In the last few years, there has been a major paradigm shift in the way in which corporate finance is practised. We have come through a sustained period of deregulation and globalization in the world’s markets. Financial innovation and the introduction of new securities has been commonplace as a result of the free markets that have spread throughout the world. However, things are very much different going into the second decade of the twenty-first century.

The financial world has seen a glut of corporate insolvencies. Governments of the major developed economies have all reduced interest rates to near zero and pumped cash into their ailing firms. Whole industries have effectively been nationalized and purchased by governments. Corporate strategies that were successful because of the availability of cheap debt are no longer possible. Finally, financial instruments that may have been viable and popular in a vibrant economy have become obsolete.

Convertible bonds are part of many companies’ capital structure. They allow bondholders to convert the debt instruments into equity during a specified window in the future. The conversion feature is an embedded option that holders will exercise if the convertible is in the money. At the turn of the century, these became exceptionally popular investment targets of hedge funds that looked for a quick return from conversion. Although, corporate convertible bond issues dipped in number after the massive price declines of recent times, their popularity has come back with a vengeance as investors anticipate economic recovery in Europe and the US.

Warrants are similar to standard call options, except that the company must issue new shares if the holder chooses to exercise them. Again, because of changes in the global financial environment they have become less popular. However, they are present in the financial structure of many companies and tend to be issued in conjunction with standard bond issues.

This chapter is concerned with valuing the option embedded in these financial instruments.

24.1 Warrants

Warrants are securities that give holders the right, but not the obligation, to buy shares of equity directly from a company at a fixed price for a given period. Each warrant specifies the number of shares of equity that the holder can buy, the exercise price and the expiration date.

From the preceding description of warrants, it is clear that they are similar to call options. The differences in contractual features between warrants and the call options that trade on Euronext Liffe are small. For example, warrants have longer maturity periods. Some warrants are actually perpetual, meaning that they never expire.
Warrants are referred to as *equity kickers* because they are usually issued in combination with privately placed bonds. In most cases, warrants are attached to the bonds when issued. The loan agreement will state whether the warrants are detachable from the bond – that is, whether they can be sold separately. Usually, the warrant can be detached immediately.

In the last few years, only a very few corporations have issued warrants and these have largely been in the United States. In recent times, governments purchased warrants from banks that needed financial assistance, and in the 2012 Greek sovereign debt bailout, bondholders received GDP-linked warrants which pay off if the Greek economy beats growth expectations.

To illustrate warrants, we will focus on a hypothetical example of a firm, Hellas Shipping, which has issued warrants. Each warrant gives the holder the right to purchase one share of equity at an exercise price of €19.32 and the warrants expire in 4 years. The share price of Hellas Shipping is €17.57, and the price of a warrant is €4.05.

The relationship between the value of Hellas Shipping’s warrants and its share price can be viewed as similar to the relationship between a call option and the share price, described in a previous chapter. Figure 24.1 depicts this relationship. The lower limit on the value of the warrants is zero if Hellas Shipping’s share price is below €19.32 per share. If the price of Hellas Shipping’s equity rises above €19.32 per share, the lower limit is the share price minus €19.32. The upper limit is the share price of Hellas Shipping. A warrant to buy one share of equity cannot sell at a price above the price of the underlying shares.

The price of Hellas Shipping’s warrants was higher than the lower limit. The height of the warrant price above the lower limit will depend on the following:

1. The variance of Hellas Shipping’s share price returns
2. The time to expiration date
3. The risk-free rate of interest
4. The share price of Hellas Shipping
5. The exercise price
6. Cash dividends.

With the exception of cash dividends, these are the same factors that determine the value of a call option.

Warrants can also have unusual features. For example, when the French specialist metals group Carbone Lorraine was unable to raise financing for solar power acquisitions, it entered...
into a financing deal with Société Générale. Under the deal, Carbone Lorraine issued convertible warrants to SocGen that allowed the French bank to buy up to 17.5 per cent of Carbone Lorraine’s equity at a 10 per cent discount whenever the metals company needed funds. This innovative financing deal was a warrant, where the decision to exercise was with the issuer, not the holder. In effect, it mimicked an equity-linked credit line that bypassed the credit squeeze in the markets at the time.

### 24.2 The Difference between Warrants and Call Options

From the holder’s point of view, warrants are similar to call options on equity. A warrant, like a call option, gives its holder the right to buy shares at a specified price. Warrants usually have an expiration date, though in most cases they are issued with longer lives than call options. From the firm’s point of view, however, a warrant is very different from a call option on the company’s equity.

The most important difference between call options and warrants is that call options are issued by individuals and warrants are issued by firms. When a warrant is exercised, a firm must issue new shares of equity. Each time a warrant is exercised, then, the number of shares outstanding increases.

To illustrate, suppose Endrun Ltd issues a warrant giving holders the right to buy one share of equity at €25. Further, suppose the warrant is exercised. Endrun must print one new share certificate. In exchange for the share certificate, it receives €25 from the holder.

In contrast, when a call option is exercised, there is no change in the number of shares outstanding. Suppose Ms Eager holds a call option on the equity of Endrun. The call option gives Ms Eager the right to buy one share of the equity of Endrun for €25. If Ms Eager chooses to exercise the call option, a seller, say Mr Swift, is obligated to give her one share of Endrun’s equity in exchange for €25. If Mr Swift does not already own a share, he must enter the stock market and buy one. The call option is a side bet between buyers and sellers on the value of Endrun shares. When a call option is exercised, one investor gains and the other loses. The total number of shares outstanding of the Endrun Company remains constant, and no new funds are made available to the company.

Warrants also affect accounting numbers. Warrants and (as we shall see) convertible bonds cause the number of shares to increase. This causes the firm’s net income to be spread over more shares, thereby decreasing earnings per share. Firms with significant amounts of warrants and convertible issues must report earnings on a primary basis and a fully diluted basis.

