16 Bonds and Sinking Funds

**Concept Questions (Section 16.2)**

1. Four variables affecting a bond’s price are:

 the face value of the bond

 the bond’s coupon rate

 the prevailing market rate of return on bonds

 the time remaining until maturity of the bond

Only the prevailing market rate of return *always* has an inverse effect on the bond’s price.

3. Yes. If, during the holding period, the capital loss (due to a rise in the prevailing market rate of return) exceeds the coupon interest paid on the bond, you will suffer a net loss on the bond investment.

5. If prevailing interest rates decline, the prices of all bonds will rise. However, the prices of long-term bonds will rise more than the prices of short-term bonds. Therefore, you will improve the portfolio’s capital gain if, prior to the interest rate decline, you increase the relative weighting of long-term bonds (by selling short-term bonds and using the proceeds to purchase long-term bonds).

**Exercise 16.2**

4.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

28 **N**

28.75 **PMT**

1000  **FV**

**CPT** **PV**

*Ans*: –1128.80

1. Given: *FV* = $1000, *b* =  = 2.875%, *i* =  = 2.25%

Time until maturity = (June 1, 2030) – (June 1, 2016)

= 14 years

*b*(*FV*) = 0.02875($1000) = $28.75 and *n* = 2(14) = 28

Bond price = $28.75 + $1000

= $1128.80

3. Given: *FV* = $1000, *b* =  = 2.375%, *i* =  = 2.95%

Time until maturity = (Dec. 15, 2023) – (June 15, 2001)

5.9 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

45 **N**

23.75 **PMT**

1000  **FV**

**CPT** **PV**

*Ans*: –857.77

= 22½ years

*b*(*FV*) = 0.02375($1000) = $23.75 and *n* = 2(22.5) = 45

Bond price = $23.75 + $1000

= $857.77

5. Given: *FV* = $1000, *b* =  = 3%, *i* =  = 2%

4 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

27 **N**

30 **PMT**

1000  **FV**

**CPT** **PV**

*Ans*: –1207.07

Time until maturity = (May 15, 2030) – (Nov. 15, 2016)

= 13½ years

*b*(*FV*) = 0.03($1000) = $30 and *n* = 2(13½) = 27

Bond price = $30 + $1000

= $1207.07

17 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

41 **N**

44 **PMT**

1000  **FV**

**CPT** **PV**

*Ans*: –534.66

7. Given: *FV* = $1000, *b* =  = 4.4%, *i* =  = 8.5%

Time until maturity = (Mar. 15, 2032) – (Sept. 15, 2011)

= 20½ years

*b*(*FV*) = 0.044($1000) = $44 and *n* = 2(20½) = 41

Bond price = $44 + $1000

= $534.66

9. Given: *FV* = $10,000, *b* =  = 3.25%, *i* =  = 2.9%

5.8 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

10 **N**

325 **PMT**

10000  **FV**

**CPT** **PV**

*Ans*: –10,300.09

*b*(*FV*) = 0.0325($10,000) = $325 and *n* = 2(5) = 10

Bond price = $325 + $10,000

= $10,300.09

11. Given: *FV* = $1000, *b* =  = 3.25%, *i* =  = 2.75%

5.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

27 **N**

32.50 **PMT**

1000  **FV**

**CPT** **PV**

*Ans*: –1094.41

*b*(*FV*) = 0.0325($1000) = $32.50 and *n* = 2(13.5) = 27

Bond price = $32.50 + $1000

= $1094.41

Bond premium = $1094.41 – $1000 = $94.41

13. Given: *FV* = $5000, *b* =  = 2.875%, *i* =  = 3.25%

6.5 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

32 **N**

143.75 **PMT**

5000  **FV**

**CPT** **PV**

*Ans*: –4630.39

*b*(*FV*) = 0.02875($5000) = $143.75 and *n* = 2(16) = 32

Bond price = $143.75 + $5000

= $4630.39

Bond discount = $5000 – $4630.39 = $369.61

7.2 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

30 **N**

600 **PMT**

20000  **FV**

**CPT** **PV**

*Ans*: –17,820.35

15. Given: *FV* = $20,000, *b* =  = 3%,

*b*(*FV*) = 0.03($20,000) = $600

On the purchase date, *i* =  = 3.6%, *n* = 2(15) = 30

Purchase price = $600 + $20,000

= $17,820.35

Same *P/Y,* *C/Y*

Same *PMT*, *FV*

4.9 **I/Y**

14 **N**

**CPT** **PV**

*Ans*: –21,290.47

Today, *i* =  = 2.45%, *n* = 2(7) = 14

Current value = $600 + $20,000

= $21,290.47

The bond is worth

$21,290.47 – $17,820.35 = $3470.12 more today.

