15 Loan Amortization: Mortgages

**Point of Interest (Section 15.1)**

***Just When You Thought You Had It All Figured Out …***

1. Jeff is missing the point that his *average* investment (including interest earned) in the GIC will exceed $10,000, whereas the *average* principal balance on the car loan will be far less than $10,000—it starts at $10,000 but declines to $0 over the 3-year term. This explains why the interest charges on the loan will be less than the interest earned on the GIC, even though the interest *rate* on the loan exceeds the interest *rate* on the GIC.

**Exercise 15.1**

15 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

6 **N**

1000  **PV**

0  **FV**

**CPTPMT**

*Ans*: –174.03

1. Given: *PV* = $1250(1 – 20%) = $1000, *n* = 6, *i* =  = 1.25%

Substitute into formula (11-2) and solve for *PMT.*

$1000 = *PMT* 

*PMT* = $174.03

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $1000.00 |
|  | 1 | $174.03 | $12.50 | $161.53 | 838.47 |
|  | 2 | 174.03 | 10.48 | 163.55 | 674.92 |
|  | 3 | 174.03 | 8.44 | 165.59 | 509.33 |
|  | 4 | 174.03 | 6.37 | 167.66 | 341.67 |
|  | 5 | 174.03 | 4.27 | 169.76 | 171.91 |
|  | 6 | 174.06 | 2.15 | 171.91 | 0.00 |
|  |  | Total: | $44.21 |  |  |

2.5 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

4 **N**

2800  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –710.97

3. Given: *PV* = $2800, *n* = 4, *i* =  = 0.625%

Substitute into formula (11-2) and solve for *PMT.*

$2800 = *PMT* 

*PMT* = $710.97

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $2800.00 |
|  | 1 | $710.97 | $17.50 | $693.47 | 2106.53 |
|  | 2 | 710.97 | 13.17 | 697.80 | 1408.73 |
|  | 3 | 710.97 | 8.80 | 702.17 | 706.56 |
|  | 4 | 710.98 | 4.42 | 706.56 | 0.00 |
|  |  | Total: | $43.89 |  |  |

5. Given: *PV* = $1000, *PMT* = $200, *i* =  = 1.25%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 |  |  |  | $1000.00 |
|  | 1 | $200.00 | $12.50 | $187.50 | 812.50 |
|  | 2 | 200.00 | 10.16 | 189.84 | 622.66 |
|  | 3 | 200.00 | 7.78 | 192.22 | 430.44 |
|  | 4 | 200.00 | 5.38 | 194.62 | 235.82 |
|  | 5 | 200.00 | 2.95 | 197.05 | 38.77 |
|  | 6 | 39.25 | 0.48 | 38.77 | 0.00 |
|  |  | Total: | $39.25 |  |  |

7. Given: *PV* = $2800, *PMT* = $600, *i* =  = 0.625%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 |  |  |  | $2800.00 |
|  | 1 | $600.00 | $17.50 | $582.50 | 2217.50 |
|  | 2 | 600.00 | 13.86 | 586.14 | 1631.36 |
|  | 3 | 600.00 | 10.20 | 589.80 | 1041.56 |
|  | 4 | 600.00 | 6.51 | 593.49 | 448.07 |
|  | 5 | 450.87 | 2.80 | 448.07 | 0.00 |
|  |  | Total: | $50.87 |  |  |

9 **I/Y**

**P/Y** 2 **ENTER**

**C/Y** 1 **ENTER**

6 **N**

9000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –1739.45

9. Given: *PV* = $9000, *n* = 6, *i* = 9%, *c* =  = 0.5

 = – 1 = 0.044030651

Solve for *PMT* in

$9000 = *PMT* 

*PMT* = $1739.45

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $9000.00 |
|  | 1 | $1739.45 | $396.28 | $1343.17 | 7656.83 |
|  | 2 | 1739.45 | 337.14 | 1402.31 | 6254.52 |
|  | 3 | 1739.45 | 275.39 | 1464.06 | 4790.46 |
|  | 4 | 1739.45 | 210.93 | 1528.52 | 3261.94 |
|  | 5 | 1739.45 | 143.63 | 1595.82 | 1666.12 |
|  | 6 | 1739.48 | 73.36 | 1666.12 | 0.00 |

8 **I/Y**

**P/Y** 4 **ENTER**

(making *C/Y =* *P/Y* = 4)

8 **N**

8000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –1092.08

11. Given: *PV* = $8000, *n* = 8, *i* =  = 2%

Substitute into formula (11-2) and solve

for *PMT.*

$8000 = *PMT* 

*PMT* = $1092.08

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $8000.00 |
|  | 1 | $1092.08 | $160.00 | $932.08 | 7067.92 |
|  | 2 | 1092.08 | 141.36 | 950.72 | 6117.20 |
|  | 3 | 2592.08 | 122.34 | 2469.74 | 3647.46 |
|  | 4 | 1092.08 | 72.95 | 1019.13 | 2628.33 |
|  | 5 | 1092.08 | 52.57 | 1039.51 | 1588.82 |
|  | 6 | 1092.08 | 31.78 | 1060.30 | 528.52 |
|  | 7 | 539.09 | 10.57 | 528.52 | 0.00 |
|  |  | Total: | $591.57 |  |  |

13. Given: *PV* = $60,000, *n* = 12(6) = 72, *i* =  = 0.625%

7.5 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

72 **N**

60000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –1037.41

Substitute into formula (11-2) and solve for *PMT.*

$60,000 = *PMT* 

*PMT* = $1037.41

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $60,000.00 |
|  | 1 | $1037.41 | $375.00 | $662.41 | 59,337.59 |
|  | 2 | 1037.41 | 370.86 | 666.55 | 58,671.04 |
|  | | | | | | | | | | |
|  | 42 | -- | -- | -- | 28,298.14 |
|  | 43 | 1037.41 | 176.86 | 860.55 | 27,437.59 |
|  | 44 | 1037.41 | 171.48 | 865.93 | 26,571.66 |
|  | | | | | | | | | | |
|  | 70 | -- | -- | -- | 2055.24 |
|  | 71 | 1037.41 | 12.85 | 1024.56 | 1030.68 |
|  | 72 | 1037.12 | 6.44 | 1030.68 | 0.00 |

