the RRSP contri-

butions form a simple annuity due. The amount in the

RRSP will be the future value of previous contributions.

With *PMT* = $1500, *n* = 2(27.5) = 55, and *i* =  = 4.25%,

**BGN mode**

8.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

55 **N**

0  **PV**

1500 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 326,252.08



= $1500(1.0425)

= $326,252.08

Interest portion = Amount in RRSP – Contributions

= $326,252.08 – 55($1500)

= $243,752.08

13. Given: *PMT* = $2000 every 6 months.

**BGN mode**

8 **I/Y**

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

22 **N**

0  **PV**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 70,551.25

For the first 11 years, *j* = 8% compounded annually.

We have a general annuity due with

*n* = 2(11) = 22, *i* = 8%, *c* =  = 0.5

 = – 1 = 0.039230485



= $2000(1.039230485)

= $70,551.25

For the next 14 years, *j* = 7% compounded semiannually.

**BGN mode**

7 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

28 **N**

70551.25 **+ / –**  **PV**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 280,678.01

We have a simple annuity due with

*n* = 2(14) = 28, *i* = 3.5%

The overall future value is

$70,551.25() + $2000(1.035)

= $184,856.407 + $95,821.599

= $280,678.01

Giorgio’s RRSP is worth $280,678.01 today.

15. *a.* For the monthly contributions, *PMT* = $400,

**BGN mode**

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

300 **N**

0  **PV**

400 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 353,097.50

*n* = 12(25) = 300, *i* =  = 0.625%

 = $400(1.00625)

= $353,097.50

For the annual contributions, *PMT* = $2000,

*n* = 25, *i* = 0.625%, and

*c* ==  = 12

**BGN mode**

Same *I/Y, PV*

**P/Y** 1 **ENTER**

**C/Y** 12 **ENTER**

25 **N**

2000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 152,217.77

and  = – 1 = 0.077632599

 = $2000(1.077632599)

= $152,217.77

The plan will be worth

$353,097.50 + $152,217.77 = $505,315

after 25 years.

*b.* Earnings =$505,315 − 300($400) − 25($2000) = $335,315

17. The amount in Fay’s RRSP on her 31st birthday was

**BGN mode**

8 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

0  **PV**

3000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 46,936.462



= $3000(1.08)

= $46,936.462

Same *I/Y,* *P/Y, C/Y*

34 **N**

46,936.462 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 642,566.44

The future value of this amount at age 65

(34 years later) will be

*FV* = $46,936.462 = $642,566.44

The amount in Fred’s RRSP on his 65th birthday will be

 = $3000(1.08) = $513,950.41

Fay will have

$642,566.44 – $513,950.41 = $128,616.03

more in her RRSP at age 65 than Fred will have at age 65.

**Concept Questions (Section 13.2)**

1. The focal date is at the beginning of the *first* payment interval. Since payments are at the *beginning* of each payment interval, the focal date coincides with the first payment.

3. Since *PV*(due) = *PV* × (1+*i*)

then *PV*(due) is *i*% larger than *PV*. In the particular case at hand, *PV*(due) will exceed *PV* by 3%.

**Exercise 13.2**

4 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

35 **N**

950 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –27,938.15

1. Given: *PMT* = $950, *j* = 4%, *m* = 4,

*n* = 4(8.75) = 35, *i* =  = 1%

*a.* 

= $950

= $27,938.15

*b.* 

**BGN mode**

Same *I/Y, P/Y, C/Y*

Same *N, PMT, FV*

**CPT** **PV**

*Ans*: –28,217.53

= $950(1.01)

= $28,217.53

*c*. Percent difference = 

= 1.0%

3. *a.* Given: *PMT* = $1000, *n* = 2(25) = 50, *i* =  = 2%

**BGN mode**

4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

50 **N**

1000 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –32,052.08



= $1000(1.02)

= $32,052.08

*b.* If instead, *i* =  = 4%

**BGN mode**

Same *P/Y, C/Y, N*

Same *PMT, FV*

8 **I/Y**

**CPT** **PV**

*Ans*: –22,341.47

= $1000(1.04)

= $22,341.47

5. Given: *PMT* = $3000, *n* = 2(4.5) = 9,

Then *i* =  = %, *c* =  = 6 and

**BGN mode**

5 **I/Y**

**P/Y** 2 **ENTER**

**C/Y** 12 **ENTER**

9 **N**

3000 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –24,486.41

 = – 1 = 0.025261868



= $3000(1.025261868)

= $24,486.41

The book value of the lease liability is $24,486.41.

7. *a*. Purchase price = Present value of payments (which form a simple annuity due.)

With *PMT* = $60.26, *n* = 30, *i* =  = 1.5%,

**BGN mode**

18 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

30 **N**

60.26 **+ / –** **PMT**

0  **FV**

**CPT** **PV**

*Ans*: 1468.90



= $60.26(1.015)

= $1468.90

The purchase price was $1468.90.

*b.* Interest paid = 30($60.26) − $1468.90 = $338.90.

9. *a.* The initial lease liability is the present value of all

**BGN mode**

9 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

60 **N**

2700 **+ / –** **PMT**

0  **FV**

**CPT** **PV**

*Ans*: 131,043.62

payments discounted at the cost of borrowing.

The payments form a simple annuity due having

*PMT* = $2700, *n* = 12(5) = 60, and *i* =  = 0.75%

Lease liability = $2700(1.0075)

= $131,043.62

*b.* After the first year, 48 payments remain,

and the lease liability will be

$2700(1.0075) = $109,312.65

**BGN mode**

Same *I/Y, P/Y, C/Y*

Same *PMT, FV*

48 **N**

**CPT** **PV**

*Ans*: 109,312.65

The reduction in the liability during the first year is

$131,043.62 – $109,312.65 = $21,730.97

11. The current economic value of Rosie’s offer

is the present value of her payments.

**BGN mode**

5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

5 **N**

1900 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –9047.75

The payments form a simple annuity due having

*PMT* = $1900, *n* = 5, *i* = =2.5%



= $1900(1.025)

= $9047.75

Rosie Senario’s offer is worth

$9047.75 – $8500 = $547.75 more in current dollars.