17. Given: For Bond C, *FV* = $1000, *b* =  = 2.1%, *n* = 2(3) = 6

For Bond D, *FV* = $1000, *b* =  = 2.1%, *n* = 2(23) = 46

At the initial market rate of *i* =  = 2.25%,

Price(Bond C) = $21 + $1000 = $991.67

Price(Bond D) = $21 + $1000 = $957.29

At the final market rate of *i* =  = 2.4%,

Price(Bond C) = $21 + $1000 = $983.42

Price(Bond D) = $21 + $1000 = $916.99

If the prevailing market rate of return rises from 4.5% compounded semi-annually to 4.8% compounded semi-annually, Bond C’s price will fall by $8.25 while Bond D’s price will fall by $40.30. This outcome demonstrates that, other things being equal,

*Bond prices fall when market rates rise, and the price of a longer-term bond falls more than the price of a shorter-term bond.*

6 **I/Y**

**P/Y** 2 **ENTER**

(making *C/Y =* *P/Y* = 2)

10 **N**

35 **PMT**

1000  **FV**

**CPT** **PV**

*Ans*: –1042.65

19. For all four bonds, *FV* = $1000, *b* =  = 3.5%,

and *i* =  = 3%. *n* = 10, 20, 30, and 50 for

bonds A, B, C, and D, respectively.

Price of A = $35 + $1000 = $1042.65

Bond Premium = $1042.65 - $1000 = $42.65

Price of B = $35 + $1000 = $1074.39

Bond Premium = $74.39

Same *I/Y,* *P/Y, C/Y*

Same *FV, PMT*

20 **N**

**CPT** **PV**

*Ans*: –1074.39

Same *I/Y,* *P/Y, C/Y*

Same *FV, PMT*

30 **N**

**CPT** **PV**

*Ans*: –1098.00

Same *I/Y,* *P/Y, C/Y*

Same *FV, PMT*

50 **N**

**CPT** **PV**

*Ans*: –1128.65

Similarly, price of C = $1098.00 with a bond premium of $98 and price of D = $1128.65 with a bond premium of $128.65

The results demonstrate that (for the same spread of the market rate *below*

the coupon rate) the *longer* maturity of a bond, the *larger the bond’s* *premium*.

21. For all three bonds, *FV* = $1000, *n* = 2(20) = 40, and *i* =  = 2.5%

*b* = 3%, 3.5%, and 4% for bonds J, K, and L, respectively.

Coupon payments = $30, $35, and $40 for bonds J, K, and L, respectively.

Price of J = $30 + $1000 = $1125.51

Bond Premium = $1125.51 - $1000 = $125.51

Price of K = $35 + $1000 = $1251.03

Bond Premium = $251.03

Price of L = $40 + $1000 = $1376.54

Bond Premium = $376.54

The results demonstrate that, for bonds having the same time to maturity, a *larger*

spread of the coupon rate *above* the market rate results in a greater bond premium.

23. Given: *FV* = $1000, *b* =  = 3.5%, and *n* = 2(15) = 30

When the market rate was *i* = 3.5%, the bond traded at face value ($1000) since *i* = *b*.

*a.* If *i* rises to  = 4%, the price will drop to

Price = $35 + $1000 = $913.54

The price change will be: ** –  = $913.54 – $1000 = – $86.46

That is, the price will drop by $86.46.

*b*. If *i* rises to  = 4.5%, the price will drop to

Price = $35 + $1000 = $837.11

The price change will be: $837.11 – $1000 = – $162.89

That is, the price will drop by $162.89.

*c*. If *i* falls to  = 3%, the price will rise to

Price = $35 + $1000 = $1098.00

The price change will be: $1098.00 – $1000 = $98.00

That is, the price will rise by $98.00.

*d*. If *i* falls to  = 2.5%, the price will rise to

Price = $35 + $1000 = $1209.30

The price change will be: $1209.30 – $1000 = $209.30

That is, the price will rise by $209.30.

*e*.  =  = 1.88

The price change for a 2% interest rate increase is less than twice the price change for a 1% interest rate increase.

*f.* The price change (– $86.46) resulting from a 1% interest rate increase is smaller in magnitude than the price change ($98.00) resulting from a 1% interest rate decrease.

25. Given: *FV* = $10,000, *b* =  = 3.25%, *n* = 2(25 – 3) = 44, *i* =  = 2.8%

*a.* Coupon payment = *b*(*FV*) = 0.0325($10,000) = $325.00

Price = $325 + $10,000 = $11,130.32

*b*. Percent capital gain = ×100%

= ×100%

= 11.30%

27. Given: *b* =  = 3.25%, *n* = 2(20 – 3) = 34

The percent capital gain will be the same for all bond denominations.

Use *FV* = $1000. Then *b*(*FV*) = 0.0325($1000) = $32.50

*a.* If *i* =  = 2.75%,

Price = $32.50 + $1000 = $1109.53

Percent capital gain = ×100% = 10.95%

*b*. If *i* =  = 3.25% (= *b*), the bond's price will be its face value (*FV* = $1000).

Then, percent capital gain = 0%

*c*. If *i* =  = 3.75%,

Price = $32.50 + $1000 = $904.80

Percent capital gain = ×100% = – 9.52%

29. The answer will not depend on the bond’s face value. Use *FV* = $1000.

When the bond was purchased, *b* =  = 5%, *n* = 2(20) = 40, and *i* =  = 9.25%.

Purchase price = $50 + $1000 = $553.89

When the bond was sold, *i* =  = 4.85% and *n* = 2(15.5) = 31.