15. Given: *PV* = $9000, *PMT* = $1800, *i* = 9%, *c* =  = 0.5

 = – 1 = 0.044030651

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $9000.00 |
|  | 1 | $1800.00 | $396.28 | $1403.72 | 7596.28 |
|  | 2 | 1800.00 | 334.47 | 1465.53 | 6130.75 |
|  | 3 | 1800.00 | 269.94 | 1530.06 | 4600.69 |
|  | 4 | 1800.00 | 202.57 | 1597.43 | 3003.26 |
|  | 5 | 1800.00 | 132.24 | 1667.76 | 1335.50 |
|  | 6 | 1394.30 | 58.80 | 1335.50 | 0.00 |

17. Given: *PV* = $8000, *PMT* = $1000, *i* =  = 2%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $8000.00 |
|  | 1 | $1000.00 | $160.00 | $840.00 | 7160.00 |
|  | 2 | 1000.00 | 143.20 | 856.80 | 6303.20 |
|  | 3 | 2000.00 | 126.06 | 1873.94 | 4429.26 |
|  | 4 | 1000.00 | 88.59 | 911.41 | 3517.85 |
|  | 5 | 1000.00 | 70.36 | 929.64 | 2588.21 |
|  | 6 | 1000.00 | 51.76 | 948.24 | 1639.97 |
|  | 7 | 1000.00 | 32.80 | 967.20 | 672.77 |
|  | 8 | 686.23 | 13.46 | 672.77 | 0.00 |
|  |  | Total: | $686.23 |  |  |

19. Given: *PV* = $60,000, *PMT* = $1000, *i* =  = 0.625%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 | -- | -- | -- | $60,000.00 |
|  | 1 | $1000.00 | $375.00 | $625.00 | 59,375.00 |
|  | 2 | 1000.00 | 371.09 | 628.91 | 58,746.09 |
|  | | | | | | | | | | |
|  | 55 | -- | -- | -- | 19,128.18 |
|  | 56 | 1000.00 | 119.55 | 880.45 | 18,247.73 |
|  | 57 | 1000.00 | 114.05 | 885.95 | 17,361.78 |
|  | | | | | | | | | | |
|  | 74 | -- | -- | -- | 1424.49 |
|  | 75 | 1000.00 | 8.90 | 991.10 | 433.39 |
|  | 76 | 436.10 | 2.71 | 433.39 | 0.00 |

21. *a.* For a 30-month term, the balance after 15 months is $5233.

That is, about 47.7% of the principal is paid off midway through the term.

For a 30-year term, the balance after 15 Years is $7543.

That is, about 24.6% of the principal is paid off midway through the term.

|  |  |  |
| --- | --- | --- |
| *b.* | Term (years) | Total Interest ($) |
|  | 5 | 2022.76 |
|  | 10 | 4244.32 |
|  | 15 | 6686.41 |
|  | 20 | 9334.03 |
|  | 25 | 12,169.22 |
|  | 30 | 15,173.16 |

*c.* Rounded to the nearest month,

(i) Total interest = Original principal if the term is 254 months.

(ii) Total interest = 1.5 times the original principal if the term is 357 months.

**Concept Questions (Section 15.2)**

1. The balance midway through the amortization period will be (i) more than half the original principal. With each successive payment, the interest component becomes smaller and the principal component becomes larger. Therefore, the total principal repaid in the first half of the amortization period will be less than the total principal repaid in the second half of the amortization period. It follows that: (1) less than half of the original principal will be repaid in the first half of the amortization period; and (2) the loan’s balance midway through the amortization period will be more than half the original principal.

3. *a*. Each regular payment includes a 0.2¢ *over*payment. Therefore, the adjusted final payment will be less than the regular payment.

*b*. The reduction in the final payment will be the future value of the 60 overpayments of 0.2¢. This will be (i) more than 60(0.2¢) = 12¢.

**Exercise 15.2**

6.6 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

120 **N**

40000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –456.2298

1. Given: *PV* = $40,000, *n* = 12(10) = 120, *i* =  = 0.55%

Substitute into formula (11-2) and solve for *PMT.*

$40,000 = *PMT* 

*PMT* = $456.23

*a.* Interest component of Payment 35

456.23 **+ / –** **PMT**

**2nd** **AMORT**

35 **ENTER**

35 **ENTER**

*INT* = –171.57

= *i* × Balance after Payment 34

= 

= 0.0055

= $171.57

*b.* Principal component of Payment 63

= Balance after Payment 62 – Balance after Payment 63

=

63 **ENTER**

63 **ENTER**

*PRN* = –331.91

-

= 

– 

= $22,603.17 – $22,271.26

= $331.91

*c.* Principal reduction in Year 1

1 **ENTER**

12 **ENTER**

*PRN* = –2922.10

= $40,000 – Balance after Year 1

= $40,000 – Balance after Payment 12

= $40,000 – $37,077.90

= $2922.10

*d.* Principal reduction in Year 10

109 **ENTER**

120 **ENTER**

*PRN* = –5283.96

= Balance after Year 9 – Balance after Year 10

= Balance after Payment 108

– Balance after Payment 120

= $5283.92 – (–$0.04)

= $5283.96

8.4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

14 **N**

14000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –1342.9245

3. Given: *PV* = $14,000, *n* = 2(7) = 14, *i* =  = 4.2%

Substitute into formula (11-2) and solve for *PMT.*

$14,000 = *PMT* 

*PMT* = $1342.92

1342.92 **+ / –** **PMT**

**2nd** **AMORT**

10 **ENTER**

10 **ENTER**

*INT* = –249.69

*a.* Interest component of Payment 10

= *i* × Balance after Payment 9

= 

= 0.042

= $249.69

*b.* Principal component of Payment 3

= Balance after Payment 2 – Balance after Payment 3

= 

– 

= 

3 **ENTER**

3 **ENTER**

*PRN* = –819.66

– 

= $12,458.45 – $11,638.79

= $819.66

*c.* Interest paid in Year 6

11 **ENTER**

12 **ENTER**

*INT* = –359.71

= Total payments in Year 6 – Principal paid in Year 6

= 2*PMT* – (Balance after Year 5 – Balance after   
Year 6)

= 2($1342.92) – (Balance after Payment 10

– Balance after Payment 12)

= 2($1342.92) – ($4851.84 – $2525.71)

