14 Annuities: Special Situations

**Point of Interest** **(Section 14.1)**

***Card Tricks—The “No-Interest, No-Payments-for-a-Year” Trap***

Given: *PV* = $5000; *j* = 28.8% compounded monthly; *n* = 12

Total amount owed after 1 year = $5000 = $6646.14

Interest portion = $6646.14 – $5000 = $1646.14

**Concept Questions (Exercise 14.1)**

1. a) For a deferred ordinary annuity, the period of deferral ends one payment interval before

the first payment. In the present case,

Period of deferral = 3.5 years – One payment interval

= 3.5 years – 3 months

= 3 years and 3 months

b) For a deferred annuity due, the period of deferral ends immediately before

the first payment. In the present case,

Period of deferral = 3.5 years

= 3 years and 6 months

**Exercise 14.1**

1. Given: *PMT* = $2000, *i* =  = 3.5%,

7 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

10 **N**

2000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –16,633.211

*n* = 10, *d* = 2(5) = 10

*PV*(5 years from now) = 

= $2000

= $16,633.211

Same *I/Y,* *P/Y, C/Y*

10 **N**

0 **PMT**

16633.211 **FV**

**CPT** **PV**

*Ans*: −11,791.60

*PV*(today) = 

= $16,633.211

= $11,791.60

3. Given: *PMT* = $500, *i* =  = 0.%,

4 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

42 **N**

500 **PMT**

0  **FV**

**CPT** **PV**

*Ans*: –19,565.90

*n* = 12(3.5) = 42, *d* = 12(2.75) = 33

*PV*(2.75 years from now) = 

= $500

= $19,565.90

Same *I/Y,* *P/Y, C/Y*

33 **N**

0 **PMT**

19565.90  **FV**

**CPT** **PV**

*Ans*: −17,531.01

*PV*(today) = 

= $19,565.90

= $17,531.01

An investment of $17,531.01 today will support the deferred annuity payments.

5. Given: *i* =  = 1.0%; *n* = 2(10) = 20; *c* =  = 2;

4 **I/Y**

**P/Y** 2 **ENTER**

**C/Y** 4 **ENTER**

20 **N**

2000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −32,671.33

*PMT* = $2000; *d* = 2(5) = 10

 = – 1 = 0.0201



= $2000

= $32,671.33

Same *I/Y*, *P/Y, C/Y*

10 **N**

0 **PMT**

32671.33 **FV**

**CPT** **PV**

*Ans*: −26,775.61

*PV*(today) = *FV*

= $32,671.33

= $26,775.61

7. Given: *PMT* = $500; *i* =  = 2.25%; *n* = 12(3.5) = 42;

9 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 4 **ENTER**

42 **N**

500 **+ / –** **PMT**

0 **FV**

**CPT** **PV**

*Ans*: 17,977.076

*d* = 12(2.75) = 33; *c* = 

 = – 1 = 0.007444443

*PV*(2.75 yrs from now) = *PMT* 

= $500

= $17,977.076

Same *I/Y*, *P/Y, C/Y*

33 **N**

0 **PMT**

17977.076 **+ / –** **FV**

**CPT** **PV**

*Ans*: 14,074.16



= $17,977.076

= $14,074.16

The original loan was for $14,074.16.

9. Today’s economic value of the future payments is

6.5 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

44 **N**

1500 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −46,891.229

their present value (discounted at the time value of

money). Viewing the payments as a deferred

ordinary simple annuity,

*PMT* = $1500, *n* = 4(11) = 44, *d* = 4(5.75) = 23,

and *i* =  = 1.625%

*PV*(5.75 years from now) = 

= $1500

Same *I/Y,* *P/Y, C/Y*

23 **N**

0 **PMT**

46891.229 **FV**

**CPT** **PV**

*Ans*: −32,365.24

= $46,891.229

*PV*(today) = 

= $46,891.229

= $32,365.23

11. Given: *PMT* = $1500, *i* = 7.9%, *n* = 1(8) = 8, and *PV* = $6383.65

At a focal date at the end of the period of deferral,

*FV* of $6383.65 = *PV* of ordinary annuity

$6383.65 = $1500 = $8652.77

Using formula (10-2) to calculate *d,*

 =  = 4.00 payment intervals

The period of deferral is 4 years.

