APPENDIX A

LINEAR PROGRAMMING USING THE EXCEL SOLVER

The key to profitable operations is making the best use of available resources of people, material, plant and equipment, and money. Today's manager has a powerful mathematical modeling tool available for this purpose with linear programming. In this appendix, we will show how the use of the Microsoft Excel Solver to solve LP problems opens a whole new world to the innovative manager and provides an invaluable addition to the technical skill set for those who seek careers in consulting. In this appendix, we use a product-planning problem to introduce this tool. Here we find the optimal mix of products that have different costs and resource requirements. This problem is certainly relevant to today's competitive market. Extremely successful companies provide a mix of products, from standard to high-end luxury models. All these products compete for the use of limited production and other capacity. Maintaining the proper mix of these products over time can significantly bolster earnings and the return on a firm's assets.

We begin with a quick introduction to linear programming and conditions under which the technique is applicable. Then we solve a simple product-mix problem. Other linear programming applications appear throughout the rest of the book.

INTRODUCTION

Linear programming (LP)

Linear programming (or simply **LP**) refers to several related mathematical techniques used to allocate limited resources among competing demands in an optimal way. LP is the most widely used of the approaches falling under the general heading of mathematical optimization techniques and has been applied to many operations management problems. The following are typical applications:

Aggregate sales and operations planning: Finding the minimum-cost production schedule. The problem is to develop a three- to six-month plan for meeting expected demand given constraints on expected production capacity and workforce size. Relevant costs considered in the problem include regular and overtime labor rates, hiring and firing, subcontracting, and inventory carrying cost.

Service/manufacturing productivity analysis: Comparing how efficiently different service and manufacturing outlets are using their resources compared to the best-performing unit. This is done using an approach called data envelopment analysis.

Product planning: Finding the optimal product mix where several products have different costs and resource requirements. Examples include finding the optimal blend of chemicals for gasoline, paints, human diets, and animal feeds. Examples of this problem are covered in this chapter.

Product routing: Finding the optimal way to produce a product that must be processed sequentially through several machine centers, with each machine in the center having its own cost and output characteristics.

Vehicle/crew scheduling: Finding the optimal way to use resources such as aircraft, buses, or trucks and their operating crews to provide transportation services to customers and materials to be moved between different locations.

Operations Management 4/ed
Robert F. Jacobs, Richard B. Chase, Alberto Grando, Andrea Sianesi

Process control: Minimizing the amount of scrap material generated by cutting steel, leather, or fabric from a roll or sheet of stock material.

Inventory control: Finding the optimal combination of products to stock in a network of warehouses or storage locations.

Distribution scheduling: Finding the optimal shipping schedule for distributing products between factories and warehouses or between warehouses and retailers.

Plant location studies: Finding the optimal location of a new plant by evaluating shipping costs between alternative locations and supply and demand sources.

Material handling: Finding the minimum-cost routings of material-handling devices (such as forklift trucks) between departments in a plant, or hauling materials from a supply yard to work sites by trucks, for example. Each truck might have different capacity and performance capabilities.

Linear programming is gaining wide acceptance in many industries due to the availability of detailed operating information and the interest in optimizing processes to reduce cost. Many software vendors offer optimization options to be used with enterprise resource planning systems. Some firms refer to these as *advanced planning option, synchronized planning,* and *process optimization.*

For linear programming to pertain in a problem situation, five essential conditions must be met. First, there must be *limited resources* (such as a limited number of workers, equipment, finances, and material); otherwise there would be no problem. Second, there must be an *explicit objective* (such as maximize profit or minimize cost). Third, there must be *linearity* (two is twice as much as one; if three hours are needed to make a part, then two parts would take six hours and three parts would take nine hours). Fourth, there must be *homogeneity* (the products produced on a machine are identical, or all the hours available from a worker are equally productive). Fifth, there must be *divisibility:* Normal linear programming assumes products and resources can be subdivided into fractions. If this subdivision is not possible (such as flying half an airplane or hiring one-fourth of a person), a modification of linear programming, called *integer programming,* can be used.

When a single objective is to be maximized (like profit) or minimized (like costs), we can use linear programming. When multiple objectives exist, *goal programming* is used. If a problem is best solved in stages or time frames, *dynamic programming* is employed. Other restrictions on the nature of the problem may require that it be solved by other variations of the technique, such as *nonlinear programming* or *quadratic programming.*

THE LINEAR PROGRAMMING MODEL

Stated formally, the linear programming problem entails an optimizing process in which nonnegative values for a set of decision variables $X_1, X_2, \ldots, X_n$ are selected so as to maximize (or minimize) an objective function in the form

$$\text{Maximize (minimize) } Z = C_1X_1 + C_2X_2 + \cdots + C_nX_n$$

subject to resource constraints in the form

$$\begin{aligned}
&A_{11}X_1 + A_{12}X_2 + \cdots + A_{1n}X_n \le B_1\\
&A_{21}X_1 + A_{22}X_2 + \cdots + A_{2n}X_n \le B_2\\
&.\\
&.\\
&.\\
&A_{m1}X_1 + A_{m2}X_2 + \cdots + A_{mn}X_n \le B_m
\end{aligned}$$

where C_n, A_{mn}, and B_m are given constants.

Depending on the problem, the constraints also may be stated with equal signs (=) or greater-than-or-equal-to signs (≥).

Tutorial: Intro to Solver

Step by Step

EXAMPLE A.1: Puck and Pawn Company

We describe the steps involved in solving a simple linear programming model in the context of a sample problem, that of Puck and Pawn Company, which manufactures hockey sticks and chess sets. Each hockey stick yields an incremental profit of \$2, and each chess set, \$4. A hockey stick requires 4 hours of processing at machine center A and 2 hours at machine center B. A chess set requires 6 hours at machine center A, 6 hours at machine center B, and 1 hour at machine center C. Machine center A has a maximum of 120 hours of available capacity per day, machine center B has 72 hours, and machine center C has 10 hours.

If the company wishes to maximize profit, how many hockey sticks and chess sets should be produced per day?

SOLUTION

Formulate the problem in mathematical terms. If H is the number of hockey sticks and C is the number of chess sets, to maximize profit the objective function may be stated as

$$\text{Maximize } Z = \$2H + \$4C$$

The maximization will be subject to the following constraints:

$$4H + 6C \leq 120 \quad \text{(machine center A constraint)}$$
$$2H + 6C \leq 72 \quad \text{(machine center B constraint)}$$
$$1C \leq 10 \quad \text{(machine center C constraint)}$$
$$H, C \geq 0$$

This formulation satisfies the five requirements for standard LP stated in the first section of this appendix:

1. There are limited resources (a finite number of hours available at each machine center).
2. There is an explicit objective function (we know what each variable is worth and what the goal is in solving the problem).
3. The equations are linear (no exponents or cross-products).
4. The resources are homogeneous (everything is in one unit of measure, machine hours).
5. The decision variables are divisible and nonnegative (we can make a fractional part of a hockey stick or chess set; however, if this were deemed undesirable, we would have to use integer programming).