### 24.3 Warrant Pricing and the Black–Scholes Model

We now wish to express the gains from exercising a call and a warrant in more general terms. The gain on a call can be written like this:

**Gain from exercising a single call**

\[
\text{Firm’s value net of debt} \div \# - \text{Exercise price} \quad (24.1)
\]

We define the *firm’s value net of debt* to be the total firm value less the value of the debt. The \# stands for the number of shares outstanding. The ratio on the left is the value of a share of equity. The gain on a warrant can be written as follows:

**Gain from exercising a single warrant**

\[
\text{Firm’s value net of debt} + \text{Exercise price \times \#_w} \div \# + \#_w - \text{Exercise price} \quad (24.2)
\]

(Value of a share of equity after warrant is exercised)
The numerator of the left term is the firm’s value net of debt after the warrant is exercised. It is the sum of the firm’s value net of debt prior to the warrant’s exercise plus the proceeds the firm receives from the exercise. The proceeds equal the product of the exercise price multiplied by the number of warrants. The number of warrants appears as \( w \). (Our analysis uses the plausible assumption that all warrants in the money will be exercised.) The denominator, \( \frac{1}{\#_w} \), is the number of shares outstanding after the exercise of the warrants. The ratio on the left is the value of a share of equity after exercise. By rearranging terms, we can rewrite Equation 24.2 as:

\[
\frac{\#_w}{\# + \#_w} \times \left( \frac{\text{Firm’s value net of debt}}{\#} - \text{Exercise price} \right) = \text{(Gain from a call on a firm with no warrants)}
\]

Formula 24.3 relates the gain on a warrant to the gain on a call. Note that the term within parentheses is Equation 24.1. Thus, the gain from exercising a warrant is a proportion of the gain from exercising a call in a firm without warrants. The proportion \( \frac{\#_w}{\# (\# + \#_w)} \) is the ratio of the number of shares in the firm without warrants to the number of shares after all the warrants have been exercised. This ratio must always be less than 1. Thus, the gain on a warrant must be less than the gain on an identical call in a firm without warrants.

The preceding implies that we can value a warrant using the Black–Scholes model, adjusted for the dilution effect:

\[
w = \frac{1}{\left(1 + \frac{\#_w}{\#} \right)} \cdot \text{cw}
\]

where \( \text{cw} \) is the value of a call option written on the equity of a firm without warrants.

### Example 24.1

#### Warrant Valuation

Veld NV is planning to issue 10,000 warrants that, when exercised, can be converted on a one-for-one basis. The proceeds of the warrant issuance will be distributed to the existing shareholders. Besides this dividend, the company is not planning to pay out any other cash dividend during the lifetime of the warrants. The company currently has 50,000 shares outstanding. If the share price of Veld NV is €2.50 and the exercise price of the warrants is €2.30, what is the value of the warrant today? The continuously compounded annual risk free rate of interest is 7 per cent, the variance of returns of Veld NV is 0.09 and the time to expiry is 1 year.

**Step 1.** First we need to calculate the value of a comparable call option on the firm’s equity. This is done by simply plugging the relevant values into the Black–Scholes Option Pricing Formula (see Chapter 22).

**Step 1a.** Calculate \( d_1 \) and \( d_2 \).

\[
d_1 = \left[ \ln \left( \frac{S}{E} \right) + (R + \sigma^2/2)t \right] / \sqrt{\sigma^2t} \\
= \left[ \ln \left( \frac{€2.50}{€2.30} \right) + (0.07 + 0.09/2)/1 \right] / \sqrt{0.09} \\
= [0.0834 + 0.115]/0.3 = 0.6613 \\
d_2 = d_1 - \sqrt{\sigma^2t} \\
= 0.3613
\]
**Step 1b.** Calculate $N(d_1)$ and $N(d_2)$ using a spreadsheet or tables.

\[
N(d_1) = N(0.6613) = 0.7458 \\
N(d_2) = N(0.3613) = 0.6411
\]

**Step 1c.** Calculate $c_w$.

\[
c_w = S \times [N(d_1)] - E \times e^{-rt} \times [N(d_2)] \\
= (€2.50 \times 0.7458) - (€2.30 \times 0.9324 \times 0.6411) \\
= €1.8645 - €1.3749 \\
= €0.49
\]

**Step 2.** The value of the Veld NV warrant is thus:

\[
w = \frac{1}{1 + \frac{\#}{\#}} \times \frac{1}{1 + \frac{10,000}{50,000}} \times €0.49 \\
= 0.8333(€0.49) \\
= €0.4083
\]

# 24.4 Convertible Bonds

A convertible bond is similar to a bond with warrants. The most important difference is that a bond with warrants can be separated into distinct securities and a convertible bond cannot. A convertible bond gives the holder the right to exchange it for a given number of shares any time up to and including the maturity date of the bond.

Preference shares can frequently be converted into equity. A convertible preference share is the same as a convertible bond except that it has an infinite maturity date.

## Example 24.2

**Convertibles**

At the end of 2008, AIG sold its Swiss bank, AIG Private Bank, to an Abu Dhabi investment group, Aabar Investments PJSC, for 307 million Swiss francs (€205 million). Aabar Investments was primarily an oil and gas investment company and this acquisition diversified their operations into the financial sector.

Aabar Investments was able to purchase AIG Private Bank because it had raised €1.3 billion by issuing a convertible bond to the International Petroleum Investment Company (IPIC), which is itself wholly owned by the Abu Dhabi Investment Company. The Abu Dhabi Investment Company is Abu Dhabi’s sovereign wealth fund, which invests overseas on behalf of the Abu Dhabi government.

On 24 September, Aabar Investments issued one convertible bond which was convertible into 2.228 billion new shares. The number of shares received for each bond (2.228 billion in this example) is called the conversion ratio.

Bond traders also speak of the conversion price of the bond. This is calculated as the ratio of the face value of the bond to the conversion ratio. The conversion price of the bond was 3 Emirati dirhams (AED), which meant that the face value of the Aabar convertible bond was AED6.684 billion ($3 \times 2.228$ billion) or €1.3 billion.
Conversion ratio, conversion price and conversion premium are well-known terms in the real world. For that reason alone, the student should master the concepts. However, conversion price and conversion premium implicitly assume that the bond is selling at par. If the bond is selling at another price, the terms have little meaning. By contrast, conversion ratio can have a meaningful interpretation regardless of the price of the bond.

24.5 The Value of Convertible Bonds

The value of a convertible bond can be described in terms of three components: straight bond value, conversion value and option value. We examine these three components next.