13. The amount required in the RRIF is

**BGN mode**

4 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

16 **N**

40000 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –484,735.50

the present value of the withdrawals.

Given: *PMT* = $40,000, *n* = 16, *i* = 4%



= $40,000(1.04)

= $484,735.50

Karsten must have $484,735.50 in his RRIF at age 65.

15. The required amount of money is the present value of the payments. The payments

form a general annuity due having *PMT* = $1200,

**BGN mode**

9 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

180 **N**

1200 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –120,339.78

*n* = 12(15) = 180, *i* =  = 4.5%, *c* =  = 

 = – 1 = 0.007363123



= $1200(1.007363123)

= $120,339.78

$120,339.78 is the initial lump amount required to

sustain the withdrawals.

17.Compare the present values of the alternative payment

streams to determine the lower cost policy. Each payment

**BGN mode**

4.8 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

120 **N**

51.75 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –4944.02

stream forms a simple annuity due with *i* =  = 0.4%.

For the Sun Life option, *PMT* = $51.75/month, *n* = 120

*PV*(due) = $51.75(1.004)

= $4944.02

For the Atlantic Life option,

*PMT* = $44.25/month for the first 5 years, and

*PMT* = $60.35/month for the subsequent 5 years.

**BGN mode**

Same *I/Y, P/Y, C/Y*

Same *FV*

60 **N**

60.35 **PMT**

**CPT** **PV**

*Ans*: –3226.42

The present value, 5 years from now, of the

final 5 years’ payments is

*PV*(due) = $60.35(1.004)

**BGN mode**

Same *I/Y, P/Y*

Same *C/Y, N*

44.25 **PMT**

3226.42  **FV**

**CPT** **PV**

*Ans*: –4904.89

= $3226.42

The present value today of all the premiums is

 + $44.25(1.004)

= $4904.89

Bram will save $39.13 by choosing the Atlantic Life policy.

19. The ranking does not depend on where the focal date is set. The choice of 8 years

from now as the focal date will minimize the calculations required to determine

each stream's economic value on the focal date.

*a.* (i) The $10,000 is already at the focal date.

(ii) Economic value =

**BGN mode**

5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

8 **N**

0  **PV**

850 **PMT**

**CPT** **FV**

*Ans*: –8522.58

= $850(1.05)

= $8522.58

(iii) Economic value =

= $1700(1.05)

= $11,536.83

The $1700 annuity has the largest economic value and the $850 annuity

has the lowest value.

*b*. (i) The $10,000 payment is already at the focal date.

(ii) Economic value = = $850(1.10) = $10,692.56

(iii) Economic value = = $1700(1.10) = $9976.31

The ranking in part *a* is reversed. The $850 annuity now has the largest economic

value and the $1700 annuity has the lowest value.

21. Heath and Company should choose the lease having the lower economic value.

The current economic value of each lease is the present value of the lease payments discounted at the time value of money.

For the lease on the current premises, *PMT* = $2100, *n* = 12(7) = 84, *i* =  = 0.625%.

*PV*(due) = $2100(1.00625) = $137,768.09

For the lease on the new premises, the payments form a deferred ordinary annuity

having *PMT* = $2500, *n* = 12(6) = 72, *i* = 0.625%, and period of deferral = 11 months.

At a focal date 11 months from now,

*PV* = = $2500= $144,591.31

*PV*(today) = *FV* = $144,591.31 = $135,013.55

Therefore, Heath should accept the lease on the new location since it represents

a saving (in current dollars) of

$137,768.09 – $135,013.55 = $2754.54

**Point of Interest (Section 13.3)**

***“Rent-to-Own” or “Rent-Too-Onerous”?***

1. The imputed interest rate paid by the customer is the discount rate that makes

the present value of the weekly rental payments equal to the retail price.

For the Samsung LCD HD TV, retail price = *PV*(due) = $650; *PMT* = $13.66; *n* = 156

The one-week periodic rate is the solution for *i* in



The solution is: *i* = 2.0549% per week.

Therefore, *j = mi* = 52(2.0549%) = 106.86% compounded weekly

and  =  − 1 = 1.8798 = 187.98%

3. With *PV*(due) = $650, *j* = 30%, n = 156, *i* =  = 2.5% and c = = 0.230769231.

 = – 1 = 0.005714561

we have $650 = *PMT* (1.005714561)

Then *PMT* = $6.27

The weekly payment to the finance company would be $6.27.

**Concept Questions (Section 13.3)**

1. If *PMT*, *n*, and *i* are the same for an ordinary annuity and an annuity due, the ordinary annuity will have the smaller *FV*. Therefore, if *FV*, *n*, and *i* are the same, the ordinary annuity has the larger *PMT*.

3. *a.* The lessee should not exercise the purchase option. If the lessee wishes to purchase the vehicle, an equivalent vehicle can be purchased at a lower price in the “used-car market”.

*b.* The lessee should exercise the purchase option. If the lessee does not wish to own the vehicle, it can be sold for more than the residual value in the “used-car market”.

*c.* If the lessee does the rational thing, the lessor *loses*

Residual value – Market value

in case *a,* but *does not gain* or capture the difference

Market value – Residual value

in case *b*. The lessor’s exposure to this market value risk is one reason why the interest rate on a lease contract is normally higher than the interest rate on a loan to purchase the same vehicle.