GRAPHICAL LINEAR PROGRAMMING

Graphical linear programming

Though limited in application to problems involving two decision variables (or three variables for three-dimensional graphing), **graphical linear programming** provides a quick insight into the nature of linear programming. We describe the steps involved in the graphical method in the context of Puck and Pawn Company. The following steps illustrate the graphical approach:

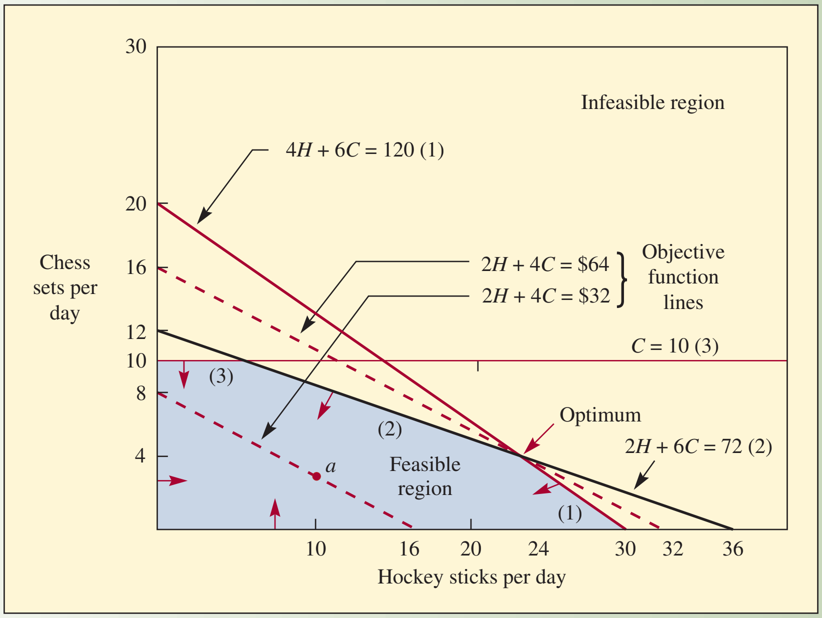
1. **Formulate the problem in mathematical terms.** The equations for the problem are given above.

2. **Plot constraint equations.** The constraint equations are easily plotted by letting one variable equal zero and solving for the axis intercept of the other. (The inequality portions of the restrictions are disregarded for this step.) For the machine center A constraint equation, when $H = 0$, $C = 20$, and when $C = 0$, $H = 30$. For the machine center B constraint equation, when $H = 0$, $C = 12$, and when $C = 0$, $H = 36$. For the machine center C constraint equation, $C = 10$ for all values of H. These lines are graphed in Exhibit A.1.

3. **Determine the area of feasibility.** The direction of inequality signs in each constraint determines the area where a feasible solution is found. In this case, all inequalities

Graph of Hockey Stick and Chess Set Problem

exhibit A.1



are of the less-than-or-equal-to variety, which means it would be impossible to produce any combination of products that would lie to the right of any constraint line on the graph. The region of feasible solutions is unshaded on the graph and forms a convex polygon. A convex polygon exists when a line drawn between any two points in the polygon stays within the boundaries of that polygon. If this condition of convexity does not exist, the problem is either incorrectly set up or is not amenable to linear programming.

4. **Plot the objective function.** The objective function may be plotted by assuming some arbitrary total profit figure and then solving for the axis coordinates, as was done for the constraint equations. Other terms for the objective function when used in this context are the *iso-profit* or *equal contribution line,* because it shows all possible production combinations for any given profit figure. For example, from the dotted line closest to the origin on the graph, we can determine all possible combinations of hockey sticks and chess sets that yield $32 by picking a point on the line and reading the number of each product that can be made at that point. The combination yielding $32 at point *a* would be 10 hockey sticks and three chess sets. This can be verified by substituting $H = 10$ and $C = 3$ in the objective function:

$$\$2(10) + \$4(3) = \$20 + \$12 = \$32$$

H	*C*	EXPLANATION
0	120/6 = 20	Intersection of Constraint (1) and *C* axis
120/4 = 30	0	Intersection of Constraint (1) and *H* axis
0	72/6 = 12	Intersection of Constraint (2) and *C* axis
72/2 = 36	0	Intersection of Constraint (2) and *H* axis
0	10	Intersection of Constraint (3) and *C* axis
0	32/4 = 8	Intersection of $32 iso-profit line (objective function) and *C* axis
32/2 = 16	0	Intersection of $32 iso-profit line and *H* axis
0	64/4 = 16	Intersection of $64 iso-profit line and *C* axis
64/2 = 32	0	Intersection of $64 iso-profit line and *H* axis

5. **Find the optimum point.** It can be shown mathematically that the optimal combination of decision variables is always found at an extreme point (corner point) of the convex polygon. In Exhibit A.1, there are four corner points (excluding the origin), and we

can determine which one is the optimum by either of two approaches. The first approach is to find the values of the various corner solutions algebraically. This entails simultaneously solving the equations of various pairs of intersecting lines and substituting the quantities of the resultant variables in the objective function. For example, the calculations for the intersection of $2H + 6C = 72$ and $C = 10$ are as follows:

Substituting $C = 10$ in $2H + 6C = 72$ gives $2H + 6(10) = 72$, $2H = 12$, or $H = 6$. Substituting $H = 6$ and $C = 10$ in the objective function, we get

$$\text{Profit} = \$2H + \$4C = \$2(6) + \$4(10)$$
$$= \$12 + \$40 = \$52$$

A variation of this approach is to read the H and C quantities directly from the graph and substitute these quantities into the objective function, as shown in the previous calculation. The drawback in this approach is that in problems with a large number of constraint equations, there will be many possible points to evaluate, and the procedure of testing each one mathematically is inefficient.

The second and generally preferred approach entails using the objective function or iso-profit line directly to find the optimum point. The procedure involves simply drawing a straight line *parallel* to any arbitrarily selected initial iso-profit line so the iso-profit line is farthest from the origin of the graph. (In cost minimization problems, the objective would be to draw the line through the point closest to the origin.) In Exhibit A.1, the dashed line labeled $\$2H + \$4C = \$64$ intersects the most extreme point. Note that the initial arbitrarily selected iso-profit line is necessary to display the slope of the objective function for the particular problem.[1] This is important since a different objective function (try Profit = $3H + 3C$) might indicate that some other point is farthest from the origin. Given that $\$2H + \$4C = \$64$ is optimal, the amount of each variable to produce can be read from the graph: 24 hockey sticks and four chess sets. No other combination of the products yields a greater profit.

LINEAR PROGRAMMING USING MICROSOFT EXCEL

Spreadsheets can be used to solve linear programming problems. Microsoft Excel has an optimization tool called *Solver* that we will demonstrate by solving the hockey stick and chess problem. We invoke the Solver from the Data tab. A dialogue box requests information required by the program. The following example describes how our sample problem can be solved using Excel.

If the Solver option does not appear in your Data tab, click on Excel Options → Add-Ins, select the Solver Add-In, and then click OK. Solver should then be available directly from the Data tab for future use.

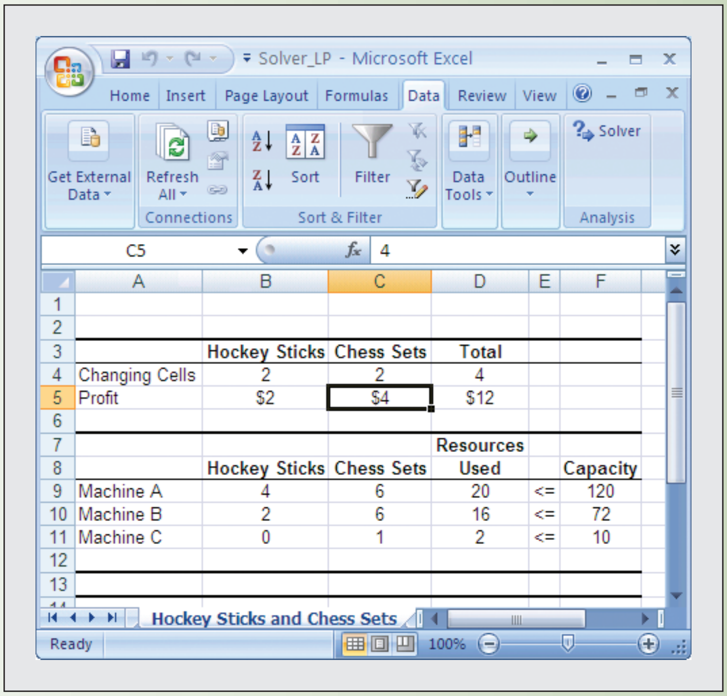
In the following example, we work in a step-by-step manner, setting up a spreadsheet and then solving our Puck and Pawn Company problem. Our basic strategy is to first define the problem within the spreadsheet. Following this, we invoke the Solver and feed it required information. Finally, we execute the Solver and interpret results from the reports provided by the program.