Straight Bond Value

The straight bond value is what the convertible bonds would sell for if they could not be converted into equity. It will depend on the general level of interest rates and on the default risk. Consider a convertible bond issued by a hypothetical firm, Cold Dawn plc. On 1 November 2013, Cold Dawn plc raised £300 million by issuing 6.75 per cent convertible subordinated debentures due in 2029. It planned to use the proceeds to invest in new plant and equipment. Like typical debentures, they had a sinking fund and were callable. Cold Dawn’s bonds differed from other debentures in their convertible feature: each bond was convertible into 2,353 shares of Cold Dawn equity any time before maturity. When Cold Dawn issued its convertible bonds, its share price was £22.625. The conversion price of £42.50 (\( \frac{5}{\£100,000/2,353} \)) was 88 per cent higher than the actual equity price. Suppose that straight debentures issued by Cold Dawn plc had been rated A, and A-rated bonds were priced to yield 4 per cent per 6 months on 1 November 2013. The straight bond value of Cold Dawn convertible bonds can be determined by discounting the £3,375 semi annual coupon payment and principal amount at 4 per cent:

\[
\text{Straight bond} = \sum_{t=1}^{32} \frac{\£3,375}{(1.04)^t} + \frac{\£100,000}{(1.04)^{32}}
\]

\[
= \£3,375 \times A_{0.04}^{32} + \frac{\£100,000}{(1.04)^{32}}
\]

\[
= \£60,323 + \£28,506
\]

\[
= \£88,829
\]

The straight bond value of a convertible bond is a minimum value. The price of Cold Dawn’s convertible could not have gone lower than the straight bond value.

Figure 24.2 illustrates the relationship between straight bond value and share price. In Figure 24.2 we have been somewhat dramatic and implicitly assumed that the convertible bond is default free. In this case, the straight bond value does not depend on the share price, so it is graphed as a straight line.
Conversion Value

The value of convertible bonds depends on conversion value. Conversion value is what the bonds would be worth if they were immediately converted into equity at current prices. Typically, we compute conversion value by multiplying the number of shares of equity that will be received when the bond is converted by the current price of the equity.

On 1 November 2013, each Cold Dawn convertible bond could have been converted into 2,353 shares of Cold Dawn equity. Cold Dawn shares were selling for £22.625. Thus, the conversion value was $2,353 \times £22.625 = £53,237$. A convertible cannot sell for less than its conversion value. Arbitrage prevents this from happening. If Cold Dawn’s convertible sold for less than £53,237, investors would have bought the bonds and converted them into equity and sold the shares. The profit would have been the difference between the value of the shares sold and the bond’s conversion value.

Thus, convertible bonds have two minimum values: the straight bond value and the conversion value. The conversion value is determined by the value of the firm’s underlying equity. This is illustrated in Figure 24.2. As the value of equity rises and falls, the conversion price rises and falls with it. When the value of Cold Dawn’s equity increased by £1, the conversion value of its convertible bonds increased by £2,353.

Option Value

The value of a convertible bond will generally exceed both the straight bond value and the conversion value. This occurs because holders of convertibles need not convert immediately. Instead, by waiting they can take advantage of whichever is greater in the future: the straight bond value or the conversion value. This option to wait has value, and it raises the value over both the straight bond value and the conversion value.

When the value of the firm is low, the value of convertible bonds is most significantly influenced by their underlying value as straight debt. However, when the value of the firm is
very high, the value of convertible bonds is mostly determined by their underlying conversion value. This is illustrated in Figure 24.3.

The bottom portion of the figure implies that the value of a convertible bond is the maximum of its straight bond value and its conversion value, plus its option value:

\[
\text{Value of convertible bond} = \text{The greater of} \ (\text{Straight bond value}, \text{Conversion value}) + \text{Option value}
\]

**Example 24.3**

**Conversion**

Suppose Avaya plc has outstanding 1,000 shares of equity and 100 bonds. Each bond has a face value of £100,000 at maturity. They are discount bonds and pay no coupons. At maturity each bond can be converted into 10 shares of newly issued equity.

What circumstances will make it advantageous for the holders of Avaya convertible bonds to convert to equity at maturity?

If the holders of the convertible bonds convert, they will receive 100 × 10 = 1,000 shares of equity. Because there were already 1,000 shares, the total number of shares outstanding becomes 2,000 upon conversion. Thus, converting bondholders own 50 per cent of the value of the firm, \(V\). If they do not convert, they will receive £10,000,000 or \(V\), whichever is less. The choice for the holders of the Avaya bonds is obvious. They should convert if 50 per cent of \(V\) is greater than £10,000,000. This will be true whenever \(V\) is greater than £20,000,000. This is illustrated as follows:
Reasons for Issuing Warrants and Convertibles

Probably there is no other area of corporate finance where real-world practitioners disagree as they do on the reasons for issuing convertible debt. To separate fact from fantasy, we present a rather structured argument. We first compare convertible debt with straight debt. Then we compare convertible debt with equity. For each comparison, we ask in what situations is the firm better off with convertible debt and in what situations is it worse off.

**Convertible Debt versus Straight Debt**

Convertible debt pays a lower interest rate than does otherwise identical straight debt. For example, if the interest rate is 10 per cent on straight debt, the interest rate on convertible debt might be 9 per cent. Investors will accept a lower interest rate on a convertible because of the potential gain from conversion.

Imagine a firm that seriously considers both convertible debt and straight debt, finally deciding to issue converts. When would this decision have benefited the firm and when would it have hurt the firm? We consider two situations.

**The Share Price Later Rises so that Conversion Is Indicated**

The firm clearly likes to see the share price rise. However, it would have benefited even more had it previously issued straight debt instead of a convertible. Although the firm paid out a lower interest rate than it would have with straight debt, it was obligated to sell the convertible holders a chunk of the equity at a below-market price.

**The Share Price Later Falls or Does Not Rise Enough to Justify Conversion**

The firm hates to see the share price fall. However, as long as the share price does fall, the firm is glad that it had previously issued convertible debt instead of straight debt. This is because the interest rate on convertible debt is lower. Because conversion does not take place, our comparison of interest rates is all that is needed.

**Summary**

Compared to straight debt, the firm is worse off having issued convertible debt if the underlying equity subsequently does well. The firm is better off having issued convertible debt if the underlying equity subsequently does poorly. In an efficient market, we cannot predict future share prices. Thus, we cannot argue that convertibles either dominate or are dominated by straight debt.