= $359.71

*d.* Principal reduction by Payments 3 to 6 inclusive

3 **ENTER**

6 **ENTER**

*PRN* = –3491.06

= Balance after Payment 2

– Balance after Payment 6

= $12,458.45 – $8967.39

= $3491.06

5. Given: *PV* = $125,000, *n* = 12(20) = 240,

6 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

240 **N**

125000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –890.2355

*i* =  = 3%, *c* = 

 = – 1 = 0.004938622

Substitute into formula (11-2) and solve for *PMT.*

$125,000 = *PMT* 

*PMT* = $890.24

*a.* Interest component of Payment 188

890.24 **+ / –** **PMT**

**2nd** **AMORT**

188 **ENTER**

188 **ENTER**

*INT* = –204.57

= *i* × Balance after Payment 187

= 0.00493862203 × $41,422.04

= $204.57

*b.* Principal component of Payment 101

101 **ENTER**

101 **ENTER**

*PRN* = –446.66

= Balance after Payment 100 – Balance after Payment 101

= 

– 

= $89,818.64 – $89,371.98

= $446.66

*c.* Principal reduction in Year 1

1 **ENTER**

12 **ENTER**

*PRN* = –3365.38

= $125,000 – Balance after Year 1

= $125,000 – Balance after Payment 12

= $125,000 – $121,634.62

= $3365.38

*d.* Principal reduction in Year 20

= Balance after Year 19 – Balance after Year 20

= Balance after Payment 228

– Balance after Payment 240

= $10,345.79 – $0

= $10,345.79

7.6 **I/Y**

**P/Y** 4 **ENTER**

**C/Y** 2 **ENTER**

40 **N**

50000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –1790.2647

7. Given: *PV* = $50,000, *n* = 4(10) = 40,

*i* =  = 3.8%, *c* =  = 0.5

 = – 1 = 0.01882285

Substitute into formula (11-2) and solve for *PMT.*

$50,000 = *PMT* 

*PMT* = $1790.26

*a.* Interest component of Payment 8

1790.26 **+ / –** **PMT**

**2nd** **AMORT**

8 **ENTER**

8 **ENTER**

*INT* = –822.74

= *i* × Balance after Payment 7

= 0.01882285016 ($43,709.81)

= $822.74

*b.* Principal component of Payment 33

33 **ENTER**

33 **ENTER**

*PRN* = –1542.15

= Balance after Payment 32

– Balance after Payment 33

= $13,181.541 – $11,639.395

= $1542.15

*c.* Total interest in Payments 21 to 30 inclusive

21 **ENTER**

30 **ENTER**

*INT* = –4474.70

= Sum of Payments 21 to 30

– Total principal in Payments 21 to 30

= 10*PMT* – (Balance after Payment 20

– Balance after Payment 30)

= 10($1790.26) – ($29,608.787 – $16,180.885)

= $4474.70

*d.* Principal paid in Year 3

9 **ENTER**

12 **ENTER**

*PRN* = –4055.64

= Balance after Year 2 – Balance after Year 3

= Balance after Payment 8

– Balance after Payment 12

= $42,742.29 – $38,686.65

= $4055.64

6 **I/Y**

**P/Y** 12 **ENTER**

(making *C/Y =* *P/Y* = 12)

15000  **PV**

275 **+ / –** **PMT**

**2nd** **AMORT**

13 **ENTER**

13 **ENTER**

*INT* = –62.66

9. Given: *PV* = $15,000, *PMT* = $275, *i* =  = 0.5%

*a.* Interest component of Payment 13

= *i* × Balance after Payment 12

Balance after Payment 12

=

= $12,532.89

Interest component of Payment 13

= 0.005($12,532.89) = $62.66

44 **ENTER**

44 **ENTER**

*PRN* = –247.84

*b.* Principal component of Payment 44

= Balance after Payment 43

– Balance after Payment 44

= $5432.09 – $5184.25

= $247.84

Same *I/Y,* *P/Y, C/Y*

Same *PV,* *PMT*

0  **FV**

**CPT** **N**

*Ans*: 63.84984

**2nd** **AMORT**

63 **ENTER**

63 **ENTER**

*BAL* = 232.63

**X**  1.005 **=**

*Ans*: 233.79

*c.* The total number of payments is



= 

= 63.84984

Final payment = (1 + *i*)× Balance after Payment 63

= 1.005($232.63)

= $233.79

11. Given: *PV* = $100,000, *PMT* = $700, *i* =  = 3.6%

7.2 **I/Y**

**P/Y** 12 **ENTER**

**C/Y** 2 **ENTER**

100000  **PV**

700 **+ / –** **PMT**

**2nd** **AMORT**

221 **ENTER**

221 **ENTER**

*INT* = –302.03

Then *c* =  and

 = – 1 = 0.005911931

*a.* Interest component of Payment 221

= *i* × Balance after Payment 220

Balance after Payment 220



= $51,088.65

Interest component of Payment 221

= 0.005911931($51,088.65)

= $302.03

156 **ENTER**

156 **ENTER**

*PRN* = –271.30

*b.* Principal component of Payment 156

= Balance after Payment 155

– Balance after Payment 156

= $72,514.34 – $72,243.04

= $271.30

*c.* The total number of payments is

Same *I/Y,* *P/Y, C/Y*

Same *PV,* *PMT*

0  **FV**

**CPT** **N**

*Ans*: 315.80251

**2nd** **AMORT**

315 **ENTER**

315 **ENTER**

*BAL* = 558.78

**X**  1.0059119 **=**

*Ans*: 562.08



= 

= 315.80251

Final payment = (1 + *i*) × Balance after Payment 315

= 1.005911931($558.78)

= $562.08

8.2 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

20 **N**

37000  **PV**

0  **FV**

**CPT** **PMT**

*Ans*: –2746.687

13. Given: *PV* = $37,000, *i* =  = 4.1%, *n* = 2(10) = 20

Substitute into formula (11-2) and solve for *PMT.*

$37,000 = *PMT* 

*PMT* = $2746.69

*a*. Principal part of Payment 6

2746.69 **+ / –** **PMT**

**2nd** **AMORT**

6 **ENTER**

6 **ENTER**

*PRN* = –1503.31

= Balance after Payment 5

– Balance after Payment 6

= $30,326.28 – $28,822.97

= $1503.31

16 **ENTER**

16 **ENTER**

*INT* = –499.93

*b*. Interest part of Payment 16

= *i* × Balance after Payment 15

= 0.041($12,193.43)

= $499.93

6 **ENTER**

15 **ENTER**

*PRN* = –18,132.85

*c*. Principal paid by Payments 6 to 15 inclusive

= Balance after Payment 5

– ­Balance after Payment 15

= $30,326.28 – $12,193.43

= $18,132.85

*d*. Interest paid in year 3

5 **ENTER**

6 **ENTER**

*INT* = –2545.96

= 2*PMT* – (Balance after Payment 4

– Balance after Payment 6)

= 2($2746.69) – ($31,770.383 – $28,822.966)