**BGN mode**

3 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

25 **N**

0  **PV**

750000  **FV**

**CPT** **PMT**

*Ans*: –19,971.75

**Exercise 13.3**

1. Given: *FV*(due) = $750,000, Term = 25 years

*a.* With *j* = 3%, *m* = 1, *n* = 25, solve for *PMT* in

$750,000 = *PMT* (1.03)

*PMT* = $19,971.75

Earnings portion = $750,000 − 25($19,971.75) = $250,706.23

*b.* With *j* = 6%, *m* = 1, *n* = 1(25) = 25, solve for *PMT* in

$750,000 = *PMT* (1.06)

**BGN mode**

Same *N, P/Y, C/Y*

Same *PV, FV*

6 **I/Y**

**CPT** **PMT**

*Ans*: –12,896.26

$750,000 = 58.156383 *PMT*

*PMT* = $12,896.26

Earnings portion = $750,000 − 25($12,896.26)

= $427,593.50

Similarly,

*c.* With *j* = 8%, *m* = 1, *n* = 25, we obtain *PMT* = $9499.15

Earnings portion = $750,000 − 25($9499.15) = $512,521.25

*d.* With *j* = 9%, *m* = 1, *n* = 25, we obtain *PMT* = $8123.57

Earnings portion = $750,000 − 25($8123.57) = $546,910.75

3. Given: *FV*(due) = $600,000, *PMT* = $2500

**BGN mode**

3 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

0  **PV**

2500 **+ / –** **PMT**

600000  **FV**

**CPT** **N**

*Ans*: 137.15

*a.* *j* = 3%, *m* = 4, *i* =  = 0.75%

*n* = 

= 137.15

It will require 138 quarterly contributions. The 138th contribution is required to reach $600,000. The time to the end of the 138th contribution is 138 x 3 months = 414 months or 414/12 = 34 years 6 months. Including accrued interest, the $600,000 is actually reached quite early in the 138th interval.

*b.* *j* = 6%, *m* = 4, *i* =  = 1.5%



**BGN mode**

Same *P/Y, C/Y*

Same *PV, PMT, FV*

6 **I/Y**

**CPT** **N**

*Ans*: 101.72

= 

= 101.72

It will require 102 quarterly contributions. The 102nd contribution is required to reach $600,000. The time to the end of the 102nd contribution is 102 x 3 = 306 or 306/12 = 25 years and 6 months. Including accrued interest, the $600,000 is actually reached in the second half of the 102nd interval

Similarly.

*c.* For *j* = 8%, *m* = 4, *i* =  = 2%, we obtain *n* = 87.94.

It will require 88 quarterly contributions. Therefore, the RRSP reaches $600,000 in 88 x 3 months = 264 months or 264/12 = 22 years after the first contribution.

*d.* For *j* = 9%, *m* = 4, *i* =  = 2.25%, we obtain *n* = 82.58.

It will require 83 quarterly contributions. Therefore, the RRSP reaches $600,000 in 83 x 3 months = 249 months or 249/12 = 20 years, 9 months after the first contribution.

5. Given: *PMT* = $500 monthly, *n* = 12(25) = 300

*a.* For a *FV* of $400,000, *b.* For a *FV* of $500,000, *c.* For a *FV* of $600,000,

**BGN mode**

**P/Y** 12 **ENTER**

(making *C/Y* = *P/Y* = 12)

300 **N**

0  **PV**

500 **+ / –** **PMT**

400000  **FV**

**CPT** **I/Y**

*Ans*: 6.8846

**BGN mode**

Same *I/Y,* *P/Y, C/Y*

Same *N, PV,* *PMT*

600000  **FV**

**CPT** **I/Y**

*Ans*: 9.360

**BGN mode**

Same *I/Y,* *P/Y, C/Y*

Same *N, PV,* *PMT*

500000  **FV**

**CPT** **I/Y**

*Ans*: 8.266

The RRSP must earn (*a*) 6.88% compounded monthly;

(*b*) 8.27% compounded monthly;

(*c*) 9.36% compounded monthly.

7. The present value of the annuity payments is $560,000.

**BGN mode**

6.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

560000 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 47,721.67

Given: *PV*(due) = $560,000, *n* = 20,

*j* = 6.5% compounded annually, *i* = 6.5%

Substitute into formula (13-2),

$560,000 = *PMT*(1.065)

*PMT* = $47,721.67

Nitesh can receive $47,721.67 per year.

9. The future value of the monthly investments is $1,000,000.

**BGN mode**

8 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

300 **N**

0  **PV**

1000000  **FV**

**CPT** **PMT**

*Ans*: –1093.09

Given: *FV*(due) = $1,000,000, *n* = 12(25) = 300,

*i* = = 8%, and *c* =  = 

 = – 1 = 0.00643403

Substitute into formula (13-1),

$1,000,000 = *PMT*(1. 00643403)

*PMT* = $1093.09

Corbin must invest $1093.09 at the

beginning of each month.

11. *a.* Given: *PV*(due) = $200,000, *n* = 20, *i* = 6%

**BGN mode**

6 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

20 **N**

200000 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 16,449.92

Substitute into formula (13-2) giving

$200,000 = *PMT* (1.06)

*PMT* = $16,449.92

*b.* Given: *PV* = $200,000, *n* = 20, *i* = 6%

Substitute into formula (11-2) giving

**END mode**

Same *I/Y,* *P/Y, C/Y*

Same *N,* *PV, PMT, FV*

**CPT** **PMT**

*Ans*: 17,436.91

$200,000 = *PMT* 

*PMT* = $17,436.91

13. We need to find the combined future value of the $20,000

**BGN mode**

7.25 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

15 **N**

20000 **+ / –**  **PV**

5000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 194,524.43

already accumulated and the additional $5000/year

to be invested after the first 15 years.

Given: *PV* = $20,000, *n* = 15, *PMT* = $5000

*j = i* = 7.25%

*FV*(due) = $20,000 + $5000(1.0725)

= $57,146.483 + $137,377.942

= $194,524.43

The payments in the next 15 years form a general annuity with PV(due) = $194,524.43, n = 12(15) = 180 and i = =0.375%

$194,524.43 = PMT 

**BGN mode**

4.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

180 **N**

194524.43  **PV**

0 **FV**

**CPT** **PMT**

*Ans*: 1,482.54

$194,524.43 = PMT (131.2103014)

PMT = $1482.54

The investment will sustain withdrawals of $1482.54 for 15 years.