**Convertible Debt versus Equity**

Next, imagine a firm that seriously considers both convertible debt and equity but finally decides to issue converts. When would this decision benefit the firm and when would it hurt the firm? We consider our two situations.
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The Share Price Later Rises so that Conversion Is Indicated
The firm is better off having previously issued a convertible instead of equity. To see this, consider the Cold Dawn case. The firm could have issued equity for £22. Instead, by issuing a convertible, the firm effectively received £42.50 for a share upon conversion.

The Share Price Later Falls or Does Not Rise Enough to Justify Conversion
No firm wants to see its share price fall. However, given that the price did fall, the firm would have been better off if it had previously issued equity instead of a convertible. The firm would have benefited by issuing equity above its later market price. That is, the firm would have received more than the subsequent worth of the equity. However, the drop in share price did not affect the value of the convertible much because the straight bond value serves as a floor.

Summary
Compared with equity, the firm is better off having issued convertible debt if the underlying equity subsequently does well. The firm is worse off having issued convertible debt if the underlying equity subsequently does poorly. We cannot predict future share prices in an efficient market. Thus, we cannot argue that issuing convertibles is better or worse than issuing equity. The preceding analysis is summarized in Table 24.1.

Modigliani–Miller (MM) pointed out that, abstracting from taxes and bankruptcy costs, the firm is indifferent to whether it issues equity or issues debt. The MM relationship is a quite general one. Their pedagogy could be adjusted to show that the firm is indifferent to whether it issues convertibles or issues other instruments. To save space (and the patience of students) we have omitted a full-blown proof of MM in a world with convertibles. However, our results are perfectly consistent with MM. Now we turn to the real-world view of convertibles.

The ‘Free Lunch’ Story
The preceding discussion suggests that issuing a convertible bond is no better and no worse than issuing other instruments. Unfortunately, many corporate executives fall into the trap of arguing that issuing convertible debt is actually better than issuing alternative instruments. This is a free lunch type of explanation, of which we are quite critical.

Example 24.4
Are Convertibles Always Better?
The share price of RW SE is €20. Suppose this company can issue subordinated debentures at 10 per cent. It can also issue convertible bonds at 6 per cent with a conversion value of €800. The conversion value means that the holders can convert a convertible bond into 40 (= €800/€20) shares of equity.

<table>
<thead>
<tr>
<th>If Firm Subsequently Does Poorly</th>
<th>If Firm Subsequently Prosper</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>No conversion because of low share price.</td>
<td>Conversion because of high share price.</td>
<td>CBs provide cheap financing because firm could have issued equity at high prices.</td>
</tr>
<tr>
<td>CBs provide cheap financing because coupon rate is lower.</td>
<td>CBs provide expensive financing because bonds are converted, which dilutes existing equity.</td>
<td>CBs provide cheap financing because firm issues equity at high prices when bonds are converted.</td>
</tr>
</tbody>
</table>

Table 24.1 The Case For and Against Convertible Bonds (CBs)
A company treasurer who believes in free lunches might argue that convertible bonds should be issued because they represent a cheaper source of financing than either subordinated bonds or equity. The treasurer will point out that if the company does poorly and the price does not rise above €20, the convertible bondholders will not convert the bonds into equity. In this case the company will have obtained debt financing at below-market rates by attaching worthless equity kickers. On the other hand, if the firm does well and the price of its equity rises to €25 or above, convertible holders will convert. The company will issue 40 shares. The company will receive a bond with face value of €1,000 in exchange for issuing 40 shares of equity, implying a conversion price of €25. The company will have issued equity at €25 per share, or 20 per cent above the €20 equity price prevailing when the convertible bonds were issued. This enables it to lower its cost of equity capital. Thus, the treasurer happily points out, regardless of whether the company does well or poorly, convertible bonds are the cheapest form of financing.

Although this argument may sound quite plausible at first, there is a flaw. The treasurer is comparing convertible financing with straight debt when the share price subsequently falls. However, the treasurer compares convertible financing with equity when the share price subsequently rises. This is an unfair mixing of comparisons. By contrast, our analysis of Table 24.1 was fair because we examined both share price increases and decreases when comparing a convertible with each alternative instrument. We found that no single alternative dominated convertible bonds in both up and down markets.

The ‘Expensive Lunch’ Story

Suppose we stand the treasurer’s argument on its head by comparing (1) convertible financing with straight debt when share prices rise, and (2) convertible financing with equity when share prices fall.

From Table 24.1, we see that convertible debt is more expensive than straight debt when share prices subsequently rise. The firm’s obligation to sell convertible holders a chunk of equity at a below-market price more than offsets the lower interest rate on a convertible.

Also from Table 24.1, we see that convertible debt is more expensive than equity when share prices subsequently fall. Had the firm issued equity, it would have received a price higher than its subsequent worth. Therefore, the expensive lunch story implies that convertible debt is an inferior form of financing. Of course, we dismiss both the free lunch and the expensive lunch arguments.

A Reconciliation

In an efficient financial market there is neither a free lunch nor an expensive lunch. Convertible bonds can be neither cheaper nor more expensive than other instruments. A convertible bond is a package of straight debt and an option to buy equity. The difference between the market value of a convertible bond and the value of a straight bond is the price investors pay for the call option feature. In an efficient market, this is a fair price.

In general, if a company prospers, issuing convertible bonds will turn out to be worse than issuing straight bonds and better than issuing equity. In contrast, if a company does poorly, convertible bonds will turn out to be better than issuing straight bonds and worse than issuing equity.

24.7 Why Are Warrants and Convertibles Issued?

From studies it is known that firms that issue convertible bonds are different from other firms. Here are some of the differences:

1. The bond ratings of firms using convertibles are lower than those of other firms.
2. Convertibles tend to be used by smaller firms with high growth rates and more financial leverage.
3. Convertibles are usually subordinated and unsecured.

The kind of company that uses convertibles provides clues to why they are issued. Here are some explanations that make sense.
Matching Cash Flows
If financing is costly, it makes sense to issue securities whose cash flows match those of the firm. A young, risky and (it hopes) growing firm might prefer to issue convertibles or bonds with warrants because these will have lower initial interest costs. When the firm is successful, the convertibles (or warrants) will be converted. This causes expensive dilution, but it occurs when the firm can most afford it.