13. Given: *PMT* = $400, *i* =  = 0.3125%, *n* = 12(15) = 180, and *PV* = $33,173.03

At a focal date at the end of the period of deferral,

*FV* of $33,173.03 = *PV* of ordinary annuity

$33,173.03 = $400 = $55,003.80

Using formula (10-2) to calculate *d,*

 =  = 162.0655435.00 payment intervals

The period of deferral is 162.0655435 months or 13 years and 6 months.

15. The price paid will be the present value of the payments discounted at the required rate of return. Since the first payment is due in 6 months, the period of deferral is 5 months (in order to treat the payments as a deferred ordinary annuity).

Given: *PMT* = $231, *n* = 15, *d* = 5, and *i* =  = 1.5%

18 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

15 **N**

231 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –3082.287

*PV*(5 months from now) = 

= $231

= $3082.287

Same *I/Y,* *P/Y, C/Y*

5 **N**

0 **PMT**

3082.287 **FV**

**CPT** **PV**

*Ans*: −2861.16

*PV*(today) = 

= $3082.29

= $2861.16

The finance company will pay $2861.16 for the contract.

17 Today’s economic value is the present value.

6.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

10 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −7188.83

For the $1000 annuity,

*PV*(today) = 

= $1000

= $7188.83

Same *I/Y,* *P/Y, C/Y*

Same *N,* *FV*

2000 **PMT**

**CPT** **PV**

*Ans*: −14,377.66

For the $2000 annuity,

*PV*(10 years from now)

= $2000

Same *I/Y,* *P/Y, C/Y, N*

0 **PMT**

14377.66 **FV**

**CPT** **PV**

*Ans*: −7659.35

= $14,377.66

*PV*(today) =  = $7659.35

The $2000 annuity has the greater (by $470.52) economic value.

19 The original loan equals the present value of the payments. Thus,

*PV* = $35,000 with *PMT* = $1573.83, *n* = 4(12) = 48, and *i* =  = 1.25%.

At a focal date at the end of the period of deferral,

*FV* of $35,000 = *PV* of ordinary annuity

$35,000= $1573.83 = $56,550.04

Using formula (10-2) to calculate *d*,

 =  = 38.62162013

The period of deferral was 38.62162013 quarters long. The interval between the date of the

loan and the first payment was 39.62162013 quarters or 9 years and 11 months.

21 Given: For the initial investment, *PV* = $10,000, *i* =  = 2.25%.

For the deferred annuity, *PMT* = $1000, *n* = 40, and *i* = 2.25%.

At a focal date at the end of the period of deferral,

4.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

40 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –26,193.52

*FV* of $10,000 = *PV* of ordinary annuity

$10,000 = $1000

= $26,193.52

Using formula (10-2) to calculate *d*,

 =  = 43.2764318

Same *I/Y,* *P/Y, C/Y*

10000 **+ / –**  **PV**

0 **PMT**

26193.52  **FV**

**CPT** **N**

*Ans*: 43.28

43.27

The period of deferral is 43.2764318 half years or

21 years and 8 months (rounded to the nearest month.)

Since, the first withdrawal occurs 6 months after the

period of deferral, the $10,000 deposit must be made

22 years and 2 months before the first withdrawal.

23 The present value (on the purchase date) of the deferred annuity payments

equals $85,000. Alternatively, at a focal date 9 years from now,

*FV* of $85,000 = *PV* of ordinary annuity

For the *FV* calculation, *PV* = $85,000, *n* = 9, and *i* = 5.5%.