15. The present value of the lease payments discounted

**BGN mode**

13.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

36 **N**

8500 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 285.24

at the lessor’s required rate of return equals

the capital cost of the equipment. Substitute

*PV*(due) = $8500, *n* = 12(3) = 36, *i* =  = 1.125%

into formula (13-2)

$8500 = *PMT* (1.01125)

*PMT* = $285.24

The lease payments will be $285.24 at the beginning

of each month for 3 years.

17. With the contributions at the beginning of each year,

**BGN mode**

9 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

27 **N**

0  **PV**

500000  **FV**

**CPT** **PMT**

*Ans*: –4465.55

*FV*(due) = $500,000, *n* = 27, *i* = 9%

$500,000 = *PMT* (1.09)

*PMT* = $4465.55

With the contributions at the end of each year,

*FV* = $500,000, *n* = 27, *i* = 9%

$500,000 = *PMT* 

**END mode**

All values

unchanged

**CPT** **PMT**

*Ans*: –4867.45

*PMT* = $4867.45

The annual contributions must be

$4867.45 – $4465.55 = $401.90

larger if made at the end of the year instead of the beginning.

19. Initial lease liability = Present value of all lease payments.

The lease payments form a simple annuity due having

*PV*(due) = $43,000, *n* = 12(5) = 60, *i* =  = 0.625%

$43,000 = *PMT* (1.00625)

*PMT* = $856.28

The monthly lease payment is $856.28.

21. 

With Down payment = $1545, *i* = ,

*PMT* = $219.40, *n* = 48, and Residual value = $6815.

**BGN mode**

3.9 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y =* 12)

48 **N**

219.40 **+ / –** **PMT**

6815 **+ / –**  **FV**

**CPT** **PV**

*Ans*: 15,600.01

Therefore,

MSRP – $1545

= $219.40 + $6815

= $15,600.01

MSRP = $15,600 + $1545 = $17,145

23. 

With the MSRP = $21,550, Down payment = $1425,

**BGN mode**

6.9 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

20125  **PV**

248 **+ / –** **PMT**

14794 **+ / –**  **FV**

**CPT** **N**

*Ans*: 36.00

*PMT* = $248, *i* = , Residual value = $14,794

we have

$21,550 – $1425

= $248 + $14,794

Using the financial calculator functions,

we obtain *n* = 36.00

The term of the lease is 36 months.

25. 

With Purchase price = $58,499, *PMT* = $697,

**BGN mode**

1.9 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y =* 12)

48 **N**

697 **+ / –** **PMT**

21000 **+ / –**  **FV**

**CPT** **PV**

*Ans*: 51,707.03

*i* = %, *n* = 48, and Residual value = $21,000,

$58,499 – (Down payment)

= $697

+ $21,000

= $51,707.03

Down payment = $58,499 – $51,707.03

= $6791.97 = $6792

27. The future value of Kim's contributions must be $15,000.

The contributions form a simple annuity due having

*FV*(due) = $15,000, *PMT* = $700, and *i* = = 0.35%.

**BGN mode**

4.2 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

0  **PV**

700 **+ / –** **PMT**

15000  **FV**

**CPT** **N**

*Ans*: 20.629

The number of contributions will be



= 

= 20.629

After 20 months, *FV*(due) = $14,526.09. Therefore, the 21st deposit (of $700)

occurring 20 months = 1 year and 8 months from today will reach the goal.

29. The purchase price is the present value of the remaining payments discounted

at the investor’s required rate of return. Viewed from the date of purchase,

the payments form a simple annuity due with

*PV*(due) = $13,372, *PMT* = $500, *i* =  = 0.8125%

**BGN mode**

9.75 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

13372 **+ / –**  **PV**

500 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 30.00

The number of remaining payments is



= 

= 30.00

The investor will receive 30 payments.

31. The monthly deposits form a simple annuity due whose future value is to be $100,000.

With *FV*(due) = $100,000, *PMT* = $220,

**BGN mode**

5.4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

0  **PV**

220 **+ / –** **PMT**

100000  **FV**

**CPT** **N**

*Ans*: 247.36

and *i* =  = 0.45%,



= 

= 247.36

That is, 248 deposits of $220 will be needed to accumulate $100,000. If the monthly deposits are only $200,

**BGN mode**

Same *I/Y,* *P/Y, C/Y*

Same *PV,* *FV*

200 **+ / –** **PMT**

**CPT** **N**

*Ans*: 261.82

*n* =  = 261.82

That is, 262 deposits of $200 will be needed to surpass

$100,000. Therefore, (262 – 248) = 14 fewer deposits

of $220 will be required.

33. The contributions form an annuity due whose

**BGN mode**

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

32 **N**

0  **PV**

2500 **+ / –** **PMT**

223000  **FV**

**CPT** **I/Y**

*Ans*: 11.54

future value is $223,000. That is,

*FV*(due) = $223,000 with *PMT* = $2500

and *n* = 2(16) = 32.

To directly obtain the effective rate of interest,

calculate *j* with *m* = *C/Y* = 1

Then *f = j* = 11.54%

35. The amount in the RRSP represents the future

**BGN mode**

**P/Y** 4 **ENTER**

(making *C/Y* = *P/Y* = 4)

80 **N**

0  **PV**

1500 **+ / –** **PMT**

327680  **FV**

**CPT** **I/Y**

*Ans*: 8.800

value of the contributions.

Hence, *FV*(due) = $327,685 with

*PMT* = $1500 and *n* = 4(20) = 80.

obtain *j* = 8.80% compounded quarterly.

**2nd** **ICONV**

8.8 **ENTER**

4 **ENTER**

**↑**  **CPT**

*Ans*: 9.0947

Then *i* = 2.2%, and



= – 1

= 9.09%

37. The purchase price is the present value

**BGN mode**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

12 **N**

1195 **+ / –**  **PV**

110 **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 24.793

of the payments. That is,

*PV*(due) = $1195, *PMT* = $110/month,

and *n* = 12.

To directly obtain the effective rate

of interest, calculate *j* with *m* = *C/Y* = 1

Then *f = j* = 24.79%

39. The present value of the lease payments discounted at

**BGN mode**

15 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 4 **ENTER**

60 **N**

25000 **+ / –** **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 585.116

the required rate of return must equal the capital cost

of the equipment.