Risk Synergy
Another argument for convertible bonds and bonds with warrants is that they are useful when it is very costly to assess the risk of the issuing company. Suppose you are evaluating a new product by a start-up company. The new product is a genetically engineered virus that may increase the yields of corn crops in northern climates. It may also cause cancer. This type of product is difficult to value properly. Thus, the risk of the company is very hard to determine: it may be high, or it may be low. If you could be sure the risk of the company was high, you would price the bonds for a high yield, say 15 per cent. If it was low, you would price them at a lower yield, say 10 per cent.

Convertible bonds and bonds with warrants can protect somewhat against mistakes of risk evaluation. Convertible bonds and bonds with warrants have two components: straight bonds and call options on the company’s underlying equity. If the company turns out to be a low-risk company, the straight bond component will have high value and the call option will have low value. However, if the company turns out to be a high-risk company, the straight bond component will have low value and the call option will have high value. This is illustrated in Table 24.2.

However, although risk has effects on value that cancel each other out in convertibles and bonds with warrants, the market and the buyer nevertheless must make an assessment of the firm’s potential to value securities, and it is not clear that the effort involved is that much less than is required for a straight bond.

Agency Costs
Convertible bonds can resolve agency problems associated with raising money. In a previous chapter, we showed that straight bonds are like risk-free bonds minus a put option on the assets of the firm. This creates an incentive for creditors to force the firm into low-risk activities. In contrast, holders of equity have incentives to adopt high-risk projects. High-risk projects with negative NPV transfer wealth from bondholders to shareholders. If these conflicts cannot be resolved, the firm may be forced to pass up profitable investment opportunities. However, because convertible bonds have an equity component, less expropriation of wealth can occur when convertible debt is issued instead of straight debt.9 In other words, convertible bonds mitigate agency costs. One implication is that convertible bonds have less restrictive debt covenants than do straight bonds in the real world. Casual empirical evidence seems to bear this out.

Backdoor Equity
A popular theory of convertibles views them as backdoor equity.10 The basic story is that young, small, high-growth firms cannot usually issue debt on reasonable terms due to high financial distress costs. However, the owners may be unwilling to issue equity if current share prices are too low.
Lewis et al. (1998) examine the risk shifting and backdoor equity theories of convertible bond debt. They find evidence for both theories.

The European Puzzle

Until very recently, there has been no research on convertible bond issuance in Europe. However, two recent papers that have explored this issue provide additional insights into why European managers issue convertibles. Bancel and Mittoo (2004) and Dutordoir and Van de Gucht (2009) both ask the question ‘Why do European firms issue convertible debt instead of straight debt or equity?’ Surprisingly, their findings differ somewhat from the established view of convertible issuance that has been propagated by US researchers.

Bancel and Mittoo (2004) surveyed 229 firms in 16 European countries that issued convertible bonds. This sample represented 295 convertible bond issues amounting to a total of €97,933 million. Only 29 firms responded to the sample and most of these were French. Normally, such a small sample would be problematic but given that this was the first study of its type, we can be forgiving. They found that there was no one reason why firms opt for convertibles over other forms of financing. French respondents argued that the liquidity of the convertibles market was a strong motivator for issuing convertibles. Clearly, this appears to be consistent with the situation in financial markets as we move into the second decade of the twenty-first century. In addition, firms whose equity appears to be overvalued opt for convertibles to avoid the share price falling because of equity dilution.

Dutordoir and Van de Gucht (2009) acts as an interesting counterpoint to the work of Bancel and Mittoo and earlier US research. Using a much larger sample of security issues, they find that European issuers of convertible bonds are actually large companies with low financing costs. This is the opposite to US convertible issuers who tend to be small and highly levered. Again, unlike the US, convertible bonds in Europe are rarely callable and only 27 per cent have been converted into equity. In Europe, it appears that convertible bonds are not used as backdoor equity but instead as ‘sweetened debt’, to reduce financing costs of raising debt.

The findings of Dutordoir and Van de Gucht (2009) raise important questions about the motivation for European firms that issue convertible bonds. Given that issuers tend to be large, financially healthy and mature, the explanations relating to financing costs that have been proposed do not appear to explain the reality in Europe. A possible reason may simply be due to market timing, as argued by Baker and Wurgler (2002) and discussed in an earlier chapter. The contrast in findings between the US and Europe highlight the need to recognize differences in environment, culture and motivations of corporate managers in these two very heterogeneous regions.

Finally, Dong et al. (2012) asked top executives from Australia, Canada, UK and US why their firms issued convertible bonds. Although the findings were not conclusive, most support was given to the ‘risk synergy’ rationale. The executives also stated that they issued convertible bonds because debt was too costly or covenant-heavy and their share price was too low.

Who Buys Convertible Bonds?

Because of their hybrid debt and equity characteristics, convertible bonds attract two main types of investors. Originally, financial institutions bought convertibles because they provided exposure to upside share price growth and limited downside credit risk. However, a new type of investor, hedge funds, now comprise a major part of the convertible market. Hedge funds buy the convertible bond and short the equity of the issue to take advantage of any undervaluation in the convertible bond price. Unfortunately, this can lead to falls in the share price of the convertible issuing firm because of the short-selling activity.11

Conversion Policy

There is one aspect of convertible bonds that we have omitted so far. Firms are sometimes granted a call option on the bond. It should be noted that in the US, call features are significantly more common than in Europe. The typical arrangements for calling a
convertible bond are simple. When the bond is called, the holder has about 30 days to choose between the following:

1. Converting the bond to equity at the conversion ratio
2. Surrendering the bond and receiving the call price in cash.

What should bondholders do? It should be obvious that if the conversion value of the bond is greater than the call price, conversion is better than surrender; and if the conversion value is less than the call price, surrender is better than conversion. If the conversion value is greater than the call price, the call is said to force conversion.

What should financial managers do? Calling the bonds does not change the value of the firm as a whole. However, an optimal call policy can benefit the shareholders at the expense of the bondholders. Because we are speaking about dividing a pie of fixed size, the optimal call policy is simple: do whatever the bondholders do not want you to do.

Bondholders would love the shareholders to call the bonds when the bonds’ market value is below the call price. Shareholders would be giving bondholders extra value. Alternatively, should the value of the bonds rise above the call price, the bondholders would love the shareholders not to call the bonds because bondholders would be allowed to hold onto a valuable asset.