= $2545.96

19 **ENTER**

19 **ENTER**

*BAL* = –2638.42

**X**  1.041 **=**

Ans: 2746.60

*e*. Final payment

= (1 + *i*) × Balance after Payment 19

= 1.041($2638.42)

= $2746.60

15. Given: *PV* = $37,000, *i* =  = 4.1%, *PMT* = $2500

*a*. Principal in Payment 16 = Balance after Payment 15 – Balance after Payment 16

= $17,169.91 – $15,373.88

= $1796.03

*b*. Interest in Payment 6 = *i* × Balance after Payment 5

= 0.041($31,665.10)

= $1298.27

*c*. Principal reduction by Payments 8 to 14 inclusive

= Balance after Payment 7 – Balance after Payment 14

= $29,212.37 – $18,895.21

= $10,317.16

*d*. Interest in Year 5 = 2*PMT* – (Principal in Year 5)

= 2*PMT* – (Balance after Payment 8 – Balance after Payment 10)

= 2($2500) – ($27,910.08 – $25,143.12)

= $2233.04

*e*. The total number of payments is

 =  = 23.230346

Final payment = (1 + *i*)× Balance after Payment 23 = 1.041($561.77) = $584.80

17. Given: *PV* = $8000, *i* =  = 0.65%, *n* = 4(4) = 16, *c* =  = 3

 = – 1 = 0.019627025

Substitute into formula (11-2) and solve for *PMT.*

$8000 = *PMT*

*PMT* = $587.46

*a*. Interest in Payment 5 = *i2* × Balance after Payment 4

= 0.019627025($6226.87)

= $122.21

*b*. Principal in Payment 11 = Balance after Payment 10 – Balance after Payment 11

= $3294.79 – $2772.00

= $522.79

*c*. Interest paid by Payments 5 to 12 inclusive

= 8*PMT* – (Balance after Payment 4 – Balance after Payment 12)

= 8($587.46) – ($6226.87 – $2238.94)

= $711.75

*d*. Principal paid in Year 2 = Balance after Payment 4 – Balance after Payment 8

= $6226.87 – $4310.38

= $1916.49

*e*. Final payment = (1 + *i2*) × Balance after Payment 15

= 1.019627025($576.18)

= $587.49

19. Given: *PV* = $8000, *PMT* = $500, *i* =  = 0.65%, *c* =  = 3

 = – 1 = 0.019627025

*a*. Interest in Payment 11 = *i*2 × Balance after Payment 10

= 0.019627025($4250.82)

= $83.43

*b*. Principal in Payment 6 = Balance after Payment 5 – Balance after Payment 6

= $6216.43 – $5838.44

= $377.99

*c*. Interest paid by Payments 3 to 9 inclusive

= 7*PMT* – (Principal in Payments 3 to 9 inclusive)

= 7*PMT* – (Balance after Payment 2 – Balance after Payment 9)

= 7($500) – ($7307.30 – $4659.37)

= $852.07

*d*. Principal paid in Year 3 = Balance after Payment 8 – Balance after Payment 12

= $5060.06 – $3409.51

= $1650.55

*e*. The total number of payments is

 =  = 19.392234

Final payment = (1 + *i*2) × Balance after Payment 19

= 1.019627025($193.48)

= $197.28

21. Given: *PV* = $40,000, *n* = 12(15) = 180, *i* =  = 0.625%

*a*. Substitute into formula (11-2) and solve for *PMT.*

$40,000 = *PMT* 

*PMT* = $370.80

*b*. Interest part of Payment 60 = *i* × Balance after Payment 59

= 0.00625($31,413.19)

= $196.33

Principal part of Payment 60 = *PMT* – Interest part of Payment 60

= $370.80 – $196.33

= $174.47

*c*. Balance after Payment 60 = $31,238.73

*d*. Interest paid in Year 5 = 12*PMT* – Principal paid in Year 5

= 12*PMT* – (Balance after Pmt 48 – Balance after Pmt 60)

= 12($370.80) – ($33,262.28 – $31,238.73)

= $2426.05

23. *a*. Given: *PV* = $12,000, Balance after Payment 24 = $10,000, *i* =  = 4.5%, *c* = 

 = – 1 = 0.007363123

The initial loan equals the combined present value of 24 payments and the $10,000 balance. That is,

$12,000 = *PMT*  + 

*PMT* = $164.85

*b*. Interest part of Payment 9 = *i2* × Balance after Payment 8

= 0.007363123($11,372.05

= $83.73

*c*. Principal in Payment 16 = Balance after Payment 15 – Balance after Payment 16

= $10,791.54 – $10,706.15

= $85.39

**Point of Interest (Section 15.3)**

***No Money Down Mortgages are No More***

1. a. in 2010 and 2016 the down payment on $250,000 would be

0.05 × $250,000 = $12,500

b. in 2010 the down payment on $750,000 would be

0.05 × $750,000 = $37,500

in 2016 the down payment on $750,000 would be

0.05 × $500,000 + 0.15 x $250,000 = $62,500

**Point of Interest (Section 15.3)**

***An Analysis of the Interest “Savings” from Choosing a Shorter Amortization Period***

1. The accumulated amount will be the future value of the periodic investments.

Substitute *PMT* = $74.75, *n* = 12(20) = 240, *i* =  into formula (11-1).

 = $74.75 = $31,444.46

3. If the rate of return on investments exceeds the interest rate on the mortgage loan, the amount calculated in question 1 will exceed the amount calculated in question 2. In this case, the 25-year amortization (and investing the difference) is economically preferred.

**Exercise 15.3**

1. The principal reduction in any 5-year period equals the decrease in the loan balance

during the period.

*Step* 1: Calculate the monthly payment.

*Step* 2: Calculate the balance owed at the end of each 5-year interval.

*Step* 3: Calculate the principal reduction in each 5-year interval using

Principal reduction = Beginning balance – Ending balance

*Step* 4: Calculate the interest paid in each 5-year interval using

Interest paid = 60*PMT* – Principal reduction

(Note: Adjust the last interval’s amount for the differing final payment.)