*a*. *PV*(due) = $25,000, *n* = 12(5) = 60,

*i* =  = 3.75%, *c* =  = 

 = – 1 = 0.012346926

$25,000 = *PMT* (1.012346926)

*PMT* = $585.12

The monthly lease payment is $585.12.

*b.* *PV*(due) = $25,000, *n* = 2(5) = 10, *i* = 3.75%, *c* =  = 2

**BGN mode**

Same *I/Y,* *PV, FV*

**P/Y** 2 **ENTER**

**C/Y** 4 **ENTER**

10 **N**

**CPT** **PMT**

*Ans*: 3405.377

 = – 1 = 0.07640625

$25,000 = *PMT* (1.07640625)

*PMT* = $3405.38

The semiannual lease payment is $3405.38.

41. Present value of installment payments = $1900.

**BGN mode**

6 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 2 **ENTER**

4 **N**

1900 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 485.58

For payments at the beginning of each quarter,

*PV*(due) = $1900, *n* = 4, *i* =  = 3.0%, *c* =  = 0.5,

 = – 1 = 0.014889157

$1900 = *PMT* (1.014889157)

*PMT* = $485.58

For payments at the beginning of each month,

*PV*(due) = $1900, *n* = 12, *i* =  = 3.0%, *c* =  = 

Same *I/Y,* *PV, FV*

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

12 **N**

**CPT** **PMT**

*Ans*: 162.66

 = – 1 = 0.004938622

$1900 = *PMT* (1.004938622)

*PMT* = $162.66

The payments are $485.58 quarterly or $162.66 monthly.

43. At a focal date 7 years from now,

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

84 **N**

10000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 13,225.14

*FV* of the $10,000 lump investment = *PV* of withdrawals

Viewed from the focal date, the withdrawals form a

simple annuity due. The future value of the $10,000 is

 = $10,000 = $13,225.14

With *PV*(due) = $13,225.14, *PMT* = $300, *i* = %,



**BGN mode**

Same *I/Y,P/Y, C/Y*

13225.14 **+ / –**  **PV**

300 **PMT**

0  **FV**

**CPT** **N**

*Ans*: 47.59

= 

= 47.5872

There can be 48 withdrawals with the last one being

less than $300. The last payment will occur 47 months

(3 years and 11 months) after the grandson starts college.

45. The future value of the regular $12,000 “draws” is $1,000,000. Hence,

*FV*(due) = $1,000,000, *PMT* = $12,000,

**BGN mode**

8.2 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 4 **ENTER**

0  **PV**

12000 **PMT**

1000000 **+ / –**  **FV**

**CPT** **N**

*Ans*: 65.910

*i* =  = 2.05%, *c* =  = 

 = – 1 = 0.006787164

The number of draws will be



= 

= 65.91

The credit limit will be reached in the 66th month. That is,

it will take 5 years and 6 months to reach the credit limit.

47. The four quarterly payments form an annuity due. The

interest rate being charged by the golf club is the discount

**BGN mode**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

4 **N**

3428  **PV**

898.80 **+ / –** **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 13.737

rate that makes the present value of the payments equal

to the single membership payment of $3428.

Given: *PV*(due) = $3428, *PMT* = $898.80 and *n* = 4

To directly obtain the effective rate

of interest, calculate *j* with *m* = *C/Y* = 1

Then *f = j* = 13.74%

49. By paying $159.80 now, the subscriber avoids paying

**BGN mode**

**P/Y** 1 **ENTER**

**C/Y** 1 **ENTER**

3 **N**

159.80  **PV**

63.80 **+ / –** **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 21.263

$63.80 at the beginning of each of the next 3 years.

The “return on investment” is the discount rate that

makes the present value of *n* = 3 payments of

*PMT* = $63.80 equal to *PV*(due) = $159.80.

To directly obtain the effective rate

of interest, calculate *j* with *m* = *C/Y* = 1

Then *f = j* = 21.26%

51. In each case, the interest rate being charged is the

discount rate that makes the present value of 1 year’s

premium payments equal to *PV*(due) = $666.96.

*a. PMT* = $341.32, *n* = 2 *b*. *PMT* = $173.62, *n* = 4 c. *PMT* = $58.85, *n* = 12

**BGN mode**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

12 **N**

58.85 **PMT**

**CPT** **I/Y**

*Ans*: 13.447

**BGN mode**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

4 **N**

173.62 **PMT**

**CPT** **I/Y**

*Ans*: 11.523

**BGN mode**

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

2 **N**

666.96 **+ / –**  **PV**

341.32 **PMT**

0  **FV**

**CPT** **I/Y**

*Ans*: 9.862

*f* = 11.52% *f* = 13.45%

*f* = 9.86%

53. With today as the focal date,

**BGN mode**

10 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

40 **N**

0  **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 69,087.62

*FV* of RRSP contributions = *PV* of the RRIF withdrawals

The amount in the RRSP 10 years ago was



= $1000(1.025)

= $69,087.62

The amount in the RRSP today is

$69,087.62 + $1000(1.03)

**BGN mode**

Same *P/Y, C/Y, N*

12 **I/Y**

69,087.62 **+ / –**  **PV**

1000 **+ / –** **PMT**

**CPT** **FV**

*Ans*: 303,029.72

= $303,029.72

For the RRIF withdrawals,

**BGN mode**

8.25 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

180 **N**

303,029.72 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 2919.74

*PV*(due) = $303,029.72, *n* = 12(15) = 180,

*i* =  = 0.6875%

The maximum beginning-of-month withdrawals

is the value of *PMT* satisfying

$303,029.72 = *PMT* (1.006875)

*PMT* = $2919.74

55. At a focal date on the purchase date of the annuity,

*FV* of RRSP contributions = *PV* of annuity payments

For the *PV* calculation, *PMT* = $3509, *n* = 12(25) = 300, *i* =  = 4%, *c* = .