There is only one policy left. This is the policy that maximizes shareholder value and minimizes bondholder value:

**Call the bond when its value is equal to the call price.**

It is a puzzle that firms do not always call convertible bonds when the conversion value reaches the call price. Ingersoll (1977) examined the call policies of 124 firms between 1968 and 1975.\(^\text{12}\) In most cases he found that the company waited to call the bonds until the conversion value was much higher than the call price. The median company waited until the conversion value of its bonds was 44 per cent higher than the call price. This is not even close to our optimal strategy. Why?

One reason is that if firms attempt to implement the optimal strategy, it may not be truly optimal. Recall that bondholders have 30 days to decide whether to convert bonds to equity or to surrender bonds for the call price in cash. In 30 days the share price could drop, forcing the conversion value below the call price. If so, the convertible is ‘out of the money’ and the firm is giving away money. The firm would be giving up cash for equity worth much less. Because of this possibility, firms in the real world usually wait until the conversion value is substantially above the call price before they trigger the call.\(^\text{13}\) This is sensible.
equity. As a consequence, bonds with warrants and convertible bonds have different effects on corporate cash flow and capital structure.

(c) Warrants and convertibles cause dilution to the existing shareholders. When warrants are exercised and convertible bonds converted, the company must issue new shares of equity. The percentage ownership of the existing shareholders will decline. New shares are not issued when call options are exercised.

4 Many arguments, both plausible and implausible, are given for issuing convertible bonds and bonds with warrants. One plausible rationale for such bonds has to do with risk. Convertibles and bonds with warrants are associated with risky companies. Lenders can do several things to protect themselves from high-risk companies:

(a) They can require high yields.
(b) They can lend less or not at all to firms whose risk is difficult to assess.
(c) They can impose severe restrictions on such debt.

Another useful way to protect against risk is to issue bonds with equity kickers. This gives the lenders the chance to benefit from risks and reduces the conflicts between bondholders and shareholders concerning risk.

5 A puzzle particularly vexes financial researchers: convertible bonds may have call provisions. Companies appear to delay calling convertibles until the conversion value greatly exceeds the call price. From the shareholders’ standpoint, the optimal call policy would be to call the convertibles when the conversion value equals the call price.

Questions and Problems

1 **Warrants**  Why are warrants sometime referred to as equity kickers? What does this mean?

2 **Warrants and Options**  What is the primary difference between a warrant and a traded call option?

3 **Warrant Pricing**  Why is the dilution factor important in warrant pricing?

4 **Convertible Bonds**  What are the main advantages of convertible bonds to issuing firms? To convertible bondholders?

5 **Reasons for Issuing Warrants and Convertibles**  Why do firms issue convertibles? What impact do convertibles have on firms with target debt to equity ratios? Discuss convertible bonds in the context of the trade-off, pecking order and market timing theories of capital structure.

6 **Reasons for Issuing Warrants and Convertibles**  Why are convertible bonds viewed as backdoor equity?

7 **Conversion Policy**  When should a firm force conversion of convertibles? Why?

8 **Warrants**  Explain the following limits on the prices of warrants:

   (a) If the share price is below the exercise price of the warrant, the lower bound on the price of a warrant is zero.
   (b) If the share price is above the exercise price of the warrant, the lower bound on the price of a warrant is the difference between the share price and the exercise price.
   (c) An upper bound on the price of any warrant is the current value of the firm’s equity.

9 **Convertible Bonds and Equity Volatility**  Assume that Barclays plc has just issued a callable convertible bond. You are concerned that the share price of Barclays is going to become more volatile over the next year. Should you buy the bond? Explain.

10 **Convertible Bond Value**  Using the same bond as in Question 9, assume that you believe interest rates are going to increase. What do you think will happen to the value of the bond? What if the bond was a puttable convertible bond? Explain.
11 Dilution What is dilution, and why does it occur when warrants are exercised?
12 Warrants and Convertibles What is wrong with the simple view that it is cheaper to
issue a bond with a warrant or a convertible feature because the required coupon is lower?
13 Warrants and Convertibles Your firm has experienced significant distress over the last
3 years and has very little cash left. The managers have decided that they will go to the
market one last time to raise financing and they have argued that the best security to issue
would be a convertible bond with a warrant attached to it. Do you agree that this would be
a sensible decision to take? Explain.
14 Convertible Bonds Why will convertible bonds not be voluntarily converted to equity
before expiration?
15 Warrant Valuation A warrant with 5 months until expiration entitles its owner to buy
100 shares of the issuing firm’s equity for an exercise price of €23 per share. If the current
market price of the equity is €10 per share, will the warrant be worthless?
16 Convertible Bonds Why do you think executives believe the risk synergy rationale for
issuing convertibles and not the other explanations? Explain.
17 Conversion Price A convertible bond with a face value of €1,000 has a conversion ratio
of 16.4. What is the conversion price?
18 Conversion Ratio A convertible bond with a face value of SKr10,000 has a conversion
price of SKr356. What is the conversion ratio of the bond?
19 Conversion Value A convertible bond has a conversion ratio of 100. If the shares are
currently priced at £9.20, what is the conversion value of the bond?
20 Conversion Premium Citic Securities recently issued bonds with a face value of 100,000
renminbi and conversion ratio of 420. If the share price at the bond issue was 124 renminbi,
what was the conversion premium?
21 Convertible Bonds Hannon Home Products recently issued £43,000,000 worth of 8 per
cent convertible debentures. Each convertible bond has a face value of £100,000. Each
convertible bond can be converted into 2,425 shares of equity any time before maturity.
The share price is £31.25, and the market value of each bond is £118,000.
   (a) What is the conversion ratio?
   (b) What is the conversion price?
   (c) What is the conversion premium?
   (d) What is the conversion value?
   (e) If the stock price increases by £2, what is the new conversion value?

22 Warrant Value A warrant gives its owner the right to purchase three shares of equity at
an exercise price of 32 Swedish krona per share. The current market price of the equity is 39
Swedish krona. What is the minimum value of the warrant?

23 Convertible Bond Value An analyst has recently informed you that at the issuance of a
company’s convertible bonds, one of the two following sets of relationships existed:

<table>
<thead>
<tr>
<th></th>
<th>Scenario A (€)</th>
<th>Scenario B (€)</th>
</tr>
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<tbody>
<tr>
<td>Face value of bond</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Straight value of convertible bond</td>
<td>950</td>
<td>950</td>
</tr>
<tr>
<td>Market value of convertible bond</td>
<td>1,000</td>
<td>900</td>
</tr>
</tbody>
</table>

Assume the bonds are available for immediate conversion. Which of the two scenarios do
you believe is more likely? Why?