Step 1: Define Changing Cells A convenient starting point is to identify cells to be used for the decision variables in the problem. These are H and C, the number of hockey sticks and the number of chess sets to produce. Excel refers to these cells as changing cells in Solver. Referring to our Excel screen (Exhibit A.2), we have designated B4 as the location for the number of hockey sticks to produce and C4 for the number of chess sets. Note that we have set these cells equal to 2 initially. We could set these cells to anything, but a value other than zero will help verify that our calculations are correct.

Step 2: Calculate Total Profit (or Cost) This is our objective function and is calculated by multiplying profit associated with each product by the number of units produced. We have placed the profits in cells B5 and C5 ($2 and $4), so the profit is calculated by the following equation: B4*B5 + C4*C5, which is calculated in cell D5. Solver refers to this as the Target Cell, and it corresponds to the objective function for a problem.

Microsoft Excel Screen for Puck and Pawn Company

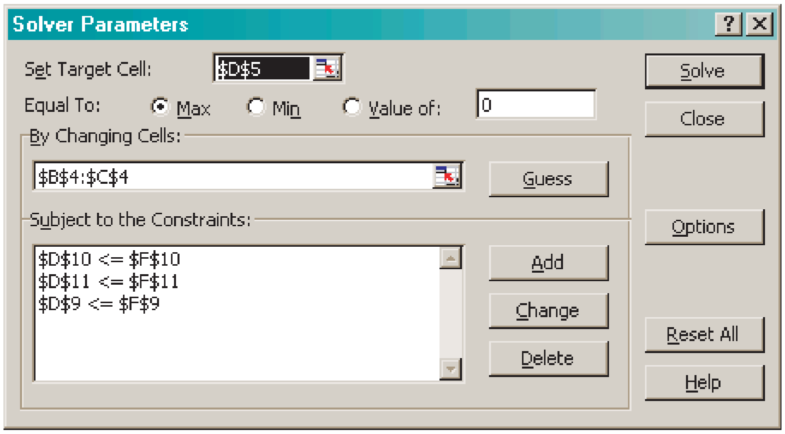
exhibit A.2



Excel: Solver LP

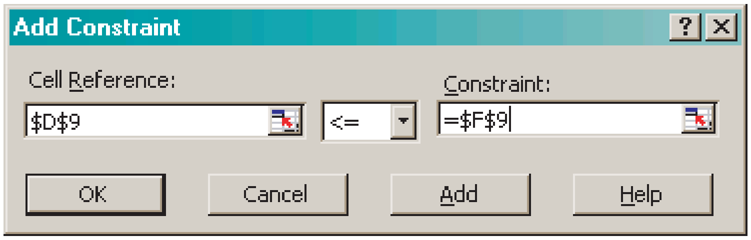
Step 3: Set Up Resource Usage Our resources are machine centers A, B, and C as defined in the original problem. We have set up three rows (9, 10, and 11) in our spreadsheet, one for each resource constraint. For machine center A, 4 hours of processing time are used for each hockey stick produced (cell B9) and 6 hours for each chess set (cell C9). For a particular solution, the total amount of the machine center A resource used is calculated in D9 (B9*B4 + C9*C4). We have indicated in cell E9 that we want this value to be less than the 120-hour capacity of machine center A, which is entered in F9. Resource usage for machine centers B and C is set up in the exact same manner in rows 10 and 11.

Step 4: Set Up Solver Go to the Data tab and select the Solver option.

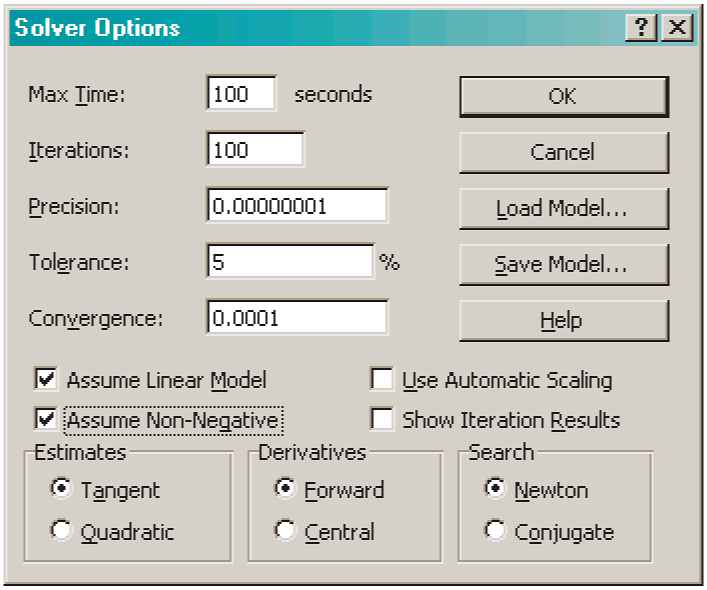


1. Set Target Cell: is set to the location where the value that we want to optimize is calculated. This is the profit calculated in D5 in our spreadsheet.
2. Equal To: is set to Max since the goal is to maximize profit.
3. By Changing Cells: are the cells that Solver can change to maximize profit. Cells B4 through C4 are the changing cells in our problem.

4. Subject to the Constraints: corresponds to our machine center capacity. Here we click on Add and indicate that the total used for a resource is less than or equal to the capacity available. A sample for machine center A follows. Click OK after each constraint is specified.



5. Clicking on Options allows us to tell Solver what type of problem we want it to solve and how we want it solved. Solver has numerous options, but we will need to use only a few. The screen is shown below.



Most of the options relate to how Solver attempts to solve nonlinear problems. These can be very difficult to solve, and optimal solutions difficult to find. Luckily our problem is a linear problem. We know this since our constraints and our objective function are all calculated using linear equations. Click on Assume Linear Model to tell Solver that we want to use the linear programming option for solving the problem. In addition, we know our changing cells (decision variables) must be numbers that are greater than or equal to zero since it makes no sense to make a negative number of hockey sticks or chess sets. We indicate this by selecting Assume Non-Negative as an option. We are now ready to actually solve the problem. Click OK to return to the Solver Parameters box.

Step 5: Solve the Problem Click Solve. We immediately get a Solver Results acknowledgment like that shown below.

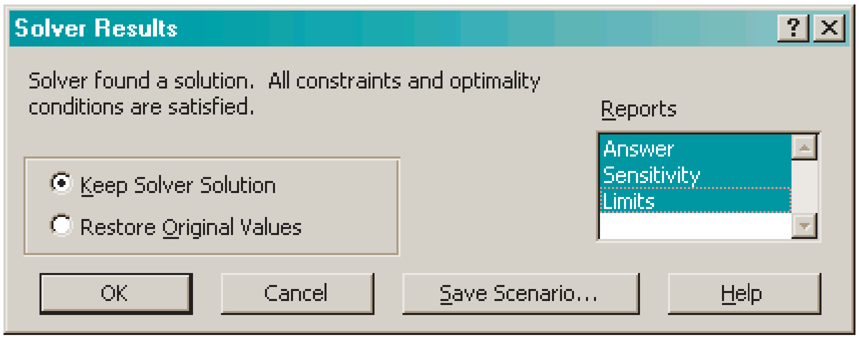


exhibit A.3

Excel Solver Answer and Sensitivity Reports

Answer Report

TARGET CELL (MAX)