For the *PV* calculation, *n* = 2(20) = 40, *i* = 5.5%, *c* =  = 0.5, and

 = – 1 = 0.027131929

Hence, $85,000 = *PMT* 

Same *I/Y*

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

40 **N**

137,623.01 **+ / –**  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: 5681.03

5.5 **I/Y**

**P/Y** 1 **ENTER**

(making *C/Y =* *P/Y* = 1)

9 **N**

85000 **+ / –**  **PV**

0 **PMT**

**CPT** **FV**

*Ans*: 137,623.01

*PMT* =  = $5681.03

The semiannual payments will be $5681.03.

25 The inheritance’s current economic value is the present value of the deferred general annuity. Since the first quarterly payment is 4.5 years from now, the period of deferral is 4.25 years (if we wish to treat the payments as a deferred ordinary annuity).

Given: *PMT* = $2000, *n* = 4(20) = 80,

6 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 12 **ENTER**

80 **N**

2000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −92,590.13

*d* = 4(4.25) = 17, *i* =  = 0.5%, *c* =  = 3

 =  – 1 = 0.015075125

*PV*(4.25 yrs. from now) =

= $2000

= $92,590.13

Same *I/Y*, *P/Y, C/Y*

17 **N**

0 **PMT**

92590.13 **FV**

**CPT** **PV**

*Ans*: −71,795.22

*PV*(today) = 

= $92,590.13

= $71,795 (to the nearest dollar)

27 Given: *PMT* = $356.83; *i* =  = 1.75%; *c* = ;

7 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 4 **ENTER**

150 **N**

356.83 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: –35,683.49

*n* = 12(12.5) = 150; *PV* = $30,000

 = – 1 = 0.005799633

At a focal date at the end of the period of deferral,

*FV* of $30,000 loan = *PV* of loan payments

Right hand side = $356.83

= $35,683.49

Left hand side = $30,000

Same *I/Y*, *P/Y, C/Y*

30000 **+ /** **–** **PV**

0 **PMT**

35683.49 **FV**

**CPT** **N**

*Ans*: 30.00

Hence, $30,000 = $35,683.49

Using formula (10-2) to calculate *d*,

=  = 30.00

The period of deferral is 30 payment intervals, that is,

30 months or 2 years and 6 months.

29 The initial investment equals the present value of the withdrawals.

3.5 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 2 **ENTER**

60 **N**

1000 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −

46573.69

Given: *PV* = $19,665, *PMT* = $1000, *n* = 60,

*i* = = 1.75%, and *c* =  =0.5.

*i*2 =  = – 1 = 0.00871205

With the focal date at the end of the period of deferral,

*FV* of $19,665 = *PV* of ordinary annuity

Right hand side = $1000= 46,573.69

Left hand side = $19,665

Hence, $19,665 = $46,573.69

Using formula (10-2) to calculate *d,*

Same *I/Y*, *P/Y, C/Y*

19665 **+ /** **–** **PV**

0 **PMT**

46573.69 **FV**

**CPT** **N**

*Ans*:

99.396

 = = 99.396

The period of deferral is 99.396 quarters long.

The investment must be made 100 quarters

or 25 years before the first withdrawal.

**Concept Questions (Section 14.2)**

1. The perpetuity has the larger present value. The greater the number of payments in an annuity, the larger the annuity’s present value. Hence, *PV*(perpetuity) > *PV*(annuity).

Alternatively, the perpetuity may be viewed as a combination of an annuity identical to the given annuity and a deferred perpetuity. Then

*PV* of given perpetuity = (*PV* of given annuity) + (*PV* of deferred perpetuity)

Therefore,

*PV* of given perpetuity > *PV* of given annuity

2. No. Each payment is the interest earned during the preceding payment interval. At a higher market interest rate, a smaller endowment can generate the same payment size.

3. The market value will rise. The rate of return (dividend yield) is the (fixed) annual dividend calculated as a percentage of the market value. If investors will accept a lower rate of return, they will pay a higher price for the shares.