Selling price = $50 + $1000 = $1023.80

In effect, the initial $553.89 investment purchased an annuity paying $50 every 6 months for 4½ years plus a lump amount of $1023.80 at the end of the 4½ years. The rate of total return is the discount rate that makes $553.89 equal to the present value of the payments received.

Solve for *i* in

$553.89 = $50 + $1023.80

The solution is *i* = 14.246% per 6 months.

The nominal annual rate of total return was

*j* = 2*i* = 28.49% compounded semi-annually.

**Exercise 16.3**

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

30 **N**

900 **+ / –**  **PV**

25 **PMT**

1000  **FV**

**CPT** **I/Y**

*Ans*: 6.022

1. Given: *FV* = $1000, *n* = 2(15) = 30, *b* =  = 2.5%,

Bond price = $900, and *b*(*FV*) = $25.

The yield to maturity (YTM) is 2*i* where *i* is the solution to

$900 = $25 + $1000

The solution is *i* = 3.011% per 6 months

YTM = 2*i* = 6.02% compounded semi-annually.

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

10 **N**

92300 **+ / –**  **PV**

2000 **PMT**

100000  **FV**

**CPT** **I/Y**

*Ans*: 5.796

3. Given: *FV* = $100,000, *n* = 2(5) = 10, Bond price = $92,300,

*b* =  = 2%, and *b*(*FV*) = $2000.

The yield to maturity (YTM) is 2*i* where *i* is the solution to

$92,300 = $2000 + $100,000

The solution is *i* = 2.898% per 6 months

YTM = 2*i* = 5.796% compounded semi-annually.

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

10 **N**

950 **+ / –**  **PV**

32.50 **PMT**

1000  **FV**

**CPT** **I/Y**

*Ans*: 7.724

5. For both bonds, *FV* = $1000, *b* =  = 3.25%,

Bond price = $950, and *b*(*FV*) = $32.50.

For bond A, *n* = 2(5) = 10 and

$950 = $32.50 + $1000

Solving for *i* gives *i* = 3.862% per 6 months. Then

YTM = 2*i* = 7.72% compounded semi-annually.

For bond C, *n* = 2(20) = 40 and

Same *P/Y, C/Y*

Same *PV, PMT, FV*

40 **N**

**CPT** **I/Y**

*Ans*: 6.967

$950 = $32.50 + $1000

Solving for *i* gives *i* = 3.484% per 6 months. Then

YTM = 2*i* = 6.97% compounded semi-annually.

7. Currently, *i* = *b*. Consequently, the bond currently trades at par ($5000).

*a.* If the price rises to $5100 with *n* = 2(3) = 6,

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

6 **N**

5100 **+ / –**  **PV**

150 **PMT**

5000  **FV**

**CPT** **I/Y**

*Ans*: 5.271

*b* =  = 3%, *FV* = $5000, and *b*(*FV*) = $150

Then, YTM = 2*i* where

$5100 = $150 + $5000

Solving for *i* gives *i* = 2.635% and

YTM = 2*i* = 5.27% compounded semi-annually.

Therefore, the YTM decreased by

6.00% – 5.27% = 0.73% compounded semi-annually.

*b*. If the price rises to $5100 with *n* = 2(15) = 30,

*b* =  = 3%, *FV* = $5000, and *b*(*FV*) = $150

Same *P/Y, C/Y*

Same *PV, PMT, FV*

30 **N**

**CPT** **I/Y**

*Ans*: 5.799

Then, YTM = 2*i* where

$5100 = $150 + $5000

Solving for *i* gives *i* = 2.899% and

YTM = 2*i* = 5.799% compounded semi-annually.

The YTM decreased by 6.00% – 5.80% = 0.20% compounded semi-annually.

**P/Y** 2 **ENTER**

(making *C/Y* = *P/Y* = 2)

13 **N**

761.50 **+ / –**  **PV**

56.25 **PMT**

1000  **FV**

**CPT** **I/Y**

*Ans*: 17.544

9. Given: *FV* = $1000, *b* =  = 5.625%, *b*(*FV*) = $56.25

*n* = 2(6.5) = 13, Bond price = $761.50.

The YTM was 2*i* where *i* is the solution to

$761.50 = $56.25 + $1000

Solving for *i* gives *i* = 8.772%

Hence, the YTM was

2*i* = 17.54% compounded semi-annually.