CELL	NAME	ORIGINAL VALUE	FINAL VALUE
$D$5	Profit Total	$12	$64

ADJUSTABLE CELLS

CELL	NAME	ORIGINAL VALUE	FINAL VALUE
$B$4	Changing Cells Hockey Sticks	2	24
$C$4	Changing Cells Chess Sets	2	4

CONSTRAINTS

CELL	NAME	CELL VALUE	FORMULA	STATUS	SLACK
$D$11	Machine C Used	4	$D$11<=$F$11	Not Binding	6
$D$10	Machine B Used	72	$D$10<=$F$10	Binding	0
$D$9	Machine A Used	120	$D$9<=$F$9	Binding	0

Sensitivity Report

ADJUSTABLE CELLS

CELL	NAME	FINAL VALUE	REDUCED COST	OBJECTIVE COEFFICIENT	ALLOWABLE INCREASE	ALLOWABLE DECREASE
$B$4	Changing Cells Hockey Sticks	24	0	2	0.666666667	0.666666667
$C$4	Changing Cells Chess Sets	4	0	4	2	1

CONSTRAINTS

CELL	NAME	FINAL VALUE	SHADOW PRICE	CONSTRAINT R.H. SIDE	ALLOWABLE INCREASE	ALLOWABLE DECREASE
$D$11	Machine C Used	4	0	10	1E+30	6
$D$10	Machine B Used	72	0.333333333	72	18	12
$D$9	Machine A Used	120	0.333333333	120	24	36

Solver acknowledges that a solution was found that appears to be optimal. On the right side of this box are options for three reports: an Answer Report, a Sensitivity Report, and a Limits Report. Click on each report to have Solver provide these. After highlighting the reports, click OK to exit back to the spreadsheet. Three new tabs have been created that correspond to these reports.

The most interesting reports for our problem are the Answer Report and the Sensitivity Report, both of which are shown in Exhibit A.3. The Answer Report shows the final answers for the total profit ($64) and the amounts produced (24 hockey sticks and 4 chess sets). In the constraints section of the Answer Report, the status of each resource is given. All of machine A and machine B are used, and there are six units of slack for machine C.

The Sensitivity Report is divided into two parts. The first part, titled "Adjustable Cells," corresponds to objective function coefficients. The profit per unit for the hockey sticks can be either up or down $0.67 (between $2.67 and $1.33) without having an impact on the solution. Similarly, the profit of the chess sets could be between $6 and $3 without changing the solution. In the case of machine A, the right-hand side could increase to 144 (120 + 24) or decrease to 84 with a resulting \$0.33 increase or decrease per unit in the objective function. The right-hand side of machine B can increase to 90 units or decrease to 60 units with the same \$0.33 change for each unit in the objective function. For machine C, the right-hand side could increase to infinity (1E+30 is scientific notation for a very large number) or decrease to 4 units with no change in the objective function.

KEY TERMS

Linear programming (LP) Refers to several related mathematical techniques used to allocate limited resources among competing demands in an optimal way.

Graphical linear programming Provides a quick insight into the nature of linear programming.

SOLVED PROBLEMS

SOLVED PROBLEM 1

A furniture company produces three products: end tables, sofas, and chairs. These products are processed in five departments: the saw lumber, fabric cutting, sanding, staining, and assembly departments. End tables and chairs are produced from raw lumber only, and the sofas require lumber and fabric. Glue and thread are plentiful and represent a relatively insignificant cost that is included in operating expense. The specific requirements for each product are as follows:

RESOURCE OR ACTIVITY (QUANTITY AVAILABLE PER MONTH)	REQUIRED PER END TABLE	REQUIRED PER SOFA	REQUIRED PER CHAIR
Lumber (4,350 board feet)	10 board feet @ \$10/foot = \$100/table	7.5 board feet @ \$10/foot = \$75	4 board feet @ \$10/foot = \$40
Fabric (2,500 yards)	None	10 yards @ \$17.50/yard = \$175	None
Saw lumber (280 hours)	30 minutes	24 minutes	30 minutes
Cut fabric (140 hours)	None	24 minutes	None
Sand (280 hours)	30 minutes	6 minutes	30 minutes
Stain (140 hours)	24 minutes	12 minutes	24 minutes
Assemble (700 hours)	60 minutes	90 minutes	30 minutes

The company's direct labor expenses are \$75,000 per month for the 1,540 hours of labor, at \$48.70 per hour. Based on current demand, the firm can sell 300 end tables, 180 sofas, and 400 chairs per month. Sales prices are \$400 for end tables, \$750 for sofas, and \$240 for chairs. Assume that labor cost is fixed and the firm does not plan to hire or fire any employees over the next month.

Required:

1 What is the most limiting resource to the furniture company?
2 Determine the product mix needed to maximize profit at the furniture company. What is the optimal number of end tables, sofas, and chairs to produce each month?

Solution

Define X_1 as the number of end tables, X_2 as the number of sofas, and X_3 as the number of chairs to produce each month. Profit is calculated as the revenue for each item minus the cost of materials (lumber and fabric), minus the cost of labor. Since labor is fixed, we subtract this out as a total sum. Mathematically we have $(400 - 100)X_1 + (750 - 75 - 175)X_2 + (240 - 40)X_3 - 75{,}000$. Profit is calculated as follows:

$$\text{Profit} = 300X_1 + 500X_2 + 200X_3 - 75{,}000$$

Constraints are the following:

Lumber: $10X_1 + 7.5X_2 + 4X_3 \leq 4{,}350$

Fabric: $10X_2 \leq 2{,}500$

Saw: $.5X_1 + .4X_2 + .5X_3 \leq 280$

Cut: $.4X_2 \le 140$

Sand: $.5X_1 + .1X_2 + .5X_3 \le 280$

Stain: $.4X_1 + .2X_2 + .4X_3 \le 140$

Assemble: $1X_1 + 1.5X_2 + .5X_3 \le 700$

Demand:

Table: $X_1 \le 300$

Sofa: $X_2 \le 180$

Chair: $X_3 \le 400$

Step 1: Define Changing Cells These are B3, C3, and D3. Note that these cells have been set equal to zero.

E4 =B4*$B$3+C4*$C$3+D4*$D$3-75000

	A	B	C	D	E	F
1	Furniture Company					
2		End Tables	Sofas	Chairs	Total	Limit
3	Changing cells	0	0	0		
4	Profit	$300	$500	$200	-$75,000	
5						
6	Lumber	10	7.5	4	0	4350
7	Fabric	0	10	0	0	2500
8	Saw	0.5	0.4	0.5	0	280
9	Cut fabric	0	0.4	0	0	140
10	Sand	0.5	0.1	0.5	0	280
11	Stain	0.4	0.2	0.4	0	140
12	Assemble	1	1.5	0.5	0	700
13	Table Demand	1			0	300
14	Sofa Demand		1		0	180
15	Chair Demand			1	0	400
16						

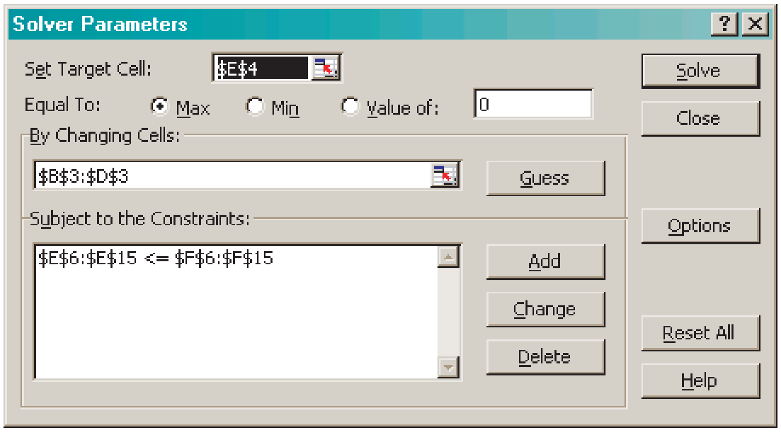
Solved Problem

Ready NUM

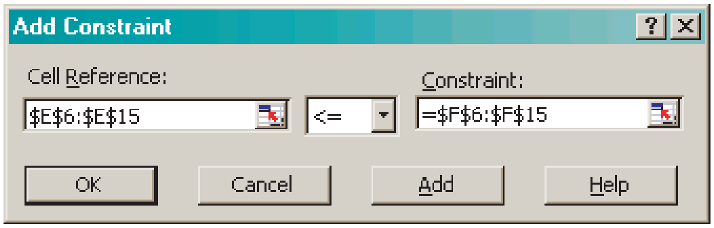
Step 2: Calculate Total Profit This is E4 (this is equal to B3 times the $300 revenue associated with each end table, plus C3 times the $500 revenue for each sofa, plus D3 times the $200 revenue associated with each chair). Note the $75,000 fixed expense that has been subtracted from revenue to calculate profit.