 = – 1 = 0.006558197

 $3509(1.006558197) = $462,781.77

On the left side of the word equation, *FV*(due) = $462,781.77, *PMT* = $2500, *i* = 8%,

*c* =  = 0.5, and  = – 1 = 0.039230485. Hence,

 =  = 54.00

Therefore, Mr. van der Linden contributed to his RRSP for 54 half years = 27 years.

**Exercise 13.4**

1. *Step* 1: Calculate the original payment size using the idea that the original loan equals

the present value of the payments.

Substitute *PV* = $20,000, *n* = 120, and *i* =  = 0.% into formula (11-2).

$20,000 = *PMT* 

*PMT* = $232.22

*Step* 2: Calculate the balance after 1 year using the idea that the original loan equals

the combined present value of the remaining 108 payments. That is,

Balance = $232.22

Balance = $18,568.28

*Step* 3: Calculate the number of monthly payments of 1.15($232.22) = $267.05

required to pay off the loan. Their present value is $18,568.28.

 =  = 89.44

The total time required to pay off the loan will be

(12 + 90) months = 8 years and 6 months.

Therefore, the loan will be paid off 1 year and 6 months sooner.

3. The combined future value of the initial $67,000 and the semiannual contributions

is to be $500,000. The contributions form an ordinary general annuity having

*PMT* = $4000, *i* =  = 1.25%, and *c* =  = 2.

**END mode**

**P/Y** 2 **ENTER**

**C/Y** 4 **ENTER**

5 I/Y

67000  **PV**

4000 **PMT**

500000 **+ / –**  **FV**

**CPT** **N**

*Ans*:43.07

 = – 1 = 0.02515625 and

$67,000 + $4000 = $500,000

We obtain *n* = 43.07.

Therefore, Sheila must make 44 contributions requiring 22 years.

5. The combined future value, 10 years from now, of the initial $133,000 and the semiannual

contributions is to be $350,000. At a focal date 7 years from now,

*FV* of $133,000 and RRSP contributions = *PV* of $350,000

The right side of the equation is

 = $350,000 = $300,193.81

The RRSP contributions form an ordinary general annuity having

*n* = 2(7) = 14, *i* = 5.25%, and *c* =  = 0.5

**END mode**

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

5.25 I/Y

133000  **PV**

14 **N**

300,193.81 **+ / –** **FV**

**CPT** **PMT**

*Ans*:6612.60

 = – 1 = 0.025914226

Putting the initial word equation into mathematics,

$133,000 + *PMT*  = $300,193.81

Solving for *PMT* gives *PMT* = $6612.60

Natalie must contribute $6612.60 every 6 months to reach her goal.

7. If the economic value, 5 years from now, of today's purchase price plus the annual

property taxes is less than the future purchase price, it is better to buy the property now.

The annual property taxes form an ordinary general annuity having

*PMT* = $9000, *n* = 5, *i* =  = 6%, *c* =  = 2, and

 = – 1 = 0.1236000

The combined future value, 5 years from now, of the purchase price and the property

tax payments is

$450,000 + $9000 = $863,467

The price would have to exceed $863,467 five years from today for it to be financially

advantageous to purchase the property today.

9. The amount in the RRSP after 25 years will be the future value of all contributions.

For the first 10 years, the contributions form an ordinary general annuity having

*PMT* = $1000, *n* = 4(10) = 40, *i* =  = 4.25%, *c* =  = 0.5, and

 = – 1 = 0.021028893.

The amount in the RRSP after 10 years will be

$1000 = $61,767.70

For the last 15 years, the contributions form an ordinary general annuity having

*PMT* = $1000, *n* = 12(15) = 180, *i* = 4.25%, *c* = , and

 = – 1 = 0.006961062.

The combined future value, 25 years from now, of these contributions

and the $61,767.70 lump amount will be

$61,767.70 + $1000 = $572,376.63

Gayle will have $572,376.63 in her RRSP after 25 years.

11. Sum of the current loan balances = *PV* of payments on the consolidated loan

*Step* 1: Calculate the monthly payment on the $8500 loan.

*PV* = $8500, *n* = 12(3) = 36, *i* =  = 0.875%

$8500 = *PMT* 

*PMT* = $276.27

*Step* 2: Calculate this loan’s balance after 11 payments (with 25 payments remaining).

Balance = $276.27 = $6179.37

*Step* 3: Calculate the balance on the second loan.

*PMT* = $313.69, *n* = 38 payments remain, *i* =  = 4.75%, *c* =  = 

 = – 1 = 0.007764383

Balance = $313.69 = $10,288.32

*Step* 4: Calculate the monthly payment on the consolidated loan.

Initial loan = $6179.37 + $10,288.32 = $16,467.69.

With *PV* = $16,467.69, *n* = 60, and *i* = 0.6875%, solve for *PMT* in

$16,467.69 = *PMT* 

*PMT* = $335.88

The monthly payment on the consolidated loan would be $335.88.

13. *a.* The total of the nominal costs is

6($12,000) + 6($11,000) + 5($10,000) + 2($15,000) = $218,000

*b.* The economic value, at the date of birth, of all costs is their present value.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | *Monthly* | *No. of* |
|  | *Years* | *cost ($)* | *months* |
|  | 1- 6 | 1000 | 72 |
|  | 7-12 | 916.67 | 72 |
|  | 13-17 | 833.33 | 60 |
|  | 18-19 | 1250 | 24 |

The present value, at the beginning of the 18th year, of the last 2 years’ costs is

 = $1250 = $28,203.58

The present value, at the beginning of the 13th year, of the last 7 years’ costs is

$833.33 +  = $64,013.81

The present value, at the beginning of the 7th year, of the last 13 years’ costs is

$916.67 +  = $100,012.42

The present value, at the birth date, of all the costs is

$1000 +  = $130,178.43

Rounded to the nearest dollar, the economic value, on the birth date, is $130,178.

*c.* The economic value, at age 19, of the expenditures is the future value of all

the expenditures. This is most easily determined by calculating the future value,

19 years later, of the result from part *b*.

 = $130,178.43 = $405,883

15. The price that Afco will pay is the present value, on the date of the sale, of the

6 payments discounted at 18% compounded semiannually.