24 Convertible Bond Value Tvep plc issued convertible bonds with a conversion price of
£20. The bonds are available for immediate conversion. The current price of the company’s
equity is £18 per share. The current market price of the convertible bonds is £990. The
convertible bonds’ straight value is not known.
   (a) What is the minimum price for the convertible bonds?
   (b) Explain the difference between the current market price of each convertible bond and
the value of the equity into which it can be immediately converted.
25 **Convertible Bonds** You own a callable, convertible bond with a conversion ratio of 500. The equity is currently selling for £22 per share. The issuer of the bond has announced a call at a call price of 105 on a face value of £100. What are your options here? What should you do?

26 **Warrant Value** General Modems has 5-year warrants that currently trade in the open market. Each warrant gives its owner the right to purchase one share of equity for an exercise price of £35.
   (a) Suppose the equity is currently trading for £33 per share. What is the lower limit on the price of the warrant? What is the upper limit?
   (b) Suppose the equity is currently trading for £39 per share. What is the lower limit on the price of the warrant? What is the upper limit?

27 **Convertible Bonds** Trichet SA has just issued a 30-year callable, convertible bond with a coupon rate of 7 per cent annual coupon payments. The bond has a conversion price of €125. The company's equity is selling for €32 per share. The owner of the bond will be forced to convert if the bond's conversion value is ever greater than or equal to €1,100. The required return on an otherwise identical non-convertible bond is 12 per cent.
   (a) What is the minimum value of the bond?
   (b) If the share price were to grow by 15 per cent per year forever, how long would it take for the bond's conversion value to exceed €1,100?

28 **Convertible Bonds** Rob Stevens is the chief executive officer of Isner Construction plc and owns 500,000 shares. The company currently has 4 million shares and convertible bonds with a face value of £20 million outstanding. The convertible bonds have a conversion price of £20, and the equity is currently selling for £25.
   (a) What percentage of the firm’s equity does Mr Stevens own?
   (b) If the company decides to call the convertible bonds and force conversion, what percentage of the firm’s equity will Mr Stevens own? He does not own any convertible bonds.

29 **Warrants** Survivor NV, an all-equity firm, has three shares outstanding. Yesterday, the firm’s assets consisted of 5 ounces of platinum, currently worth €1,000 per ounce. Today, the company issued Ms Wu a warrant for its fair value of €1,000. The warrant gives Ms Wu the right to buy a single share of the firm’s equity for €2,100 and can be exercised only on its expiration date one year from today. The firm used the proceeds from the issuance to immediately purchase an additional ounce of platinum.
   (a) What was the price of a single share of equity before the warrant was issued?
   (b) What was the price of a single share of equity immediately after the warrant was issued?
   (c) Suppose platinum is selling for €1,100 per ounce on the warrant’s expiration date in one year. What will be the value of a single share of equity on the warrant’s expiration date?

30 **Warrants** The capital structure of Ricketti Enterprises plc consists of 10 million shares of equity and 1 million warrants. Each warrant gives its owner the right to purchase one share of equity for an exercise price of £15. The current share price is £17, and each warrant is worth £3. What is the new share price if all warrant holders decide to exercise today?

31 **Convertible Calculations** You have been hired to value a new 25-year callable, convertible bond. The bond has a 6.80 per cent coupon rate, payable annually. The conversion price is £150, and the equity currently sells for £44.75. The stock price is expected to grow at 12 per cent per year. The bond is callable at £1,200 but based on prior experience it will not be called unless the conversion value is £1,300. The required return on this bond is 10 per cent. What value would you assign to this bond?

32 **Warrant Value** Superior Clamps AB has a capital structure consisting of 4 million shares of equity and 500,000 warrants. Each warrant gives its owner the right to purchase one share of newly issued equity for an exercise price of €20. The warrants are European and will expire one year from today. The market value of the company’s assets is €88 million, and the annual variance of the returns on the firm’s assets is 0.04. Treasury bills that mature in one year yield a continuously compounded interest rate of 7 per cent. The company does not pay a dividend. Use the Black–Scholes model to determine the value of a single warrant.
33 **Warrant Value** Omega Airline’s capital structure consists of 1.5 million shares of equity and zero coupon bonds with a face value of $10 million that mature in 6 months. The firm just announced that it will issue warrants with an exercise price of $95 and 6 months until expiration to raise the funds to pay off its maturing debt. Each warrant can be exercised only at expiration and gives its owner the right to buy a single newly issued share of equity. The firm will place the proceeds from the warrant issue immediately into Treasury bills.

The market value balance sheet shows that the firm will have assets worth $160 million after the announcement. The company does not pay dividends. The standard deviation of the returns on the firm’s assets is 65 per cent, and Treasury bills with a 6-month maturity yield 6 per cent. How many warrants must the company issue today to be able to use the proceeds from the sale to pay off the firm’s debt obligation in 6 months?

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**Exam Question (45 minutes)**

1. You have been hired to value a new 10-year callable, convertible bond. The bond has a 5.6 per cent coupon rate, payable annually. The conversion price is £150, and the equity currently sells for £44.75. The share price is expected to grow at 8 per cent per year. The bond is callable at £1,100 but based on prior experience it will not be called unless the conversion value is £1,200. The required return on this bond is 6 per cent. What value would you assign to this bond? (30 marks)

2. Your firm has 3 million shares of equity and 100,000 warrants. Each warrant gives its owner the right to purchase one share of newly issued equity for an exercise price of €15. The warrants are European and will expire one year from today. The market value of the company’s assets is €60 million, and the annual standard deviation of the returns on the firm’s assets is 24 per cent. Treasury bills that mature in one year yield a continuously compounded interest rate of 2 per cent. The company does not pay a dividend. Use the Black–Scholes model to determine the value of a single warrant. (30 marks)

3. Review the reasons given for why firms issue convertible bonds. Which one do you think is the most valid? Explain. (40 marks)

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**Mini Case**

**S&S Air’s Convertible Bond**

Kartner Meister was recently hired by S&S Air AB to assist the company with its short-term financial planning and to evaluate the company’s performance. Kartner graduated from university 5 years ago with a finance degree. He has been employed in the finance department of a major German company since then.