Step 3: Set Up Resource Usage In cells E6 through E15, the usage of each resource is calculated by multiplying B3, C3, and D3 by the amount needed for each item and summing the product (for example, E6 = B3*B6 + C3*C6 + D3*D6). The limits on these constraints are entered in cells F6 to F15.

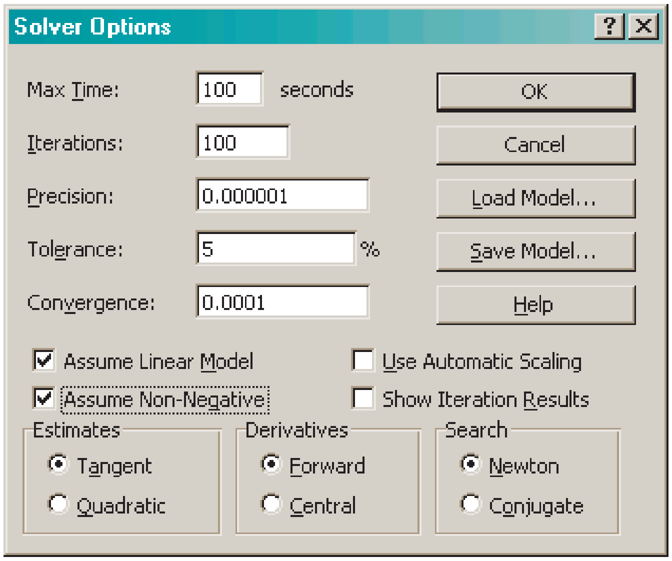
Step 4: Set Up Solver Go to Tools and select the Solver option.



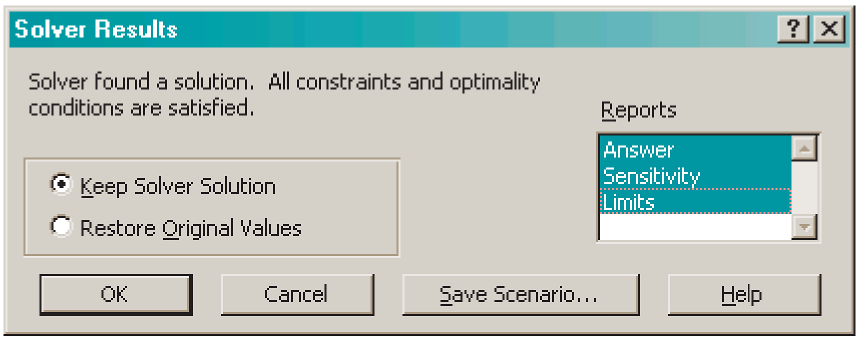
a. Set Target Cell: is set to the location where the value that we want to optimize is calculated. This is the profit calculated in E4 in this spreadsheet.
b. Equal To: is set to Max since the goal is to maximize profit.
c. By Changing Cells: are the cells that Solver can change to maximize profit (cells B3 through D3 in this problem).
d. Subject to the Constraints: is where a constraint set is added; we indicate that the range E6 to E15 must be less than or equal to F6 to F15.



Step 5: Set Options There are many options here, but for our purposes we just need to indicate Assume Linear Model and Assume Non-Negative. Assume Linear Model means all of our formulas are simple linear equations. Assume Non-Negative indicates that changing cells must be greater than or equal to zero. Click OK and we are ready to solve our problem.



Step 6: Solve the Problem Click Solve. We can see the solution and two special reports by highlighting items on the Solver Results acknowledgment that is displayed after a solution is found. Note that in the following report, Solver indicates that it has found a solution and all constraints and optimality conditions are satisfied. In the Reports box on the right, the Answer, Sensitivity, and Limits options have been highlighted, indicating that we would like to see these items. After highlighting the reports, click OK to exit back to the spreadsheet.



Note that three new tabs have been created: an Answer Report, a Sensitivity Report, and a Limits Report. The Answer Report indicates in the Target Cell section that the profit associated with this solution is $93,000 (we started at −$75,000). From the Target Cell section, we should make 260 end tables, 180 sofas, and no chairs. From the Constraints section, notice that the only constraints limiting profit are the staining capacity and the demand for sofas. We can see this from the column indicating whether a constraint is binding or nonbinding. Nonbinding constraints have slack, as indicated in the last column.

Target Cell (Max)

CELL	NAME	ORIGINAL VALUE	FINAL VALUE
$E$4	Profit Total	−$75,000	$93,000

Adjustable Cells

CELL	NAME	ORIGINAL VALUE	FINAL VALUE
$B$3	Changing cells End Tables	0	260
$C$3	Changing cells Sofas	0	180
$D$3	Changing cells Chairs	0	0

Constraints

CELL	NAME	CELL VALUE	FORMULA	STATUS	SLACK
$E$6	Lumber Total	3950	$E$6<=$F$6	Not Binding	400
$E$7	Fabric Total	1800	$E$7<=$F$7	Not Binding	700
$E$8	Saw Total	202	$E$8<=$F$8	Not Binding	78
$E$9	Cut fabric Total	72	$E$9<=$F$9	Not Binding	68
$E$10	Sand Total	148	$E$10<=$F$10	Not Binding	132
$E$11	Stain Total	140	$E$11<=$F$11	Binding	0
$E$12	Assemble Total	530	$E$12<=$F$12	Not Binding	170
$E$13	Table Demand Total	260	$E$13<=$F$13	Not Binding	40
$E$14	Sofa Demand Total	180	$E$14<=$F$14	Binding	0
$E$15	Chair Demand Total	0	$E$15<=$F$15	Not Binding	400

Of course, we may not be too happy with this solution since we are not meeting all the demand for tables, and it may not be wise to totally discontinue the manufacturing of chairs.

The Sensitivity Report (shown below) gives additional insight into the solution. The Adjustable Cells section of this report shows the final value for each cell and the reduced cost. The reduced cost indicates how much the target cell value would change if a cell that was currently set to zero were brought into the solution. Since the end tables (B3) and sofas (C3) are in the current solution, their reduced cost is zero. For each chair (D3) that we make, our target cell would be reduced $100 (just round these numbers for interpretation purposes). The final three columns in the adjustable cells section of the report are the Objective Coefficient from the original spreadsheet and columns titled Allowable Increase and Allowable Decrease. Allowable Increase and Decrease show by how much the value of the corresponding coefficient could change so there would not be a change in the changing cell values (of course, the target cell value would change). For example, revenue for each end table could be as high as $1,000 ($300 + $700) or as low as $200 ($300 − $100), and we would still want to produce 260 end tables. Keep in mind that these values assume nothing else is changing in the problem. For the allowable increase value for sofas, note the value 1E+30. This is a very large number, essentially infinity, represented in scientific notation.