**Exercise 16.4**

**2nd** **Bond**

6.1519 **ENTER**

8 **ENTER**

6.0130 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

5.25 **ENTER**

**CPT**

*Ans*: *PRI* = 122.6973

*Ans*: *AI* = 0.3060

Flat price = 10($122.6973

+ $0.3060) = $1230.03

1. Given: *FV* = $1000, *b* =  = 4%, *i* =  = 2.625%

Time from preceding interest payment date to maturity

= (June 1, 2030) – (June 1, 2019)

= 11 years = 22 half-years

Price (on June 1, 2019)

= $40 + $1000

= $1227.597

Time from preceding interest payment date

to the purchase date

= (June 15, 2019) – (June 1, 2019) = 14 days

Number of days in current interest payment interval

= (Dec. 1, 2019) – (June 1, 2019) = 183 days

Price (on June 15, 2019) = $1227.597

= $1230.03

3. Given: *FV* = $1000, *b* =  = 2%, *i* =  = 2.25%

**2nd** **Bond**

4.1506 **ENTER**

4 **ENTER**

1.0121 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

4.5 **ENTER**

**CPT**

*Ans*: *PRI* = 94.6566

*Ans*: *AI* = 1.1492

Flat price = 10($94.6566

+ $1.1492) = $958.06

Time from preceding interest payment date to maturity

= (Jan. 1, 2021) – (Jan. 1, 2006) = 30 half-years

Price (on Jan. 1, 2006)

= $20 + $1000

= $945.88

Time from preceding interest payment date

to the purchase date

= (Apr. 15, 2006) – (Jan. 1, 2006) = 104 days

Number of days in current interest payment interval

= (July 1, 2006) – (Jan. 1, 2006) = 181 days

Price (on Apr. 15, 2006) = $945.88 = $958.06

5. Given: *FV* = $1000, *b* =  = 3.05%, *i* =  = 2.45%

**2nd** **Bond**

12.1519 **ENTER**

6.1 **ENTER**

8.0135 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

4.9 **ENTER**

**CPT**

*Ans*: *PRI* = 112.9913

*Ans*: *AI* = 2.2543

Flat price = 10($112.9913

+ $2.2543) = $1152.46

Time from preceding interest payment date to maturity

= (Aug. 1, 2035) – (Aug. 1, 2019) = 32 half-years

Price (on Aug. 1, 2019)

= $30.50 + $1000

= $1132.022

Time from preceding interest payment date to purchase date

= (Dec. 15, 2019) – (Aug. 1, 2019) = 136 days

Number of days in current interest payment interval

= (Feb. 1, 2020) – (Aug. 1, 2019) = 184 days

Price (on Dec. 15, 2019) = $1132.022

= $1152.46

7. Given: *FV* = $1000, *b* =  = 2.6%, *i* =  = 2.85%

**2nd** **Bond**

3.2514 **ENTER**

5.2 **ENTER**

12.0132 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

5.7 **ENTER**

**CPT**

*Ans*: *PRI* = 94.2884

*Ans*: *AI* = 1.6286

Flat price = 10($94.2884

+ $1.6286) = $959.17

Time from preceding interest payment date to maturity

= (Dec. 1, 2032) – (Dec. 1, 2013) = 38 half-years

Price (on Dec. 1, 2013)

= $26 + $1000

= $942.43

Time from preceding interest payment date to purchase date

= (March 25, 2014) – (Dec. 1, 2013) = 114 days

Number of days in current interest payment interval

= (June 1, 2014) – (Dec. 1, 2013) = 182 days

Price (on March 25, 2014)

= $942.43

= $959.17

9. Given: *FV* = $1000, *b* =  = 3.25%, *i* =  = 2.375%

**2nd** **Bond**

6.1120 **ENTER**

6.5 **ENTER**

10.1539 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

4.75 **ENTER**

**CPT**

*Ans*: *PRI* = 121.9761

*Ans*: *AI* = 1.0123

Flat price = 10($121.9761

+ $1.0123) = $1229.88

Time from preceding interest payment date to maturity

= (Oct. 15, 2039) – (Apr. 15, 2020) = 39 half-years

Price (on Apr. 15, 2020)

= $32.50 + $1000

= $1220.92

Time from preceding interest payment date to purchase date

= (June 11, 2020) – (Apr. 15, 2020) = 57 days

Number of days in current interest payment interval

= (Oct. 15, 2020) – (Apr. 15, 2020) = 183 days

Price (on June 11, 2020)

= $1220.92

= $1229.88

11. Given: *FV* = $1000, *b* =  = 5%, *i* =  = 2.75%

**2nd** **Bond**

6.0120 **ENTER**

10 **ENTER**

7.1536 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

5.5 **ENTER**

**CPT**

*Ans*: *PRI* = 147.6878

*Ans*: *AI* = 3.7912

Flat price = 10($147.6878

+ $3.7912) = $1514.79

Time from preceding interest payment date to maturity

= (July 15, 2036) – (January 15, 2020)

= 33 half-years

Price (on Jan. 15, 2020)

= $50 + $1000

= $1483.95

Time from preceding interest payment date to purchase date

= (June 1, 2020) – (January 15, 2020) = 138 days

Number of days in current interest payment interval

= (July 15, 2020) – (January 15, 2020) = 182 days

Price (on June 1, 2020)

= $1483.95

= $1514.79

13. Given: *FV* = $1000, *b* =  = 2.6%, *i* =  = 3%

Time from preceding interest payment date to maturity

= (Mar. 15, 2039) – (Mar. 15, 2020) = 38 half-years

Price (on Mar. 15, 2020) = $26 + $1000 = $910.03

Number of days in current interest payment interval

= (Sept. 15, 2020) – (Mar. 15, 2020) = 184 days

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | *No. days since* |  |
|  | *Date* | *Mar. 15, 2020* | *Price* |
|  | Apr. 15, 2020 | 31 | $910.03 = $914.57 |
|  | May 15, 2020 | 61 | $910.03 = $918.99 |
|  | June 15, 2020 | 92 | $910.03 = $923.58 |
|  | July 15, 2020 | 122 | $910.03 = $928.04 |
|  | Aug. 15, 2020 | 153 | $910.03 = $932.67 |