**Exercise 14.2**

1. The $500,000 gift is the present value of the payments.

*PMT* =  = 0.045 × $500,000 = $22,500

3. Given: *PMT* = $10,000

With  = 2.5%,  =  = $400,000

With  = 2%,  = = $500,000

Therefore, $500,000 – $400,000 = $100,000 less is required to fund

the perpetuity at the higher rate of return.

5. The amount required to fund each case is the present value of the payments.

For the perpetuity,  =  = $400,000

For the annuity,  = $5000 = $309,914.24

You need $400,000 – $309,914.24 = $90,085.76 more money to fund the perpetuity.

7. The owner should be willing to accept the current economic value of the stream of future payments. This economic value equals the present value of the future payments. On the date of the next scheduled payment, the payments consist of a simple perpetuity plus the payment due on that date. Hence,

*PV* = $500 +  = $500 +  = $9120.69

The landowner should be willing to accept $9120.69.

9. Price = $1000 + Present value of quarterly payments

The quarterly payments form an ordinary general perpetuity having

*PMT* = $25, *i* = 4.8%, and *c* =  = 0.25

 = – 1 = 0.011789855

*PV* =  =  = $2120.47

Price of a plot = $1000 + $2120.47 = $3120.47

11. *a.* The price equals the present value of the dividends discounted at the required rate

of return. Since the next dividend is about to be paid, the dividends form a simple

perpetuity *after* the first payment. With *PMT* = $1.50 and *i* = = 2.25%.

Price = $1.50 +  = $1.50 +  = $68.17

*b*. Price = Present value of the payments

= *PMT* + 

$70 = $1.50 + 

*i* = = 0.02190 = 2.190% per half year

*j = mi* = 2(2.190%) = 4.38% compounded semiannually

13. *a*. The payments form a general perpetuity having

*PMT* = $1000, *i* =  = 2.75%, *c* =  = 0.5, and

 = -1 = 0.013656747

The initial amount needed to sustain the perpetuity is

 =  = $73,223.88

*b.* The present value of the payments 9 months from now is the amount calculated

in part *a*, that is, $73,223.88. The present value today is

 = $73,223.88 = $70,303.99

15. At a focal date 2 years and 9 months from now,

*FV* of $1 million donation = *PV* of ordinary simple perpetuity

For the *FV* calculation, *PV* = $1,000,000, *n* = 4(2.75) = 11, and *i* =  = 1.5%

 = $1,000,000 = $1,177,948.94

For the perpetuity, *PV* = $1,177,948.94 and *i* = 1.5%. Hence,

*PMT* = *i* × *PV* = 0.015 × $1,177,948.94 = $17,669.23

Quarterly payments of $17,669.23 will be made.

17. At a focal date today,

*FV* of the $500,000 bequest = *PV* of the ordinary perpetuity

For the *FV* calculation, *PV* = $500,000, *i* =  = 2.5%, and *n* = 2(1.5) = 3. Hence,

= $500,000 = $538,445.31

The quarterly payments form an ordinary general perpetuity having

*PV* = $538,445.31, *i* =  = 2.6%, *c* =  = 0.5, and

 = – 1 = 0.012916581

Then *PMT* = *i*2× *PV* = 0.012916581($538,445.31) = $6954.87

The hospice will receive quarterly payments of $6954.87.

**Point of Interest (Section 14.3)**

***Such a Deal!***

1. *a*. Given: *PMT* = $5336.10 *n* = 40, *g* = 2.3%, *i* = 3%. Then

 = $5336.10 = $593,648.56

*b.* If instead, *i* = 4%, then

*FV* = $5336.10 = $727,512.13

3. *a.* Given: *PMT* = $34,405.82, *n* = 86 – 65 = 21, *g* = 2.3%, *i* = 3%. Then

 = $34,405.82 = $655,795.26

*b.* If instead, *i* = 4%, then

 = $34,405.82 = $592,108.64

**Postscript:** The estimated 3% rate of return required for CPP contributions to fund the future CPP pension is only about half the compound annual rate of return earned over the past 50 years on a balanced portfolio of bonds and stocks. Contribution rates before 1990 were far below those needed to actuarially fund accrued CPP benefits. During the first 2 decades of the 21st century, contributors will be subsidizing pension benefits being paid to the preceding generation of contributors. This “subsidy” is the cause of the low imputed rate of return on current workers’ contributions.