Adjustable Cells

CELL	NAME	FINAL VALUE	REDUCED COST	OBJECTIVE COEFFICIENT	ALLOWABLE INCREASE	ALLOWABLE DECREASE
$B$3	Changing cells End Tables	260	0	299.9999997	700.0000012	100.0000004
$C$3	Changing cells Sofas	180	0	500.0000005	1E+30	350.0000006
$D$3	Changing cells Chairs	0	−100.0000004	199.9999993	100.0000004	1E+30

Constraints

CELL	NAME	FINAL VALUE	SHADOW PRICE	CONSTRAINT R.H. SIDE	ALLOWABLE INCREASE	ALLOWABLE DECREASE
$E$6	Lumber Total	3950	0	4350	1E+30	400
$E$7	Fabric Total	1800	0	2500	1E+30	700
$E$8	Saw Total	202	0	280	1E+30	78
$E$9	Cut fabric Total	72	0	140	1E+30	68
$E$10	Sand Total	148	0	280	1E+30	132
$E$11	Stain Total	140	749.9999992	140	16	104
$E$12	Assemble Total	530	0	700	1E+30	170
$E$13	Table Demand Total	260	0	300	1E+30	40
$E$14	Sofa Demand Total	180	350.0000006	180	70	80
$E$15	Chair Demand Total	0	0	400	1E+30	400

For the Constraints section of the report, the actual final usage of each resource is given in Final Value. The Shadow Price is the value to our target cell for each unit increase in the resource. If we could increase staining capacity, it would be worth $750 per hour. The Constraint Right-Hand Side is the current limit on the resource. Allowable Increase is the amount the resource could be increased while the shadow price is still valid. Another 16 hours' work of staining capacity could be added with a value of $750 per hour. Similarly, the Allowable Decrease column shows the amount the resource could be reduced without changing the shadow price. There is some valuable information available in this report.

The Limits Report provides additional information about our solution.

CELL	TARGET NAME	VALUE
$E$4	Profit Total	$93,000

CELL	ADJUSTABLE NAME	VALUE	LOWER LIMIT	TARGET RESULT	UPPER LIMIT	TARGET RESULT
$B$3	Changing cells End Tables	260	0	15000	260.0000002	93000
$C$3	Changing cells Sofas	180	0	3000	180	93000
$D$3	Changing cells Chairs	0	0	93000	0	93000

Total profit for the current solution is $93,000. Current value for B3 (end tables) is 260 units. If this were reduced to 0 units, profit would be reduced to $15,000. At an upper limit of 260, profit is $93,000 (the current solution). Similarly, for C3 (sofas), if this were reduced to 0, profit would be reduced to $3,000. At an upper limit of 180, profit is $93,000. For D3 (chairs), if this were reduced to 0, profit is $93,000 (current solution), and in this case the upper limit on chairs is also 0 units.

Acceptable answers to the questions are as follows:

1 *What is the most limiting resource to the furniture company?*
In terms of our production resources, staining capacity is really hurting profit at this time. We could use another 16 hours of capacity.

2 *Determine the product mix needed to maximize profit at the furniture company.*
The product mix would be to make 260 end tables, 180 sofas, and no chairs.

Of course, we have only scratched the surface with this solution. We could actually experiment with increasing staining capacity. This would give insight into the next most limiting resource. We also could run scenarios where we are required to produce a minimum number of each product, which is probably a more realistic scenario. This could help us determine how we could possibly reallocate the use of labor in our shop.

SOLVED PROBLEM 2

It is 2:00 on Friday afternoon and Joe Bob, the head chef (grill cook) at Bruce's Diner, is trying to decide the best way to allocate the available raw material to the four Friday night specials. The decision has to be made in the early afternoon because three of the items must be started now (Sloppy Joes, Tacos, and Chili). The table below contains the information on the food in inventory and the amounts required for each item.

Food	Cheese Burger	Sloppy Joes	Taco	Chili	Available
Ground Beef (lbs.)	0.3	0.25	0.25	0.4	100 lbs.
Cheese (lbs.)	0.1	0	0.3	0.2	50 lbs.
Beans (lbs.)	0	0	0.2	0.3	50 lbs.
Lettuce (lbs.)	0.1	0	0.2	0	15 lbs.
Tomato (lbs.)	0.1	0.3	0.2	0.2	50 lbs.
Buns	1	1	0	0	80 buns
Taco Shells	0	0	1	0	80 shells

One other fact relevant to Joe Bob's decision is the estimated market demand and selling price.

	Cheese Burger	Sloppy Joes	Taco	Chili
Demand	75	60	100	55
Selling Price	$2.25	$2.00	$1.75	$2.50

Joe Bob wants to maximize revenue since he has already purchased all the materials that are sitting in the cooler.

Required:

1 What is the best mix of the Friday night specials to maximize Joe Bob's revenue?
2 If a supplier offered to provide a rush order of buns at $1.00 a bun, is it worth the money?

Solution

Define X_1 as the number of Cheese Burgers, X_2 as the number of Sloppy Joes, X_3 as the number of Tacos, and X_4 as the number of bowls of chili made for the Friday night specials.

$$\text{Revenue} = \$2.25\,X_1 + \$2.00\,X_2 + \$1.75\,X_3 + \$2.50\,X_4$$

Constraints are the following:

Ground Beef: $0.30\,X_1 + 0.25\,X_2 + 0.25\,X_3 + 0.40\,X_4 \leq 100$
Cheese: $0.10\,X_1 + 0.30\,X_3 + 0.20\,X_4 \leq 50$
Beans: $0.20\,X_3 + 0.30\,X_4 \leq 50$
Lettuce: $0.10\,X_1 + 0.20\,X_3 \leq 15$
Tomato: $0.10\,X_1 + 0.30\,X_2 + 0.20\,X_3 + 0.20\,X_4 \leq 50$
Buns: $X_1 + X_2 \leq 80$
Taco Shells: $X_3 \leq 80$

Demand

Cheese Burger $X_1 \leq 75$
Sloppy Joes $X_2 \leq 60$
Taco $X_3 \leq 100$
Chili $X_4 \leq 55$

Step 1: Define the Changing Cells These are B3, C3, D3, and E3. Note the values in the changing cell are set to 10 each so the formulas can be checked.

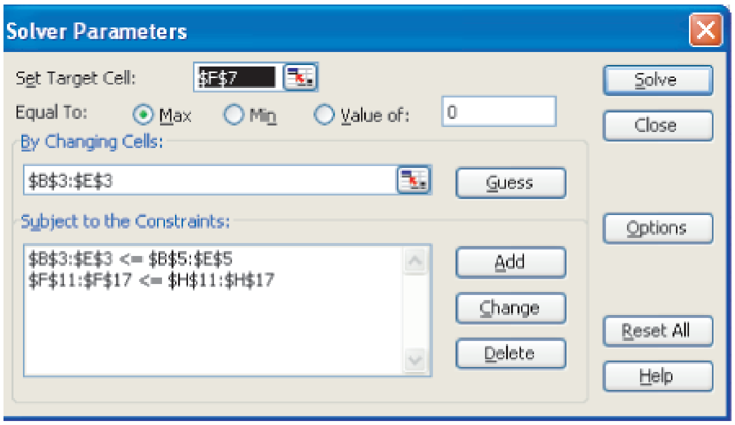
F7 =SUMPRODUCT($B$3:$E$3,B7:E7)