Price after interest payment on Sept. 15, 2020

= $26 + $1000 = $911.33

15. Quoted price = Flat price – Accrued interest

= $1065.50 – Accrued interest

Time from preceding interest payment date to quotation date

= August 1 – May 15

= 78 days

Number of days in current interest payment interval

= November 15 – May 15

= 184 days

Accrued interest = *Prt* = (*FV*)*bt* = $1000(0.038) = $16.11

Quoted price = $1065.50 – $16.11 = $1049.39

The quoted price is 104.94% of face value.

17. Given: Quoted price (on Oct. 23) = 108.50% of face value

*FV* = $1000, *b* =  = 3.6% paid on Mar. 1 and Sept. 1

Time from preceding interest payment date to quotation date

= October 23 – September 1

= 52 days

Number of days in current interest payment interval

= March 1 – September 1

= 184 days

Accrued interest = (Coupon payment) × (Fraction of payment interval that has elapsed)

= $36

= $10.17

Flat price = Quoted price + Accrued interest = $1085.00 + $10.17 = $1095.17

19. In the solution to Problem 4, it was determined that the flat

price on June 1, 2011 was $923.19, and that 73 days of the

184-day interest payment interval had elapsed. Hence,

Accrued interest = $25 = $9.92

Quoted price = Flat price – Accrued interest = $923.19 – $9.92 = $913.27

**Note regarding Problems 20-22: Bond yields in Table 16.2 are rounded to 4 figures. Therefore, we can expect bond prices calculated using these yields to have no better than 3-figure accuracy. Keep this in mind when comparing the “quoted prices” calculated in Problems 20–22 to the “quoted prices” presented in Table 16.2.**

21. Given: Purchase date = April 15, 2019

**2nd** **Bond**

4.1519 **ENTER**

1.35 **ENTER**

3.0822 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

1.718 **ENTER**

**CPT**

*Ans*: *PRI* =98.96

Coupon rate = 1.35%, *FV* = $100,

Maturity date = March 8, 2022

*YTM* = 1.718%

The computed price agrees with the given quoted price of $98.97 (within 3 figure accuracy in calculations)

23. Given: Purchase date = April 15, 2019

**2nd** **Bond**

4.1519 **ENTER**

8.9 **ENTER**

8.1822 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

122.74 **ENTER**

**CPT**

*Ans*: *YLD* = 1.856

Coupon rate = 8.9%, *FV* = $100,

Maturity date = August 18, 2022

Quoted price = 122.74

The computed yield agrees with the

quoted yield of 1.849%.

(within 3 figure accuracy in calculations)

25. Given: *FV* = $5000, *b* =  = 3.5%, *b*(*FV*) = $175,

**2nd** **Bond**

1.2516 **ENTER**

7 **ENTER**

1.2135 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

6.5 **ENTER**

**CPT**

*Ans*: *PRI* = 105.4079

*Ans*: *AI* = 0.0769

Flat price = 50($105.4079

+ $0.0769) = $5274.24

Maturity date = January 21, 2035

On the date of purchase (Jan. 25, 2016), *i* =  = 3.25%

On the date of sale (January 13, 2017), *i* =  = 2.6%

*a.* *Step* 1: Determine the purchase price.

Time from preceding interest payment date to maturity

= (January 21, 2035) – (January 21, 2016)

= 38 half-years

Price (Jan. 21, 2016)

= $175 + $5000

= $5270.537

Time from preceding interest payment date to purchase date

= (January 25, 2016) – (January 21, 2016) = 4 days

Number of days in current interest payment interval

= (July 21, 2016) – (January 21, 2016) = 182days

Price (January 25, 2016) = $5270.537

= $5274.24

*Step* 2: Determine the selling price.

**2nd** **Bond**

1.1317 **ENTER**

7 **ENTER**

1.2135 **ENTER**

100 **ENTER**

Set at *“ACT”*

Set at *“2/Y”*

5.2 **ENTER**

**CPT**

*Ans*: *PRI* = 120.8895

*Ans*: *AI* = 3.3478

Flat price = 50($120.8895

+ $3.3478) = $6211.87

Time from preceding interest payment date to maturity

= (January 21, 2035) – (July 21, 2016)

= 37 half-years

Price (July 21, 2016)

= $175 + $5000

= $6061.213

Time from preceding interest payment date to date of sale

= (Jan. 13, 2017) – (July 21, 2016)

= 176 days

Number of days in current interest payment interval

= (Jan. 21, 2017) – (July 21, 2016)

= 184 days

Price (Jan. 13, 2017) = $6061.213

= $6211.87

*Step* 3: Calculate the capital gain.