*Step* 1: *PV* = $100,000, *n* = 12(25) = 300, *i* =  = 2.6%, *c* =  = 

 = –1 = 0.004287121

Substitute into formula (11-2) and solve for *PMT.*

$100,000 = *PMT* 

*PMT* = $593.04

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | *Balance at* | *Principal* | *Interest* |
|  |  | *Interval* | *end* (*Step* 2) | *reduction* (*Step* 3) | *paid* (*Step* 4) |
|  |  | 0 – 5 years | $88,783.39 | $ 11,216.61 | $24,365.79 |
|  |  | 6 – 10 years | 74,284.49 | 14,498.90 | 21,083.50 |
|  |  | 11 – 15 years | 55,542.80 | 18,741.69 | 16,840.71 |
|  |  | 16 – 20 years | 31,316.76 | 24,226.04 | 11,356.36 |
|  |  | 21 – 25 years | 0 | 31,316.76 | 4267.15 ➀ |

➀ Final payment = (1 + *i2*) × Balance after payment 299

= 1.004287121($592.01)

= $594.55

Total interest = 59($593.04) + $594.55 – $31,316.76 = $4267.15

3. Given: *PV* = $100,000, *n* = 12(25) = 300, *c* = 2÷12 = 

*a*. For each of *i* =  = 2.0%, *i* =  = 3%, and *i* =  = 4%, calculate  and *PMT*

|  |  |  |  |
| --- | --- | --- | --- |
|  | *i*(%) | *i*2 (%) | *PMT*($) |
|  | 2 | 0.330589033 | $526.02 |
|  | 3 | 0.493862203 | $639.81 |
|  | 4 | 0.655819694 | $763.21 |

*b*. Percent difference = ×100% = 45.09%

|  |  |  |  |
| --- | --- | --- | --- |
|  | *c*. | *j* (% *csa*) | *Total interest paid* |
|  |  | 4 | 300($526.02) – $100,000 = $57,806 |
|  |  | 6 | 300($639.81) – $100,000 = $91,943 |
|  |  | 8 | 300($763.21) – $100,000 = $128,963 |

(These answers assume the 300th payment does not differ from the others.)

5. The maximum loan equals the present value of the $1000 payments discounted

at the interest rate on the loan.

Given: *PMT* = $1000, *n* = 12(25) = 300, *c* = = 

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *j* | *i* | *i*2 (%) | Maximum loan |
|  | *a.* | 4.5% csa | 2.25% | 0.37153196% | *PV* = $180,677.30 |
|  | *b*. | 7.5% csa | 3.75% | 0.615452392% | *PV* = $136,695.14 |

7. *Step* 1: Calculate the payment size for the first 5-year term.

*Step* 2: Calculate the balance after 5 years.

*Step* 3: Calculate the payment upon renewal.

*Step* 4: Calculate the difference between the *Step* 1 and *Step* 3 results.

*Step* 1: Given: *PV* = $100,000, *n* = 12(25) = 300, *i* =  = 3.25%, *c* =  = 

 = – 1 = 0.00534474

Substitute into formula (11-2) and solve for *PMT.*

$100,000 = *PMT* 

*PMT* = $669.82

*Step* 2: Balance after payment 60 = $90,455.83

*Step* 3: Now *PV* = $90,455.83, *n* = 12(20) = 240, *i* =  = 1.75%, *c* =  = 

 = – 1 = 0.002895624

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $523.43

*Step* 4: Decrease in payment size = $669.82 - $523.43= $146.39

9. *a*. Given: *PV* = $80,000, *i* =  = 3.7%, *n* = 12(25) = 300, *c* =  = 

 = – 1 = 0.006073692

Substitute into formula (11-2) and solve for *PMT.*

$80,000 = *PMT* 

*PMT* = $580.23

Balance after payment 36 = $76,216.85

*b*. Now *PV* = $76,216.85; *i* =  = 2.4%; *n* = 300 – 36 = 264; *c* =  = 

 = – 1 = 0.003960577

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $465.99

11. *a.* Given: *PV* = $40,000; *i* =  = 0.55%; *n* = 12(15) = 180

Substitute into formula (11-2) and solve for *PMT.*

$40,000 = *PMT* 

*PMT* = $350.65

Rounded monthly payment = $360

Balance after 4 years = Balance after 48 payments of $360 = $32,333.43

*b*. On renewal, *PV* = $32,333.43; *n* = 12(11) = 132; *i* =  = 0.35%

Substitute into formula (11-2) and solve for *PMT.*

$32,333.43 = *PMT* 

*PMT* = $306.30

Rounded monthly payment = $310.00

13. *a*. Given: *PV* = $27,000; *i* =  = 1.625%; *n* = 12(10) = 120; *c* =  = 

 = – 1 = 0.002690176

Substitute into formula (11-2) and solve for *PMT.*

$27,000 = *PMT* 

*PMT* = $263.57

Rounded payment = $270.00

Balance after 60 payments of $270 = $14,167.50

*b*. For the second 5‑year term,

*PV* = $14,167.50; *i* =  = 2.875%; *n* = 60; *c* =  =  and

 = – 1 = 0.004735255

Solve for *PMT* giving *PMT* = $271.81

Rounded monthly payment = $280

The number of monthly payments of $280 needed to pay off the balance owed is

 =  = 57.9805

Final payment = (1 + *i2*) × Balance after 57 payments

= 1.004735255 ($273.27)

= $274.56

15. *a.* GDS ratio must be  32%. Let *PMT* represent the monthly mortgage payment.

Then GDS ratio =   0.32

That is, *PMT* + $150 + $200  0.32($5000)

*PMT*  0.32($5000) – $350 = $1250

The TDS ratio must be  40%. That is,

TDS ratio =   0.40

Therefore, *PMT*  0.40($5000) – $950 = $1050

The TDS ratio is the more restrictive ratio for the Archibalds.

With *PMT* = $1050, *n* = 12(25) = 300, *i* =  = 3.3%, and *c* =  = 

 = – 1 = 0.00542586533

The maximum mortgage loan is

*PV* = $1050 = $155,349

Rounded to the nearest $100, the maximum mortgage loan (based on

income) is $155,300.

*b*. For the maximum CMHC‑insured mortgage loan and minimum down payment,

$155,300 = 0.95(Maximum home price)

Maximum home price =  = $163,474

Rounded to the nearest $100, the maximum price they can pay is $163,500.

17. *a*. Given: *PV* = $100,000; *i* =  = 2.0%; *n* = 12(25) = 300; *c* =  = 

 = – 1 = 0.00330589

Substitute into formula (11-2) and solve for *PMT.*

$100,000 = *PMT* 

*PMT* = $526.02

*b*. The weekly payment is *PMT* = 0.25($526.02) = $131.51.

*PV* = $100,000; *i* = 2.0%; and *c* =  = 0.038461538.

 = – 1 = 0.00076193

Substituting into formula (11-2n), we obtain

 =  = 1137.023

Therefore, 1138 weekly payments will be required. The loan will be paid off after

21 years and 46 weeks. The amortization period is reduced by a little more than 3 years.