	A	B	C	D	E	F	G	H
1								
2		Cheese Burger	Sloppy Joes	Taco	Chili			
3	Changing Cells	10	10	10	10			
4		>=	>=	>=	>=			
5	Demand	75	60	100	55			
6						Total		
7	Revenue	$ 2.25	$ 2.00	$ 1.75	$ 2.50	$ 85.00		
8								
9								
10	Food	Cheese Burger	Sloppy Joes	Taco	Chili	Total		Available
11	Ground Beef (lbs.)	0.3	0.25	0.25	0.4	12.00	<=	100
12	Cheese (lbs.)	0.1	0	0.3	0.2	6.00	<=	50
13	Beans (lbs.)	0	0	0.2	0.3	5.00	<=	50
14	Lettuce (lbs.)	0.1	0	0.2	0	3.00	<=	15
15	Tomato (lbs.)	0.1	0.3	0.2	0.2	8.00	<=	50
16	Buns	1	1	0	0	20.00	<=	80
17	Taco Shells	0	0	1	0	10.00	<=	80
18								

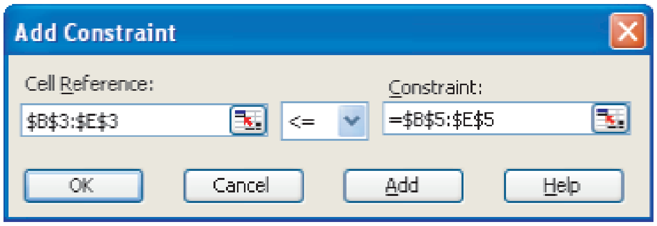
Step 2: Calculate Total Revenue This is in cell F7 (this is equal to B3 times the $2.25 for each cheese burger, plus C3 times the $2.00 for a Sloppy Joe, plus D3 times the $1.75 for each taco, plus E3 times the $2.50 for each bowl of chili; the SUMPRODUCT function in Excel was used to make this calculation faster). Note that the current value is $85, which is a result of selling 10 of each item.

Step 3: Set Up the Usage of the Food In cells F11 to F17, the usage of each food is calculated by multiplying the changing cells row times the per item use in the table and then summing the result. The limits on each of these food types are given in H11 through H17.

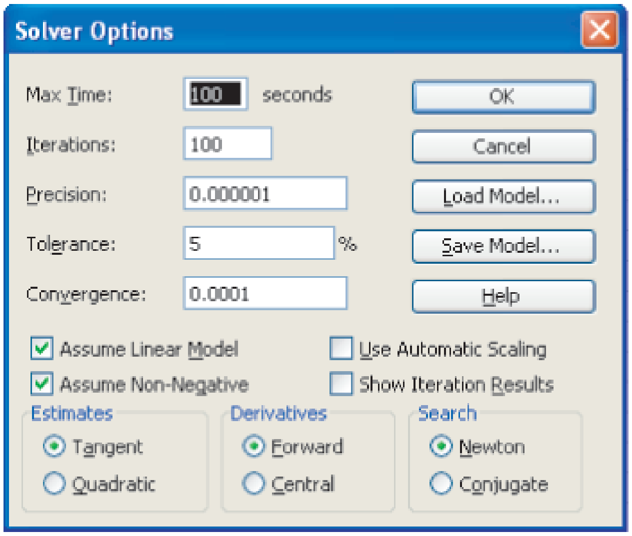
Step 4: Set Up Solver and Select the Solver Option



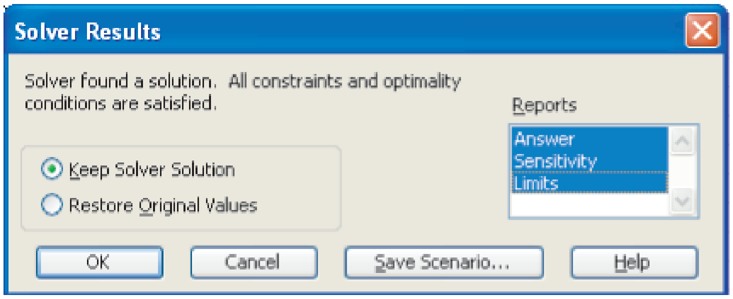
a. Set Target Cell: is set to the location where the value that we want to optimize is calculated. The revenue is calculated in F7 in this spreadsheet.
b. Equal to: is set to Max since the goal is to maximize revenue.
c. By Changing Cells: are the cells that tell how many of each special to produce.
d. Subject to the Constraints: is where we add two separate constraints, one for demand and one for the usage of food.



Step 5: Set Options Click on Options. We will leave all the settings as the default values and only need to make sure of two changes: (1) check the Assume Linear Model option and (2) check the Assume Non-Negative option. These two options make sure that Solver knows that this is a linear programming problem and that all changing cells should be nonnegative. Click OK to return to the Solver Parameters screen.



Step 6: Solve the Problem Click Solve. We will get a Solver Results box. Make sure it says that it has the following statement: "Solver found a solution. All constraints and optimality conditions are satisfied."



On the right-hand side of the box, there is an option for three reports: Answer, Sensitivity, and Limit. Click on all three reports and then click OK; this will exit you back to the spreadsheet, but you will have three new worksheets in your workbook.

The answer report indicates that the target cell has a final solution of $416.25 and started at $85. From the adjustable cells area we can see that we should make 20 cheese burgers, 60 Sloppy Joes, 65 tacos, and 55 bowls of chili. This answers the first requirement from the problem of what the mix of Friday night specials should be.

Target Cell (Max)

CELL	NAME	ORIGINAL VALUE	FINAL VALUE
$F$7	Revenue Total	$85.00	$416.25

Adjustable Cells

CELL	NAME	ORIGINAL VALUE	FINAL VALUE
$B$3	Changing Cells Cheese Burger	10	20
$C$3	Changing Cells Sloppy Joes	10	60
$D$3	Changing Cells Taco	10	65
$E$3	Changing Cells Chili	10	55

Constraints

CELL	NAME	CELL VALUE	FORMULA	STATUS	SLACK
$F$11	Ground Beef (lbs.) Total	59.25	$F$11<=$H$11	Not Binding	40.75
$F$12	Cheese (lbs.) Total	32.50	$F$12<=$H$12	Not Binding	17.5
$F$13	Beans (lbs.) Total	29.50	$F$13<=$H$13	Not Binding	20.5
$F$14	Lettuce (lbs.) Total	15.00	$F$14<=$H$14	Binding	0
$F$15	Tomato (lbs.) Total	44.00	$F$15<=$H$15	Not Binding	6
$F$16	Buns Total	80.00	$F$16<=$H$16	Binding	0
$F$17	Taco Shells Total	65.00	$F$17<=$H$17	Not Binding	15
$B$3	Changing Cells Cheese Burger	20	$B$3<=$B$5	Not Binding	55
$C$3	Changing Cells Sloppy Joes	60	$C$3<=$C$5	Binding	0
$D$3	Changing Cells Taco	65	$D$3<=$D$5	Not Binding	35
$E$3	Changing Cells Chili	55	$E$3<=$E$5	Binding	0

The second required answer was whether it is worth it to pay a rush supplier $1 a bun for additional buns. The answer report shows us that the buns constraint was binding. This means that if we had more buns, we could make more money. However, the answer report does not tell us whether a rush order of buns at $1 a bun is worthwhile. In order to answer that question, we have to look at the sensitivity report.