*Step* 1: Calculate the customer's monthly payments. Since no interest is charged during

the first 6 months, $1800 is owed when the first of the 6 monthly payments is

made. Then, *PV*(due) = $1800 with *n* = 6 and *i* =  = 1.25%.

$1800 = *PMT* (1.0125)

*PMT* = $309.39

*Step* 2: Calculate the present value, 6 months from the date of sale, discounting

the payments at  = – 1 = 0.014466592

 $309.39(1.014466592) = $1791.42

*Step* 3: Calculate the present value, on the date of sale, of the *Step* 2 amount.

 = $1791.42 = $1643.50

Pioneer will receive $1643.50 from Afco on the sale of the contract.

17. With the focal date at Reg’s 60th birthday,

*FV* of RRSP contributions = *PV* of all annuity payments

The right side can be calculated in two steps.

*Step* 1: Calculate the present value at age 68 of the 20‑year annuity having

*PMT* = $6000, *n* = 12(20) = 240, and *i* =  = 0.625%. Then

*PV* = $6000 = $744,792.79

*Step* 2: Calculate the *PV* at age 60 of the 8‑year annuity (*n* = 96) and the *Step* 1 result.

*PV* = $5000 +  = $769,645.01

*Step* 3: For the left side of the initial word equation,

*FV*(due) = $769,645.01, *n* = 4(30) = 120, *i* = 0.625%, *c* =  = 3, and

 = – 1 = 0.018867432. Solve for *PMT* in

$769,645.01 = *PMT* (1.018867432)

*PMT* = $1692.37

The quarterly RRSP contributions must be $1692.37.

19. The car buyer will be indifferent between the alternatives if the payments

during the 4-year term of each loan are the same.

*Step* 1: Calculate the monthly payment on $35,000 financed at 1.9%

compounded monthly for 4 years.

Solve for *PMT* when *PV* = $35,000, *n* = 48, and *i* =  = %.

$35,000 = *PMT* 

*PMT* = $757.80

*Step* 2: Calculate the initial 4-year loan at 6.6% compounded monthly that

would have the same payments. This amount is the present value of

the payments discounted at *i* =  = 0.55%.

Initial loan = $757.80 = $31,892.46

*Step* 3: Calculate the cash rebate. For indifference, the rebate must reduce

the net price to $31,892.46.

Rebate = $35,000 – $31,892.46 = $3107.54

21. *Step* 1: Calculate the amount in the RRSP at age 58.

*PMT* = $5000, *n* = 2(12) = 24, *i* =  = 0.625%, *c* =  = 6, and

 = – 1 = 0.0380908432

The combined *FV* of the initial $97,000 and subsequent contributions will be

*FV* = $97,000 + $50001.0380908432

= $237,914.21 + $197,955.62

= $435,869.83

*Step* 2: Calculate the amount in the RRSP at age 63 (just before the purchase of

the second annuity). This will be the future value, after a further 5 years, of half

the *Step* 1 result. That is,

 = $217,934.91 = $316,723.59

*Step* 3: Calculate the amount available at age 68 to purchase the third annuity. This will

be the future value, after a further 5 years, of half the *Step* 2 result. That is,

 = $158,361.80 = $230,146.31

*Step* 4: Calculate the payments in the first annuity purchased for $217,934.91.

Solve for  in

$217,934.91 = 

 = $1755.67 per month

*Step* 5: Calculate the payments in the second annuity purchased for $158,361.80.

Solve for  in

$158,361.80 = 

 = $1275.75

*Step* 6: Calculate the payments in the third annuity purchased for $230,146.31.

Solve for  in

$230,146.31 = 

 = $1854.04

*Step* 7: Calculate the total monthly incomes at ages 65 and 70.

Monthly income at age 65 =  +  = $3031.42

Monthly income at age 70 =  +  +  = $4885.46.

**Review Problems**

1. *a.* Initial liability = Present value of all lease payments

With *PMT* = $1900, *i* =  = 0.5208%, *n* = 12(5) = 60

Initial liability = $1900(1.005208) = $98,198.89

*b*. Liability after the first year = Present value of the remaining 48 payments

= $1900(1.005208)

= $80,928.41

The reduction in the liability during the first year will be

$98,198.89 – $80,928.41 = $17,270.48

3. The $300,000 in the fund represents the present value of the withdrawals.

*a*. Given: *PV*(due) = $300,000, *n* = 25, *i* = 7.75%

Substitute into formula (13-2) and solve for *PMT*.

$300,000 = *PMT* (1.0775)

*PMT* = $25,527.54

*b*. *PV* = $300,000, *n* = 25, *i* = 7.75%

Substitute into formula (11-2) and solve for *PMT*.

$300,000 = *PMT* 

*PMT* = $27,505.93

5. The contributions form an annuity due having

*FV*(due) = $316,000, *PMT* = $3500, *n* = 2(17) = 34

Substitute into formula (13-1). Solve for *i* using the trial-and-error method of

Appendix 11B. We obtain *i* = 5.0513% per half year. Therefore,

 = – 1 = 10.36%

**BGN mode**

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

34 N

0  **PV**

3500 **PMT**

316000 **+ / –**  **FV**

**CPT** **I/Y**

*Ans*:43.07

7. Choose the payment plan with the lower economic value.

For the monthly premiums, *PMT* = $33.71, *n* = 12, and *i* =  = 0.625%

*PV*(due) = $33.71(1.00625) = $390.98

This is $3.48 more than the single payment falling on the focal date.

Therefore, choose the single payment plan.

9. The contributions form a simple annuity due.

*a*. To accumulate *FV*(due) = $500,000 with *PMT* = $5000 and *i* = 5%,

 =  = 35.89

That is, 36 contributions are required.

*b.* To accumulate $1,000,000, we similarly obtain *n* = 48.24. That is, 49 contributions

in total will be required. Therefore, only 49 – 36 = 13 additional contributions are

required to accumulate the second $500,000.