19. This problem is the same as problem 18 except that the initial amortization period

is now 20 years. A briefer solution will be presented in this case.

Given: *PV* = $200,000; *i* =  = 3.3%; *n* = 12(20) = 240; *c* =  = 

 = – 1 = 0.005425865

Solve for *PMT* giving *PMT* = $1492.45

Balance after 12 payments = $194,964.16.

*a.* Balance after the $10,000 prepayment = $184,964.16

Solve for *n* giving *n* = 206.26.

A total of 207 + 12 = 219 payments (months) will be required to pay off the loan.

The amortization period will be reduced by

240 – 219 = 21 months = 1 year and 9 months

*b*. Balance after the $20,000 prepayment = $174,964.16

Solve for *n* giving *n* = 186.81

A total of 187 + 12 = 199 payments (months) will be required to pay off the loan.

The amortization period will be reduced by

240 – 199 = 41 months = 3 years and 5 months

21. Given: *PV* = $100,000, *i* =  = 2.45%, *n* = 12(20) = 240, *c* =  = 

 = – 1 = 0.004042263

Solve for *PMT* giving *PMT* = $651.74

Balance after 12 payments = $96,962.90

Increased payment = 1.10($651.74) = $716.91

Balance after 12 more payments = $92,975.50

Payment after second increase = 1.10($716.91) = $788.60

Solve for the remaining *n* giving *n* = 160.47

Total time to pay off loan = 161 + 12 + 12 = 185 months

Reduction of the amortization period = (240 – 185) months

= 55 months

= 4 years and 7 months

23. Given: PV = $100,000, i =  = 3.4%, n = 12(20) = 240, c =  = 

 = – 1 = 0.005588018

Solve for *PMT* giving *PMT* = $757.73

Balance after the 8th payment = $98,377.10

Balance after the extra payment = $98,377.10 – $757.73 = $97,619.37

Solve for the remaining *n* giving *n* = 228.38

Total time to pay off loan = 229 + 8 = 237 months

Amortization period will be shortened by 240 – 237 = 3 months.

25. Given: *PV* = $100,000; *i* =  = 2.15%; *n* = 12(25) = 300; *c* =  = 

 = – 1 = 0.003551648

Solve for *PMT* giving *PMT* = $542.41

Balance after the 11th payment = $97,903.33

Interest for the 12th month = 0.003551648($97,903.33) = $347.72

Balance after the 12th month = $97,903.33 + $347.72 = $98,251.05

Substitute into formula (11-2n) to obtain the remaining *n* giving *n* = 290.79

Total time to pay off loan = 291 + 12 = 303 months

The amortization period will be lengthened by 3 months.

27. Given: *PV* = $100,000; *i* =  = 1.9%; *n* = 12(25) = 300; *c* =  = 

 = – 1 = 0.003141884

Solve for *PMT* giving *PMT* = $515.23

Balance after 12 payments = $97,545.37

Monthly payments beginning in the 2nd year = 1.1($515.23 = $566.75

Balance after 12 more payments = $94,367.54

Balance after the lump payment = $84,367.54

Substitute into formula (11-2n) to obtain the remaining *n* giving *n* = 201.01

Total time to pay off the loan = 202 + 24 = 226 months

The amortization period will be reduced by (300 – 226) months = 74 months

or 6 years and 2 months.

29. Given: *PV* = $120,000; *i* =  = 2.075%; *n* = 12(25) = 300; *c* =  = 

 = – 1 = 0.003428807

Solve for *PMT* giving *PMT* = $641.02

Balance after 2 years = Balance after 24 payments = $114,267.67

Balance after the $5000 prepayment = $109,267.67

*a.* Solve for the remaining *n* using formula (11-2n) giving *n* = 256.56

Total time to repay loan = 257 + 24 = 281 months

The amortization period will be shortened by

(300 – 281) months = 19 months = 1 year and 7 months

*b*. Balance after 5 years = Balance 3 years after the $5000 prepayment

= $99,080.24

31. Given: *PV* = $200,000; *i* =  = 3.15%; *n* = 12(30) = 360; *c* =  = 

 = – 1 = 0.005182391

Solve for *PMT* giving *PMT* = $1227.39

Balance after 3 years = Balance after 36 payments = $192,465.64

Payment size beginning in Year 4 = 1.10($1227.39) = $1350.13

*a.* Use formula (11-2n) to obtain the number of payments of $1350.13 required

to pay off $192,465.64. We obtain *n* = 259.69.

Total time to pay off the loan = 260 + 36 = 296 months

Reduction in amortization period = 360 – 296 = 64 months or 5 years and 4 months.

*b*. Balance after 5 years

= Balance 24 payments after the balance reaches $192,465.64

= $183,476.71

33. Originally, *PV* = $266,000; *n* = 12(25) = 300; *i* =  = 2.25%; *c* =  = 

 = – 1 = 0.00371532

Substitute into formula (11-2) to obtain *PMT* = $1472.24

Balance after 2 years = $253,874.78

Monthly payment starting in year 3 = $1472.24 + $150 = $1622.24

With *PV* = $253,874.78 and *PMT* = $1622.24,

Balance an additional 24 payments later = $236,862.82

Balance after $10,000 prepayment = $226,862.82

*a.* Number of payments of $1622.24 to pay off the $226,862.82 balance is

 =  = 197.68

Overall, the amortization period has been reduced by

300 – (198 + 24 + 24) = 54 months = 4 years and 6 months

*b*. Balance after 5 years = Balance 12 payments after balance reached $226,862.82

= $217,316.86

**Point of Interest**

***Mortgaging Your Future!***

1. Gross amount of loan = *PV* of 300 payments discounted at *i* = %

Gross amount of $50,000 *net* loan = = $58,200.18

Gross amount of $75,000 *net* loan = = $85,499.22

For the $100,000 *net* loan,

$113,000 =

*PMT* = $693.92

For the $125,000 *net* loan,

$139,500 =

*PMT* = $856.65

3. The effective cost of borrowing is the discount rate than makes

Net amount of the loan = (*PV* of 12 payments) + (*PV* of the end-of-term balance)

For the net loan of $25,000, solve for *i* in

$25,000 =  + $29,817.30

The solution is: *i* = 2.16976% per month

 =  = 29.38%

Similarly, for the net loan of $50,000,

*i* = 1.7841% and *f* = 23.64%

For the net loan of $75,000,

*i* = 1.6008% and *f* = 20.99%

For the net loan of $100,000,

*i* = 1.5236% and *f* = 19.90%

For the net loan of $125,000,

*i* = 1.4143% and *f* = 18.36%

**Exercise 15.4**

1. The monthly payments are based on

*PV* = $21,500, *n* = 60, and *i* =  = %.