Capital gain = $6211.87 – $5274.24 = $937.63

*b*. Percent capital gain =  × 100% = 17.78%

**Exercise 16.5**

1. Given: *FV* = $12,000,000, *n* = 2(10) = 20, *i* =  = 3.5%

*a*. Substitute into formula (11-1) and solve for *PMT*.

$12,000,000 = *PMT*

*PMT* = $424,333

*b*. The balance after the 12th interval (and payment) will be

*FV* = $424,333 = $6,196,094

3. Given: *FV* = $15,000,000, *n* = 15, *i* = 6.5%

*a*. Substitute into formula (11-1) and solve for *PMT*.

$15,000,000 = *PMT*

*PMT* = $620,292

*b*. The balance after the 11th interval will be

*FV* = $620,292 = $9,534,856

5. Given: *FV*(due) = $6,000,000, *i* =  = 0.4375%, *n* = 12(5) = 60

*a*. Substitute into formula (13-1) and solve for *PMT*.

$6,000,000 = *PMT*(1.004375)

*PMT* = $87,284

*b*. The balance at the end of the 27th interval will be

*FV*(due) = $87,284(1.004375) = $2,506,640

7. Given: *FV*(due) = $18,000,000, *n* = 2(15) = 30, *i* =  = 1.375%

*a*. Substitute into formula (13-1) and solve for *PMT*.

$18,000,000 = *PMT*(1.01375)

*PMT* = $482,162

*b*. The balance at the end of the 19th interval will be

*FV*(due) = $482,162(1.01375) = $10,530,997

9. Given: *FV* = $10,000,000, *n* = 2(10) = 20, *i* =  = 3.5%, 2*b* = 10%

*a*. Substitute into formula (11-1) and solve for *PMT*.

$10,000,000 = *PMT*

*PMT* = $353,611

*b*. Annual cost of the debt = 2*PMT* + (2*b*)$10,000,000

= 2($353,611) + $1,000,000

= $1,707,222

*c*. Balance after 12th interval = *FV* = $353,611 = $5,163,414

Book value of debt = $10,000,000 – $5,163,414 = $4,836,586

11. Given: *FV* = $15,000,000, *n* = 2(15) = 30, *i* =  = 3.25%, 2*b* = 9%

*a.* Substitute into formula (11-1) and solve for *PMT*.

$15,000,000 = *PMT*

*PMT* = $302,726

*b.* Annual cost of the debt = 2*PMT* + (2*b*)($15,000,000)

= 2($302,726) + 0.09($15,000,000)

= $1,955,452

*c.* Book value of debt after the 21st interval = $15,000,000 – $302,726

= $6,081,667

13. Given: *FV* = $7,000,000, *n* = 2(5) = 10, *i* =  = 2.875%, 2*b* = 8%

*a*. Substitute into formula (11-1) and solve for *PMT*.

$7,000,000 = *PMT*

*PMT* = $614,137

*b*. Annual cost of the debt = 2*PMT* + 2*b*($7,000,000)

= 2($614,137) + 0.08($7,000,000)

= $1,788,274

*c*. Book value of debt after the 7th interval = $7,000,000 – $614,137

= $2,311,969

15. Given: *FV* = $11,000,000, *n* = 2(15) = 30, *i* =  = 3.75%, 2*b* = 10.25%

*a*. Substitute into formula (11-1) and solve for *PMT*.

$11,000,000 = *PMT*

*PMT* = $204,464

*b*. Annual cost of the debt = 2*PMT* + 2*b*($11,000,000)

= 2($204,464) + 0.1025($11,000,000)

= $1,536,428

*c*. Book value of debt after the 19th interval = $11,000,000 – $204,464

= $5,478,509

17. Given: *FV* = $800,000, *n* = 2(3) = 6, *i* =  = 3.5%

Substitute into formula (11-1) and solve for *PMT*.

$800,000 = *PMT* 

*PMT* = $122,135

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Payment* |  |  |  | *Balance in* |
|  | *interval* | *Payment* | *Interest* | *Increase* | *fund (end* |
|  | *number* | *(at end)* | *earned* | *in the fund* | *of interval)* |
|  | 0 | — | — | — | $0 |
|  | 1 | $122,135 | $0 | $122,135 | 122,135 |
|  | 2 | 122,135 | 4275 | 126,410 | 248,545 |
|  | 3 | 122,135 | 8699 | 130,834 | 379,379 |
|  | 4 | 122,135 | 13,278 | 135,413 | 514,792 |
|  | 5 | 122,135 | 18,018 | 140,153 | 654,945 |
|  | 6 | 122,135 | 22,923 | 145,058 | 800,003 |
|  |  | Total: | $67,193 | $800,003 |  |

Total interest = $800,003 – 6($122,135) = $67,193

19. Given: *FV*(due) = $1,000,000, *n* = 5, *i* = 6.75%

Substitute into formula (13-1) and solve for *PMT*

$1,000,000 = *PMT*(1.0675)