11. The present value of the lease payments discounted at the required rate of return must

equal the capital cost of the equipment. The proposed lease payments form a general

annuity due having

*PV*(due) = $7650, *n* = 12(4) = 48, *i* =  = 3.75%, *c* = 

 = – 1 = 0.012346926

Substitute into formula (13-2) and solve for *PMT*.

$7650 = *PMT* (1.012346926)

*PMT* = $209.61

The beginning‑of‑month lease payment should be $209.61.

13. The contributions form a simple annuity due having *FV*(due) = $200,000 and *PMT* = $300.

If *i* =  = 0.625%,

 =  = 262.77

That is, 263 contributions will be required.

If *i* =  = , we similarly obtain *n* = 246.31.

That is, 247 contributions will be required. Therefore, at the lower rate of return,

263 – 247 = 16 more contributions will be required.

15. The interest rate being charged is the discount rate that makes the present value

of the payments equal to the purchase price. The payments form an annuity due having

*PV*(due) = $1395, *n* = 12, and *PMT* = $125.

Using the calculator’s financial functions to obtain *j* = 16.176% compounded monthly.

Alternatively, substitute into formula (13-2) and solve for *i* using the trial-and-error

method of Appendix 11B to obtain *i* = 1.348% per month.

**BGN mode**

**P/Y** 12 **ENTER**

**C/Y** 1 **ENTER**

12 N

0  **FV**

125 **PMT**

1395 **+ / –**  **PV**

**CPT** **I/Y**

*Ans*:17.43

Then  = – 1 = 17.43%

17. *a.* The contributions form a simple annuity due having

*FV*(due) = $1,000,000, *n* = 31, *i* = 8%

Substitute into formula (13-1) and solve for *PMT*.

$1,000,000 = *PMT* (1.08)

*PMT* = $7506.74 on each birthday

*b*. If instead, *n* = 26, we similarly obtain *PMT* = $11,580.67

19. The total amount in the RRSP after 30 years will be the future value of all contributions.

The future value, 10 years from now, of the $4000 contributions will be

*FV*(due) = $4000(1.0825) = $63,476.43

The future value, 30 years from now (an additional *n* = 20 years), of this amount

and the $6000 contributions will be

$63,476.43 + 6000(1.0825) = $615,447.79

21. With the focal date at Ms. Bowers 62nd birthday,

*FV* of RRSP contributions = *PV* of the annuity payments

Viewed from the focal date, both payment streams form simple annuities due.

For the *PV* calculation,

*PMT* = $3500, *n* = 12(20) = 240, *i* =  = 0.4%

*PV*(due) = $3500(1.004) = $541,483.87

For the RRSP contributions, *FV*(due) = $541,483.87, *n* = 4(27) = 108, and *i* =  = 2%.

Substitute into formula (13-1) and solve for *PMT*.

$541,483.87 = *PMT* (1.02)

*PMT* = $1417.86

Each quarterly contribution should be $1417.86.

23. *a.* The single equivalent payment at the beginning of the year is the present

value of the schedule payments.

*PV*(due) = $1000 = $11,572.42

The landlord should accept a single payment of $11,572.42.

*b*. Compare the amounts accumulated after 1 year under the two alternatives

if all rental payments are invested at 8% compounded monthly.

For the normal monthly payments

*FV*(due) = $1000 = $12,532.93

For the alternative initial “lump” payment of $11,572.42

*FV* = $11,572.42 = $12,532.93

In either case, the landlord will be in the same financial position at the end of the year.

25. The amount required to purchase the annuities is the present value, on the

purchase date, of all payments.

*Step* 1: Calculate the present value, 5 years from now, of the $2500‑payment annuity.

The $2500 payments form a general annuity due having

*PMT* = $2500, *n* = 12(15) = 180, *i* =  = 1.4%, *c* =  = , and

 = – 1 = 0.0046450567

*PV*(due) = $2500(1.0046450567) = $305,914.00

*Step* 2: Calculate today's present value of the amount from *Step* 1 and

the $4000‑payment annuity.

The $4000 payments form an ordinary simple annuity having

*PMT* = $4000, *n* = 4(5) = 20, *i* = 1.4%

*PV*(Today) =  + $4000= $301,010.58

The amount required to purchase the annuity is $301,010.58.

27. At a focal date at 25 years from now,

*FV* of the RRSP contributions = *PV* of the annuity payments

Both cash flow streams are simple annuities due.

For the RRSP contributions, *PMT* = $2500, *n* = 50, *i* =  = 4%, and

*FV*(due) = $2500(1.04) = $396,934.42

For the income annuity, *PV*(due) = $396,934.42, *PMT* = $2500, and *i* =  = 0.425%.

 =  = 262.80

The annuity will deliver 263 payments with the last payment occurring 262 months

or 21 years and 10 months after the start of the annuity.

29. The implied interest rate is the discount rate that makes the present value of four

3-month memberships equal to the price of a one-year membership.

Substitute *PV*(due) = $500, *n* = 4, and *PMT* = $160 into formula (13-2) or use the calculator TVM function. Solve for *i* using the trial-and-error method of Appendix 11B. We obtain *i* = 19.443% per quarter. Then

**BGN mode**

**P/Y** 4 **ENTER**

**C/Y** 1 **ENTER**

4 N

0  **FV**

160 **PMT**

500 **+ / –**  **PV**

**CPT** **I/Y**

*Ans*:103.54

 = – 1 = 103.54%

**Case (Chapter 13)**

***A “Lotto” Money***

1. Nominal sum = 30($24,946,667) = $748,400,010.

When Powerball reports a $748.4 million jackpot, most people will think of it in terms of *current* dollars. Therefore, it is misleading to promote this figure, particularly when the prize’s cash value ($477 million) is known.

3. With *PMT* = $24,946,700, *n* = 30, and *i* =  = 6%, the initial economic value is

*PV*(due) = $24,946,700(1.06) = $363,990,340

In this case, choose the lump payment option because it has the greater economic value.

5. As we saw in the answer to Question 1, MSLA reports the simple sum of the *nominal* payments. For the 25-year annuity, this would be

25($27,852,324) = $696,308,100