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $420.67

Since the borrower receives only $20,000, the effective interest rate per

payment interval is the value of *i* satisfying

$20,000 = $420.67

The solution is *i* = 0.00797059 per month.

The effective cost of borrowing is

= – 1 = 0.099953 = 9.995%

3. Monthly payments are based on

*PV* = $105,000, *n* = 12(15) = 180, *i* =  = 5.4%, and *c* = ,

 = – 1 = 0.008803937

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $1164.89.

*a*. Balance after *n* = 60 payments is $86,097.64

The effective interest rate per payment interval is the value of *i* satisfying

$100,000 = $1164.89 + $86,097.64

The solution is *i* = 0.00994354 per month

The effective annual cost of borrowing is

 = – 1 = 12.607%

*b*. Balance after *n* = 120 payments = $54,114.42

Solve for *i* in

$100,000 = $1164.89 + $54,114.42

The solution is *i* = 0.00959692 per month

The effective cost of borrowing is

*f* = – 1 = 12.144%.

*c*. Solve for *i* in

$100,000 = $1164.89

The solution is *i* = 0.00954005 per month

The effective cost of borrowing is

*f* = – 1 = 12.068%

5. The payments on the loan from the mortgage broker are based on

*PV* = $82,200, *i* =  = 5.125%, *n* = 120, and *c* = 

 = – 1 = 0.00836478

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $1088.00

*a.* Balance after 60 payments = $51,162.08

The effective rate per payment interval on the brokered loan is the solution to

$80,000 = $1088.00 + $51,162.08

The solution is *i* = 0.009049832.

The effective cost of borrowing on the brokered loan is

 = – 1 = 11.417%

The effective annual rate on the trust company loan (*i* =  = 5.375%) is

*f* = – 1 = 11.039%

The trust company loan's effective rate is 0.38% lower.

*b*. The effective rate per payment interval on the brokered loan is the solution to

$80,000 = $1088.00

The solution is *i* = 0.00890817.

The effective cost of borrowing is

*f* = – 1 = 11.229%

compared to *f* = 11.039% for the trust company loan.

The effective cost of borrowing from the trust company is 0.19% lower.

7. With *PV* = $40,000, *n* = 4(10) = 40, *i* =  = 3.5%, *c* =  = 0.5, and

 = – 1 = 0.017349497,

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $1395.12.

*a*. The fair market value (FMV) of the mortgage equals the present value of

the 40 payments discounted at the prevailing market rate of interest.

With *i* =  = 5.25%, the market rate per payment interval is

 = – 1 = 0.025914226

FMV = *PV* = $1395.12 = $34,488.28

*b*. With *i* =  = 4.5%, the market rate per payment interval is

 = – 1 = 0.022252415

FMV = *PV* = $1395.12 = $36,699.09

9. The fair market value (FMV) of a mortgage equals the present value of the remaining

cash flows discounted at the prevailing market rate.

*Step* 1: Calculate the payments on the mortgage having

*PV* = $60,000, *i* =  = 3.4%, *n* = 12(20) = 240, and *c* = 

 = – 1 = 0.005588018

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $454.64

*Step* 2: Calculate the balance at the end of the 5-year term.

Balance after *n* = 60 payments is $51,519.54.

*Step* 3: Calculate the combined present value of the *Step* 2 balance and the

remaining *n* = 39 payments discounted at the prevailing market rate.

*a*. At a market rate of *i* = 3%, *i*2 = – 1 = 0.004938622

FMV = $454.64 + 

= $58,605.77

*b*. Similarly, at a market rate of *i* =  = 3.4%,

 = – 1 = 0.005588018and FMV = $57,348.31

*c*. Similarly, at a market rate of *i* =  = 3.75%,

 = – 1 = 0.006154524and FMV = $56,276.22

11. Equivalent cash value = Cash payment + Fair market value of vendor take-back mortgage

*Step* 1: Calculate the monthly payment on the take-back mortgage.

*PV* = $95,000, *n* = 120, *i* =  = 4%, *c* = 

 = – 1 = 0.006558197

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $1146.09.

*Step* 2: Calculate the present value of these payments discounted at the prevailing

market rate (10.25% compounded semiannually).

 = – 1 = 0.00836478

*PV* = $1146.09 = $86,588.99

Equivalent cash value = $75,000 + $86,588.99 = $161,588.99

13. Equivalent cash value = Cash payment + FMV of the vendor take-back mortgage

*Step* 1: Calculate the balance at the end of the 5-year term.

*PV* = $100,000, *i* =  = 4%, *PMT* = $750, *c* = 

 = – 1 = 0.006558197

Balance after *n* = 60 payments is $93,103.36

*Step* 2: Calculate the combined present value of the *Step* 1 balance and the first

60 payments discounted at  = – 1 = 0.00836478

FMV = $750 + = $91,749.57

Equivalent cash value = $50,000 + $91,749.57 = $141,749.57

Offer M is worth $1749.50 more (in current dollars).

15. *Step* 1: Calculate the size of the mortgage payments.

*PV* = $80,000, *n* = 300, *i* =  = 3.75%, and *c* = 

 = – 1 = 0.006154524

Substitute into formula (11-2) and solve for *PMT* giving *PMT* = $585.24

*Step* 2: Calculate the balance after 2 years.

Balance after *n* = 24 payments is $77,605.80.

*Step* 3: *a*. Calculate 3 months' interest on the *Step* 2 balance.

Penalty = 3(0.0061545239)$77,605.80 = $1432.88

*Step* 4: *b.* Calculate the current market value of the mortgage by discounting

the remaining 36 payments and the end-of-term balance at

 = – 1 = 0.004938622

Balance at the end of the 5-year term (*n* = 60) will be $73,283.83

FMV = $585.24 + = $80,632.61

*Step* 5: Calculate the part *b* prepayment penalty.

Penalty = $80,632.61 – $77,605.80 = $3026.81

The larger of the alternative penalties is $3026.81.

**Review Problems**

1. Amount to be paid off = $1150(1 – 0.25) = $862.50.

The present value of the six payments is $862.50.