*PMT* = $163,710

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Payment* |  |  |  | *Balance in* |
|  | *interval* | *Payment* | *Interest* | *Increase* | *fund (end* |
|  | *number* | *(at start)* | *earned* | *in the fund* | *of interval)* |
|  | 0 | — | — | — | $0 |
|  | 1 | $163,710 | $11,050 | $174,760 | 174,760 |
|  | 2 | 163,710 | 22,847 | 186,557 | 361,317 |
|  | 3 | 163,710 | 35,439 | 199,149 | 560,466 |
|  | 4 | 163,710 | 48,882 | 212,592 | 773,058 |
|  | 5 | 163,710 | 63,232 | 226,942 | 1,000,000 |
|  |  | Total: | $181,450 | $1,000,000 |  |

Total interest = $1,000,000 – 5($163,710) = $181,450

21. From the solution to Problem 2, *PMT* = $302,720, *n* = 2(10) = 20, *i* = 3%/2 = 1.5%

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Payment* |  |  |  | *Balance in* |
|  | *interval* | *Payment* | *Interest* | *Increase* | *fund (end* |
|  | *number* | *(at end)* | *earned* | *in the fund* | *of interval)* |
|  | 0 | — | — | — | $0 |
|  | 1 | $302,720 | $0 | $302,720 | 302,720 |
|  | 2 | 302,720 | 4541 | 307,261 | 609,981 |
|  | | | | | | | | | | |
|  | 10 |  |  |  | 3,239,928 |
|  | 11 | 302,720 | 48,599 | 351,319 | 3,591,247 |
|  | 12 | 302,720 | 53,869 | 356,589 | 3,947,836 |
|  | | | | | | | | | | |
|  | 18 |  |  |  | 6,202,544 |
|  | 19 | 302,720 | 93,038 | 395,758 | 6,598,302 |
|  | 20 | 302,720 | 98,975 | 401,695 | 6,999,997 |

23. From the solution to Problem 9,

*PMT* = $353,611, *n* = 2(10) = 20, *i* = 7%/2 = 3.5%

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *Payment* |  |  |  | *Balance in* |  |
|  | *interval* |  | *Interest* | *Increase* | *fund (end* | *Book value* |
|  | *number* | *Payment* | *earned* | *in the fund* | *of interval)* | *of the debt* |
|  | 0 | — | — | — | $0 | $10,000,000 |
|  | 1 | $353,611 | $0 | $353,611 | 353,611 | 9,646,389 |
|  | 2 | 353,611 | 12,376 | 365,987 | 719,598 | 9,280,402 |
|  | | | | | | | | | | | | |
|  | 18 |  |  |  | 8,663,360 | 1,336,640 |
|  | 19 | 353,611 | 303,218 | 656,829 | 9,320,189 | 679,811 |
|  | 20 | 353,611 | 326,207 | 679,818 | 10,000,007 | (7) |

25. *a*. Given: *FV*(due) = $600,000, *n* = 12(5) = 60, *i* =  = 0.625%

$600,000 = *PMT*(1.00625)

*PMT* = $8221

*b*. Interest earned in Year 4

= Balance after Year 4 – Balance after Year 3 – 12*PMT*

= $8221(1.00625) – $8221(1.00625) –12($8221)

= $461,399 – $332,809 – $98,652

= $29,938

*c*. Solve for *n* in $300,000 = $8221(1.00625)

*n* = 32.79

Hence, the fund will pass the halfway point in the 33rd month.

*d*. Interest earned in the 35th month

= *i* × (Balance after the 34th month + *PMT*)

= 0.00625

= 0.00625($312,297 + $8221) = $2003

27. Given: *FV* = $20,000,000, *n* = 2(20) = 40, *i* =  = 2.25%

*a*. $20,000,000 = *PMT*

*PMT* = $313,548

*b*. Interest earned in Year 6

= Balance after Year 6 – Balance after Year 5 – 2*PMT*

= $313,548 – $313,548 – 2($313,548)

= $4,264,949 – $3,472,766 – $627,096

= $165,087

*c*. Increase in the fund during Interval 27

= Balance after Interval 27 – Balance after Interval 26

= $313,548 – $313,548

= $559,178

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *d.* | *Payment* |  |  |  | *Balance in* |  |
|  | *interval* | *Payment* | *Interest* | *Increase* | *fund (end* | *Book value* |
|  | *number* | *(at end)* | *earned* | *in the fund* | *of interval)* | *of the debt* |
|  | 0 | --- | --- | --- | $0 | $20,000,000 |
|  | 1 | $313,548 | $0 | $313,548 | 313,548 | 19,686,452 |
|  | 2 | 313,548 | 7,055 | 320,603 | 634,151 | 19,365,849 |
|  | | | | | | | | | | | | |
|  | 38 | --- | --- | --- | 18,522,966 | 1,477,034 |
|  | 39 | 313,548 | 416,767 | 730,315 | 19,253,281 | 746,719 |
|  | 40 | 313,548 | 433,199 | 746,747 | 20,000,028 | (28) |
|  |  | Total: | $7,458,108 |  |  |  |

Total Interest = $20,000,028 − 40($313,548) = $7,458,108

29. Given: *FV*(due) = $800,000, *n* = 4(10) = 40, *i* =  = 1.75%

*a*. $800,000 = *PMT*(1.0175)