**Exercise 14.3**

1. The amount in the RRSP will be the future value of the contributions, which form a constant-growth ordinary simple annuity.

Given: *PMT* = $3000, *g* = 2.5%, *n* = 25, and *i* = 9%. Then,

 = $3000 = $312,421.69

3. Given: *FV* = $750,000, *n* = 30, *g* = 3%, and *i* = 10%. Substitute into



$750,000 = *PMT*  = *PMT*(214.601997)

*PMT* = $3494.84

5. For the fixed-payment annuity,

*PMT* = $1000, *n* = 12(25) = 300, *i* =  = 0.45%, and

Price =  = $1000 = $164,438.55

For the indexed annuity, *g* =  = 0.2% per month and

 = $1000 = $210,593.83

Indexation of the annuity costs $210,593.83 – $164,438.55 = $46,155.28 more.

7. The current economic value of future pension payments is their present value.

Given: *PMT* = $20,000, *n* = 25, *i* = 5%

With indexation (at *g* = 2.5% per year),

 = $20,000 = $362,020.14

Without indexation (*g* = 0%),

 = $20,000 = $281,878.89

Hence, $362,020.14 – $281,878.89 = $80,141.25 (representing 22.1%) of the

pension’s economic value comes from the indexation feature.

9. At a focal date at the end of the 30-year contributory period,

*FV* of RRSP contributions = *PV* of RRIF withdrawals

For the *FV* calculation,

*PMT* = $2000, *n* = 30, *g* = 3%, and *i* = 9%. Then,

 = $2000 = $361,347.20

For the *PV* calculation,

*PV* = $361,347.20, *n* = 12(25) = 300, *g* =  = 0.15%, and *i* =  = 0.5%

Substitute into



$361,347.20 = *PMT* 

= *PMT*(185.391515)

*PMT* = $1949.10

The initial RRIF withdrawal will be $1949.10.

11. Market value of shares = *PV* of dividends

The dividends in the first 5 years form a constant growth ordinary simple annuity.

The dividends in the next 25 years form another constant growth ordinary simple annuity.

For the first 5 years,

*PMT* = $2.00(1.10) = $2.20, *n* = 5, *g* = 10%, and *i* = 9%.

For the next 25 years,

*PMT* = $2.00(1.10)5(1.03) = $3.318, *n* = 25, *g* = 3%, and *i* = 9%.

At a focal date 5 years from now, the *PV* of the subsequent 25 years’ dividends is

 = $3.318 = $41.873

The *PV* today of this amount and the first 5 years’ dividends is

*PV* = $41.873 + $2.20

= $27.215 + $10.279

= $37.49

The fair market value of the shares is $37.49.

**Review Problems**

1. The required amount is the present value of the deferred annuity.

With *PMT* = $500, *n* = 12(5) = 60, *d* = 12(4) = 48, and *i* =  = 0.6%,

*PV*(4 years from now) = = $500= $25,131.065

*PV*(today) =  = $25,131.065 = $18,858.53

Therefore, $18,858.53 invested now will provide the desired payments.

3. The amount required to fund an annuity or a perpetuity is the present value of

the payments. For a perpetuity having *PMT* = $1000 and *i* = 6%,

*PV* =  =  = $16,666.67

For the annuity having *PMT* = $1000, *n* = 25, and *i* = 6%,

 = $1000 = $12,783.36

Therefore,

 × 100% = 30.38%

more funds are needed to fund the perpetuity.