Adjustable Cells

CELL	NAME	FINAL VALUE	REDUCED COST	OBJECTIVE COEFFICIENT	ALLOWABLE INCREASE	ALLOWABLE DECREASE
$B$3	Changing Cells Cheese Burger	20	0	2.25	0.625	1.375
$C$3	Changing Cells Sloppy Joes	60	0.625	2	1E+30	0.625
$D$3	Changing Cells Taco	65	0	1.75	2.75	1.25
$E$3	Changing Cells Chili	55	2.5	2.5	1E+30	2.5

Constraints

CELL	NAME	FINAL VALUE	SHADOW PRICE	CONSTRAINT R.H. SIDE	ALLOWABLE INCREASE	ALLOWABLE DECREASE
$F$11	Ground Beef (lbs.) Total	59.25	0.00	100	1E+30	40.75
$F$12	Cheese (lbs.) Total	32.50	0.00	50	1E+30	17.5
$F$13	Beans (lbs.) Total	29.50	0.00	50	1E+30	20.5
$F$14	Lettuce (lbs.) Total	15.00	8.75	15	3	13
$F$15	Tomato (lbs.) Total	44.00	0.00	50	1E+30	6
$F$16	Buns Total	80.00	1.38	80	55	20
$F$17	Taco Shells Total	65.00	0.00	80	1E+30	15

We have highlighted the buns row to answer the question. We can see that buns have a shadow price of $1.38. This shadow price means that each additional bun will generate $1.38 of profit. We also can see that other foods such as ground beef have a shadow price of $0. The items with a shadow price of $0 add nothing to profit since we are currently not using all that we have now. The other important piece of information that we have on the buns is that they are only worth $1.38 up until the next 55 buns and that is why the allowable increase is 55. We also can see that a pound of lettuce is worth $8.75. It might be wise to also look for a rush supplier of lettuce so we can increase our profit on Friday nights.

Acceptable answers to the questions are as follows:

1 *What is the best mix of the Friday night specials to maximize Joe Bob's revenue?*
20 cheese burgers, 60 Sloppy Joes, 65 tacos, and 55 bowls of chili.

2 *If a supplier offered to provide a rush order of buns at $1.00 a bun, is it worth the money?*
Yes, each additional bun brings in $1.38, so if they cost us $1, then we will net $0.38 per bun. However, this is true only up to 55 additional buns.

PROBLEMS

1 Solve the following problem with Excel Solver:

$$\text{Maximize } Z = 3X + Y.$$
$$12X + 14Y \leq 85$$
$$3X + 2Y \leq 18$$
$$Y \leq 4$$

2 Solve the following problem with Excel Solver:

$$\text{Minimize } Z = 2A + 4B.$$
$$4A + 6B \geq 120$$
$$2A + 6B \geq 72$$
$$B \geq 10$$

3 A manufacturing firm has discontinued production of a certain unprofitable product line. Considerable excess production capacity was created as a result. Management is considering devoting this excess capacity to one or more of three products: X_1, X_2, and X_3.

Machine hours required per unit are

	Product		
Machine Type	X_1	X_2	X_3
Milling machine	8	2	3
Lathe	4	3	0
Grinder	2	0	1

The available time in machine hours per week is

	Machine Hours per Week
Milling machines	800
Lathes	480
Grinders	320

The salespeople estimate they can sell all the units of X_1 and X_2 that can be made. But the sales potential of X_3 is 80 units per week maximum.

Unit profits for the three products are

	Unit Profits
X_1	$20
X_2	6
X_3	8

a. Set up the equations that can be solved to maximize the profit per week.
b. Solve these equations using the Excel Solver.
c. What is the optimal solution? How many of each product should be made, and what should the resultant profit be?

d. What is this situation with respect to the machine groups? Would they work at capacity, or would there be unused available time? Will X_3 be at maximum sales capacity?

e. Suppose that an additional 200 hours per week can be obtained from the milling machines by working overtime. The incremental cost would be $1.50 per hour. Would you recommend doing this? Explain how you arrived at your answer.

4 A diet is being prepared for the University of Arizona dorms. The objective is to feed the students at the least cost, but the diet must have between 1,800 and 3,600 calories. No more than 1,400 calories can be starch, and no fewer than 400 can be protein. The varied diet is to be made of two foods: *A* and *B*. Food *A* costs $0.75 per pound and contains 600 calories, 400 of which are protein and 200 starch. No more than two pounds of food *A* can be used per resident. Food *B* costs $0.15 per pound and contains 900 calories, of which 700 are starch, 100 are protein, and 100 are fat.

a. Write the equations representing this information.

b. Solve the problem graphically for the amounts of each food that should be used.

5 Repeat Problem 4 with the added constraint that not more than 150 calories shall be fat and that the price of food has escalated to $1.75 per pound for food *A* and $2.50 per pound for food *B*.

6 Logan Manufacturing wants to mix two fuels, *A* and *B*, for its trucks to minimize cost. It needs no fewer than 3,000 gallons to run its trucks during the next month. It has a maximum fuel storage capacity of 4,000 gallons. There are 2,000 gallons of fuel *A* and 4,000 gallons of fuel *B* available. The mixed fuel must have an octane rating of no less than 80.

When fuels are mixed, the amount of fuel obtained is just equal to the sum of the amounts put in. The octane rating is the weighted average of the individual octanes, weighted in proportion to the respective volumes.

The following is known: Fuel *A* has an octane of 90 and costs $1.20 per gallon. Fuel *B* has an octane of 75 and costs $0.90 per gallon.

a. Write the equations expressing this information.

b. Solve the problem using the Excel Solver, giving the amount of each fuel to be used. State any assumptions necessary to solve the problem.

7 You are trying to create a budget to optimize the use of a portion of your disposable income. You have a maximum of $1,500 per month to be allocated to food, shelter, and entertainment. The amount spent on food and shelter combined must not exceed $1,000. The amount spent on shelter alone must not exceed $700. Entertainment cannot exceed $300 per month. Each dollar spent on food has a satisfaction value of 2, each dollar spent on shelter has a satisfaction value of 3, and each dollar spent on entertainment has a satisfaction value of 5.

Assuming a linear relationship, use the Excel Solver to determine the optimal allocation of your funds.

8 C-town brewery brews two beers: Expansion Draft and Burning River. Expansion Draft sells for $20 per barrel, while Burning River sells for $8 per barrel. Producing a barrel of Expansion Draft takes 8 pounds of corn and 4 pounds of hops. Producing a barrel of Burning River requires 2 pounds of corn, 6 pounds of rice, and 3 pounds of hops. The brewery has 500 pounds of corn, 300 pounds of rice, and 400 pounds of hops. Assuming a linear relationship, use Excel Solver to determine the optimal mix of Expansion Draft and Burning River that maximizes C-town's revenue.

9 BC Petrol manufactures three chemicals at their chemical plant in Kentucky: BCP1, BCP2, and BCP3. These chemicals are produced in two production processes known as zone and man. Running the zone process for an hour costs $48 and yields three units of BCP1, one unit of BCP2, and one unit of BCP3. Running the man process for one hour costs $24 and yields one unit of BCP1 and one unit of BCP2. To meet customer demands, at least 20 units of BCP1, 10 units of BCP2, and 6 units of BCP3 must be produced daily. Assuming a linear relationship, use Excel Solver to determine the optimal mix of processes zone and man to minimize costs and meet BC Petrol daily demands.

10 A farmer in Wood County has 900 acres of land. She is going to plant each acre with corn, soybeans, or wheat. Each acre planted with corn yields a $2,000 profit; each with soybeans yields $2,500 profit; and each with wheat yields $3,000 profit. She has 100 workers and 150 tons of fertilizer. The table below shows the requirement per acre of each of the crops. Assuming a linear relationship, use Excel Solver to determine the optimal planting mix of corn, soybeans, and wheat to maximize her profits.

	CORN	SOYBEANS	WHEAT
Labor (workers)	0.1	0.3	0.2
Fertilizer (tons)	0.2	0.1	0.4

Selected Bibliography

Anderson, D. R.; D. J. Sweeney; and T. A. Williams. *An Introduction to Management Science*. 11th ed. Mason, OH. South-Western, 2005.

Kelly, Julia, and Curt Simmons. *The Unofficial Guide to Microsoft Excel 2007*. New York: John Wiley & Sons, 2007.

Winston, W. L., and S. C. Albright. *Practical Management Science*. 3rd ed. Mason, OH: South-Western, 2006.

Footnote

1 The slope of the objective function is -2. If P = profit, $P = \$2H + \$4C$; $\$2H = P + \$4C$; $H = P/2 - 2C$. Thus, the slope is -2.