Given: *PV* = $862.50, *n* = 6, *i* =  = 0.9375%

Substitute into formula (11-2) to obtain *PMT* = $148.50.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Payment* |  | *Interest* | *Principal* | *Principal* |
|  | *number* | *Payment* | *portion* | *portion* | *balance* |
|  | 0 |  |  |  | $862.50 |
|  | 1 | $148.50 | $8.09 | $140.41 | 722.09 |
|  | 2 | 148.50 | 6.77 | 141.73 | 580.36 |
|  | 3 | 148.50 | 5.44 | 143.06 | 437.30 |
|  | 4 | 148.50 | 4.10 | 144.40 | 292.90 |
|  | 5 | 148.50 | 2.75 | 145.75 | 147.15 |
|  | 6 | 148.53 | 1.38 | 147.15 | 0.00 |
|  |  | Total: | $28.53 |  |  |

3. Given: *PV* = $28,000, *i* =  = 2%, *n* = 4(7) = 28

Substitute into formula (11-2) to obtain *PMT* = $1315.71

*a.* Principal part of Payment 6 = Balance after Payment 5 – Balance after Payment 6

= $24,067.25 – $23,232.89

= $834.36

*b*. Interest part of Payment 22 = *i* × Balance after Payment 21

= 0.02($8515.29)

= $170.31

*c*. Principal paid by Payment 10 to Payment 15 inclusive

= Balance after Payment 9 – Balance after Payment 15

= $20,628.33 – $14,931.19

= $5697.14

*d.* Interest paid in Year 2 = 4*PMT* – (Balance after Payment 4 – Balance after Payment 8)

= 4($1315.71) – ($24,885.26 – $21,513.76)

= $1891.34

5. Given: *PV* = $60,000, *i* =  = 5.25%, *PMT* = $10,000

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Payment** |  | **Interest** | **Principal** | **Principal** |
|  | **number** | **Payment** | **portion** | **portion** | **balance** |
|  | 0 |  |  |  | $60,000.00 |
|  | 1 | $10,000.00 | $3150.00 | $6850.00 | 53,150.00 |
|  | 2 | 10,000.00 | 2790.38 | 7209.62 | 45,940.38 |
|  | 3 | 10,000.00 | 2411.87 | 7588.13 | 38,352.25 |
|  | 4 | 10,000.00 | 2013.49 | 7986.51 | 30,365.74 |
|  | 5 | 10,000.00 | 1594.20 | 8405.80 | 21,959.94 |
|  | 6 | 10,000.00 | 1152.90 | 8847.10 | 13,112.84 |
|  | 7 | 10,000.00 | 688.42 | 9311.58 | 3801.26 |
|  | 8 | 4000.83 | 199.57 | 3801.26 | 0.00 |
|  |  | Total: | $14,000.83 |  |  |

7. Given: *PV* = $45,000, *i* =  = 0.4%, *PMT* = $400

*a*. Interest in Payment 37 = *i* × Balance after Payment 36

= 0.004($36,499.62)

= $146.00

*b*. Principal part of Payment 92 = Balance after Payment 91 – Balance after Payment 92

= $20,908.36 – $20,591.99

= $316.37

*c*. Interest paid by Payment 85 to Payment 96 inclusive

= 12*PMT* – (Balance after Payment 84 – Balance after Payment 96)

= 12($400) – ($23,087.92 – $19,313.83)

= $1025.91

*d*. Principal paid in Year 5 = Balance after Payment 48 – Balance after Payment 60

= $33,383.64 – $30,114.76

= $3268.88

*e.* The total number of payments is

 =  = 149.75797

Final payment = (1 + *i*) × Balance after Payment 149 = 1.004($302.13) = $303.34

9. Given: *PV* = $255,000; *PMT* = $6000; *i* = 2.5%; *c* =  = 0.25

 = –1 = 0.006192246

*a.* The number of payments needed to pay off the loan is

 =  = 49.47207809

Final payment = (1 + *i*2) × Balance after Payment 49

= 1.006192246($2819.62)

= $2837.08

*b*. Interest in Payment 27 = *i*2 × Balance after Payment 26

= 0.006192246($130,700.59)

= $809.33

*c*. Principal part of Payment 33 = Balance after Payment 32 – Balance after Payment 33

= $99,070.44 – $93,683.91

= $5386.53

*d.* Principal paid by Payments 14 to 20 inclusive

= Balance after Payment 13 – Balance after Payment 20

= $195,342.74 – $161,180.62

= $34,162.12

*e.* Interest paid in Year 6

= 4*PMT* – (Balance after Payment 20 – Balance after Payment 24)

= 4($6000) – ($161,180.62 – $140,986.29)

= $3805.67

11. *PV* = $87,000; *i* =  = 2.2%; *n* = 12(20) = 240; *c* = 

 = – 1 = 0.003633501

Substitute into formula (11-2) to obtain *PMT* = $543.86

*a.* Balance after 16 payments = $83,255.07

Balance after $4000 prepayment = $79,255.07

The number of additional payments required to pay off this balance is

 =  = 207.88

Total time to pay off loan = 208 + 16 = 224 months. Hence, the $4000 prepayment

shortens the amortization period by 240 – 224 = 16 months or 1 year and 4 months.

*b*. Balance after 3 years = Balance 20 months after the $4000 prepayment = $73,956.77

13. Given: *PV* = $90,000, *i* =  = 2.625%, *n* = 12(20) = 240, *c* = 

 = – 1 = 0.004327902

Substitute into formula (11-2) to obtain *PMT* = $603.62

*a.* Balance after 5 years = Balance after Payment 60 = $75,367.19

*b*. For renewal of the loan after 5 years,

*PV* = $75,367.19, *n* = 12(15) = 180, *i* =  = 3.25%, *c* = 

 = – 1 = 0.00534474

Substitute into formula (11-2) to obtain *PMT* = $652.96

15. Given: *PV* = $25,000, *i* =  = 0.55%, *n* = 12(10) = 120

Substitute into formula (11-2) to obtain *PMT* = $285.14.

The “rounded” payment is $290.

Balance after 36 payments of $290 = $18,947.10

17. Given: *PV* = $110,000; *n* = 12(25) = 300; *i* =  = 3.35%; *c* = 

 = – 1 = 0.005506958

*a.* Substitute into formula (11-2) to obtain *PMT* = $750.19.

Balance after 24 payments = $106,305.17

Monthly payment after 10% increase = 1.1($750.19) = $825.21

The number of these payments required to pay off the balance is

 =  = 225.04

The total time required to pay off the loan is 226 + 24 = 250 months.

Hence, the larger payments reduce the amortization period by

(300 – 250) months = 50 months or 4 years and 2 months

*b*. Balance at end of the 5-year term = Balance 3 years after the payment increase

= Balance after 36 payments of $825.21

= $96,786.36