5. We have a perpetuity with *PV* = $200,000 and *i* =  = 0.45%

The monthly payments will be

*PMT* = *i* × *PV* = 0.0045 × $200,000 = $900.00

7. Purchase price = *PV* of annuity payments

Given: *PMT* = $10,000, *g* = 2.0%, *n* = 1(15) = 15, and *i* = 5.5%. Then,

 = $10,000 = $113,468.88

It will cost $113,468.88 to purchase the annuity.

9. The selling price (original loan) equals the present

value of the payments. The payments form a

16.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

12 **N**

226.51 **PMT**

0 **FV**

**CPT** **PV**

*Ans*: −2490.006

deferred simple annuity having

*PMT* = $226.51, *n* = 12, *d* = 3, and *i* =  = 1.375%.

*PV*(3 months from now) = 

= $226.51

= $2490.006

Same *I/Y*, *P/Y, C/Y*

3 **N**

0 **PMT**

2490.006 **FV**

**CPT** **PV**

*Ans*: −2390.05

*PV*(today) = 

= $2490.006

= $2390.05

The selling price of the stereo system was $2390.05.

11. The amount that must be dedicated today is the present value of the future payments.

*a*. The payments form an ordinary simple perpetuity having

*PMT* = $2000 and *i* =  = 1.4%.

*PV* =  =  = $142,857.14

*b*. The payments form a deferred ordinary simple perpetuity having a 4-year plus

9-month period of deferral. The *PV* of the payments on that date is the amount

calculated in part *a*. The *PV* of this amount today is

 = $142,857.14 = $109,693.48

13. The amount required to fund an annuity or a perpetuity is the present value of

the payments. For the perpetuity, *PMT* = $500, *i* =  = 1.75%, *c* =  = , and

 = – 1 = 0.005799633

*PV* =  =  = $86,212.36

For the annuity having *PMT* = $500, *n* = 12(25) = 300, and = 0.57996326%,

 = $500 = $71,002.41

Therefore, $86,212.36 – $71,002.41 = $15,209.95 more is required to fund the perpetuity.

15. The amount required to fund an annuity or a perpetuity is the present value of

the payments. For a perpetuity having *PMT* = $1000 and *i* =  = 2.9%,

*PV* =  =  = $34,482.76

For the annuity having *PMT* = $1000, *n* = 2(30) = 60, and *i* = 2.9%,

 = $1000 = $28,278.65

Therefore,

×100% = 21.94%

more funds are needed to fund the perpetuity.

17. With the focal date 1 month before the first payment,

(Future value of original principal) = (Present value of loan payments)

For the present value calculation,

*PMT* = $425.10, *n* = 12(10) = 120, and *i* =  = 0.75%

 = $425.10 = $33,558.11

On the left side of the equation, we have *FV* = $33,558.11, *PV* = $30,000, and *i* = 0.75%.

Then the number of compounding intervals is

 =  = 15.00

Therefore, the time interval between the date of the loan and the first payment

was 15 + 1 = 16 months.

**Case (Chapter 14)**

***Should You Choose to Start Receiving the CPP Retirement***

***Pension at Age 60 Instead of Age 65?***

1. The economic value of the reduced pension is the present value of the expected

pension payments. With *PMT* = $8867.20, *i* = 6%, *g* = 2.3%, and *n* = 21,

 = $8867.20 = $126,009.16

3. The present value of 16 years of the full pension at age 65 is

 = $15,523.32 = $181,920.33

The present value of the preceding amount at age 60 (end of 2018) 5 years earlier is

 = $181,920.33 = $135,941.45

5. With *i* = 4.5%, the economic value of the reduced pension at age 60 is

*PV* = $8867.20 = $145,238.45

The present value of the full pension at age 65 is

*PV* = $15,523.32 = $203,596.55

The present value of the preceding amount at age 60 (5 years earlier) is

 = $203,596.55 = $163,376.26

At a discount rate of 4.5% compounded annually, the economic value of the full pension

is $18,137.81 more than the economic value of the reduced pension. Therefore, Neil

should again choose the full pension beginning at age 65.