S&S Air was founded 10 years ago by two friends, Stephan Lochner and Hans Multscher. The company has manufactured and sold light airplanes over this period, and the company’s products have received high reviews for safety and reliability. The company has a niche market in that it sells primarily to individuals who own and fly their own airplanes. The company has two models: the Birdie, which sells for €53,000, and the Eagle, which sells for €78,000.

S&S Air is not publicly traded, but the company needs new funds for investment opportunities. In consultation with Conrad Witz of underwriter Koerbecke and Pleydenwurff, Kartner decided that a convertible bond issue with a 20-year maturity is the way to go. He met with the owners, Stephan and Hans, and presented his analysis of the convertible bond issue. Because the company is not publicly traded, Kartner looked at comparable publicly traded companies and determined that the average PE ratio for the industry is 12.5. Earnings per share for the company are €1.60. With this in mind, Kartner concluded that the conversion price should be €25 per share.

Several days later Stephan, Hans and Kartner met again to discuss the potential bond issue. Both Hans and Stephan have researched convertible bonds and have questions for Kartner. Hans begins by asking Kartner if the convertible bond issue will have a lower coupon rate than a comparable bond without a conversion feature. Kartner replies that to sell the bond at par value, the convertible bond issue would require a 6 per cent coupon rate with a conversion value of €800, while a plain vanilla bond would have a 7 per cent coupon rate. Hans nods in agreement, and he explains that the convertible bonds are a win–win form of financing.
He states that if the value of the company equity does not rise above the conversion price, the company has issued debt at a cost below the market rate (6 per cent instead of 7 per cent). If the company’s equity does rise to the conversion value, the company has effectively issued shares at above the current value.

Stephan immediately disagrees, arguing that convertible bonds are a no-win form of financing. He argues that if the value of the company equity rises to €25, the company is forced to sell shares at the conversion price. This means the new shareholders (those who bought the convertible bonds) benefit from a bargain price. Put another way, if the company prospers, it would have been better to have issued straight debt so that the gains would not be shared.

Kartner has gone back to Conrad for help. As Conrad’s assistant, you have been asked to prepare another memo answering the following questions:

1. Why do you think Kartner is suggesting a conversion price of €25? Given that the company is not publicly traded, does it even make sense to talk about a conversion price?
2. What is the floor value of the S&S Air convertible bond?
3. What is the conversion ratio of the bond?
4. What is the conversion premium of the bond?
5. What is the value of the option?
6. Is there anything wrong with Hans’ argument that it is cheaper to issue a bond with a convertible feature because the required coupon is lower?
7. Is there anything wrong with Stephan’s argument that a convertible bond is a bad idea because it allows new shareholders to participate in gains made by the company?
8. How can you reconcile the arguments made by Hans and Stephan?
9. During the debate, a question comes up concerning whether the bonds should have an ordinary (not make-whole) call feature. Kartner confuses everybody by stating, ‘The call feature lets S&S Air finance conversion, thereby minimizing the problem Stephan has identified.’ What is he talking about? Is he making sense?

Practical Case Study

Download the financial accounts of ten companies and look for any issues of warrants or convertibles. You may find examples of bonds that have conversion options or warrant-like properties. Are convertibles an important part of your firms’ capital structures? Write a brief report on the use of convertibles and warrants for your sample of firms.

Relevant Accounting Standards

The most important accounting standard for warrants and convertibles is IAS 39 Financial Instruments: Recognition and Measurement. IAS 39 provides definitions for different types of financial securities and states how the different components of a security should be valued and presented in a firm’s financial accounts. Visit the IASPlus website (www.iasplus.com) for more information.

References

Additional Reading

Much of the recent research in this area has already been discussed in the main text. The following papers will add to your understanding.

1 Warrants are also issued with publicly distributed bonds and new issues of equity.

2 At the turn of the twenty-first century many banks began issuing options on other companies under the name ‘call warrants’. These are not warrants in the real sense because the companies on which the call warrants are written do not issue new equity in response to an exercise of the call warrants. In this chapter, we will not consider the valuation of these call warrants. See Ter Horst and Veld (2008) for more information on call warrants.

3 Just like call options, warrants are protected against stock splits and stock dividends, but not against cash dividends. The latter generally does not cause a problem for the valuation of call options, since these only have short maturities and therefore a relatively small amount of dividends are paid during their lifetime. Given that warrants generally have maturities of several years, the effect of cash dividends can be significant.

4 To derive Formula 24.3, we separate ‘Exercise price’ in Equation 24.2. This yields

\[
\frac{\text{Firm’s value net of debt}}{# + #_w} - \frac{#}{# + #_w} \times \text{Exercise price}
\]

By rearranging terms, we can obtain Formula 24.3.

5 Equation 24.4 is not exactly correct. After a warrant is issued, the share price and volatility of returns will change to reflect the warrant’s existence. However, Schulz and Trautmann (1994) report that Equation 24.4 performs just as well as more complex valuation models, except when the warrant is deep out of the money.

6 The most plausible exception is when conversion would provide the investor with a dividend much greater than the interest available prior to conversion. The optimal strategy here could very well be to convert immediately, implying that the market value of the bond would exactly equal the conversion value. Other exceptions occur when the firm is in default or the bondholders are forced to convert.

7 Brigham (1966).


9 Barnea et al. (1985), Chapter VI.

10 Stein (1992). See also Lewis et al. (1998); Lewis and Verwijmeren (2011).

11 See Duca et al. (2012); Cho et al. (2010).

12 See also Harris and Raviv (1985). Harris and Raviv describe a signal equilibrium that is consistent with Ingersoll’s result. They show that managers with favourable information will delay calls to avoid depressing stock prices.

13 See Altintig and Butler (2005); Asquith (1995). On the other hand, the stock market usually reacts negatively to the announcement of a call. For example, see Singh et al. (1991); Mazzeo and Moore (1992). Ederington et al. (1997) tested various theories about when it is optimal to call convertibles. They found evidence consistent for the preceding 30-day ‘safety margin’ theory. They also found that calls of in-the-money convertibles are highly unlikely if dividends to be received (after conversion) exceed the company’s interest payment.