*PMT* = $13,737

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *b.* | *Payment* |  |  |  | *Balance in* |  |
|  | *interval* |  | *Interest* | *Increase* | *fund (end* | *Book value* |
|  | *number* | *Payment* | *earned* | *in the fund* | *of interval)* | *of the debt* |
|  | 0 | — | — | — | $0 | $800,000 |
|  | 1 | $13,737 | $240 | $13,977 | 13,977 | 786,023 |
|  | 2 | 13,737 | 485 | 14,222 | 28,199 | 771,801 |
|  | | | | | | | | | | | | |
|  | 38 |  |  |  | 745,465 | 54,535 |
|  | 39 | 13,737 | 13,286 | 27,023 | 772,488 | 27,512 |
|  | 40 | 13,737 | 13,759 | 27,496 | 799,984 | 16 |
|  |  | Total: | $250,504 |  |  |  |

Total interest = $799,984 – 40($13,737) = $250,504

**Review Problems**

1. Given: *FV* = $1000, *n* = 2(19.5) = 39, *b* =  = 3.75%, *i* =  = 4.3%

Bond price = $37.50 + $1000 = $896.86

Bond discount = $1000 – $896.86 = $103.14

3. Given: For each bond, *FV* = $1000 and *b* =  = 4.25%

At the time the bonds were purchased, *n* = 2(18) = 36 and *i* =  = 4.9%

Purchase price = $42.50 + $1000 = $891.05

Now, *n* = 2(13.5) = 27 and *i* =  = 4.0%

Current price = $42.50 + $1000 = $1040.82

Capital gain = 15($1040.82 – $891.05) = $2246.55

5. The answer will be the same for any face value. Use *FV* = $1000.

Currently, *n* = 2(22.5) = 45, *b* =  = 4%, and *i* =  = 3.45%.

Current price = $40 + $1000 = $1124.77

Percent capital gain = ×100% = 12.48%

7. Given: *FV* = $1000, *b* =  = 3.8%, *i* =  = 2.95%

Time from preceding interest payment date to maturity

= (Nov. 1, 2031) – (May 1, 2019) = 12½ years = 25 half-years

Price (on May 1, 2019) = $38 + $1000 = $1148.84

Time from preceding interest payment date to June 10, 2019

= (June 10, 2019) – (May 1, 2019) = 40 days

Number of days in current interest payment interval

= (Nov. 1, 2019) – (May 1, 2019) = 184 days

Price (on June 10, 2019) = $1148.84 = $1156.12

9. Given: *FV* = $500,000, *n* = 2(7) = 14, *i* = 7%/2 = 3.5%

Substitute into formula (11-1) and solve for *PMT*.

$500,000 = *PMT*

*PMT* = $28,285

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *Payment* |  |  |  | *Balance in* |  |
|  | *interval* |  | *Interest* | *Increase* | *fund (end* | *Book value* |
|  | *number* | *Payment* | *earned* | *in the fund* | *of interval)* | *of the debt* |
|  | 0 | — | — | — | $0 | $500,000 |
|  | 1 | $28,285 | $0 | $28,285 | 28,285 | 471,715 |
|  | 2 | 28,285 | 990 | 29,275 | 57,560 | 442,440 |
|  | | | | | | | | | | | | |
|  | 12 |  |  |  | 413,016 | 86,984 |
|  | 13 | 28,285 | 14,456 | 42,741 | 455,757 | 44,243 |
|  | 14 | 28,285 | 15,951 | 44,236 | 499,993 | 7 |

11. Given: *FV* = $1000, *b* =  = 3.25%, *i* = = 2.6%

Time from preceding interest payment date to maturity

= (June 15, 2036) – (June 15, 2020) = 32 half-years

Price (on June 15, 2020) = $32.50 + $1000 = $1140.04

Time from preceding interest payment date to purchase date

= (Dec. 10, 2020) – (June 15, 2020) = 178 days

Number of days in current interest payment interval

= (Dec. 15, 2020) – (June 15, 2020) = 183 days

Price (on Dec. 10, 2020) = $1140.04 = $1168.86

13. Given: *FV*(due) = $750,000, *n* = 2(3) = 6, *i* =  = 3%

Substitute into formula (13-1) and solve for *PMT*.

$750,000 = *PMT*(1.03)

*PMT* = $112,571

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Payment* |  |  |  | *Balance in* |
|  | *interval* | *Payment* | *Interest* | *Increase* | *fund (end* |
|  | *number* | *(at start)* | *earned* | *in the fund* | *of interval)* |
|  | 0 | — | — | — | $0 |
|  | 1 | $112,571 | $3377 | $115,948 | 115,948 |
|  | 2 | 112,571 | 6856 | 119,427 | 235,375 |
|  | 3 | 112,571 | 10,438 | 123,009 | 358,384 |
|  | 4 | 112,571 | 14,129 | 126,700 | 485,084 |
|  | 5 | 112,571 | 17,930 | 130,501 | 615,585 |
|  | 6 | 112,571 | 21,845 | 134,416 | 750,001 |
|  |  | Total: | $74,575 | $750,001 |  |