## **List of Symbols**

γ	the <i>n</i> -dimensional vector of gravitational accelerations
$\Pi^{C}$	power due to the constraint wrenches, $\mathbf{w}^{C}$
$\theta$	the <i>n</i> -dimensional vector of generalized coordinates
θ, θ	the <i>n</i> -dimensional vectors of generalized rates and accelerations, respectively
ρ	a Spline function
$ au_i$	torque at each joint
τ	the <i>n</i> -dimensional vector of generalized forces due to external forces and
	moments
Ω	the $6n \times 6n$ matrix of angular velocities different than <b>W</b>
$\mathbf{\Omega}_i$	the 6 $\times$ 6 angular velocity matrix for the <i>i</i> <sup>th</sup> body different than $\mathbf{W}_i$
$\omega_e$	the 3-dimensional angular velocity vector of the end-effector
ω	the 3-dimensional angular velocity vector of the <i>i</i> <sup>th</sup> body
$\boldsymbol{\omega}_i,  \dot{\boldsymbol{\omega}}_i$	the 3-dimensional vectors of angular velocity and acceleration of link <i>i</i> ,
	respectively
$\mathbf{a}_i$	the 3-dimensional vector denoting point $O_{i+1}$ from $O_i$
$\mathbf{B}_{ii}$	the $6 \times 6$ twist propagation matrix from # <i>i</i> to # <i>j</i>
b	damping coefficient
$b_i, \theta_i, a_i, \alpha_i$	Four DH parameters
С	the $n \times n$ matrix for convective inertia terms (MCI)
С	total number of constraints imposed by $p$ joints
$c_i$	number of constraints imposed by each joint
<b>c</b> <sub><i>i</i></sub>	the 3-dimensional vector denoting the mass center $C_i$ from the origin of fixed
	frame, F
<b>ċ</b> , <b>ċ</b> <sub><i>i</i></sub>	The 3-dimensional vectors of linear velocity and acceleration of the mass
	center of link <i>i</i> , respectively
$\mathbf{d}_i$	the 3-dimensional vector of the mass center, $C_i$ , from the origin point, $O_i$
ess	steady-state error
$\mathbf{e}_i$	the 3-dimensional unit vector along the axis of the $i^{\text{in}}$ joint
$\mathbf{f}_{ij}$	the 3-dimensional vector of resulting force exerted on link $i$ by link $j$ at $O_i$
g	the 3-dimensional vector of acceleration due to gravity
h	the <i>n</i> -dimensional vector of convective inertia terms (VCI)
I	the $n \times n$ Generalized Inertia Matrix (GIM)
$\mathbf{I}_i$	the 3×3 inertia tensor of the $i^{\text{th}}$ body or link about its mass center, $C_i$
$\tilde{\mathbf{I}}_i$	the $3\times3$ inertia tensor of the <i>i</i> <sup>th</sup> composite body consisting of rigidly connected
	links, # <i>i</i> to # <i>n</i>
J	the $6 \times n$ Jacobian matrix for <i>n</i> -DOF robot
k	spring constant

$k_p, k_v, k_i$	proportional, derivative, and integral gains, respectively
Ĺ	Lagrangian
Μ	the $6n \times 6n$ generalized mass matrix
$\widetilde{\mathbf{M}}$	the $6n \times 6n$ matrix of composite mass matrix
$\mathbf{M}_i$	the 6 $\times$ 6 mass matrix for the <i>i</i> <sup>th</sup> body
$\widetilde{\mathbf{M}}_{i}$	the $6 \times 6$ mass matrix of the <i>i</i> <sup>th</sup> 'composite body'
$\widetilde{\mathbf{M}}_{o}, \widetilde{\mathbf{M}}_{l}, \widetilde{\mathbf{M}}_{e}$	the $6n \times 6n$ matrices, which constitute MCI
$m_i$	mass of <i>i</i> <sup>th</sup> body
m	linear momentum
m	angular momentum
Ν	the $6n \times n$ NOC matrix
$\mathbf{N}_d$	the $6n \times n$ block diagonal matrix of the DeNOC matrices
$\mathbf{N}_l$	the $6n \times 6n$ lower block triangular matrix of the DeNOC matrices
n	degree of freedom of the whole system
n <sub>i</sub>	relative degree of freedom of each joint
n <sub>ij</sub>	the 3-dimensional vector of resulting moment exerted on link $i$ by link j at $O_i$ .
p	number of kinematic pairs or joints in the system
р	position vector of a point P. Position vector of any other point is similarly
	represented.
$\mathbf{p}_{i}$	the 6-dimensional joint-motion propagation vector
[ <b>p</b> ] <sub>F</sub>	representation of vector $P$ in the fixed frame $F$ . Similar representation for other
0	frames.
Q	the $3 \times 3$ rotation matrix
$\mathbf{Q}_i$	the 3 $\times$ 3 rotation matrix transforming frame <i>i</i> to frame <i>i</i> +1, i.e., any vector
	representation in frame $i+1$ is pre-multiplied by this matrix to find the
r	number of rigid hodies or links in the system
, r	the 3-dimensional vector of the origin $\Omega_{\rm eff}$ from the mass center C
i i	dimension of working space (For planar, $s = 3$ ; spatial, $s = 6$ )
з Т	kinetic energy
Т	the $4 \times 4$ homogeneous transformation matrix
t	the $6n$ -dimensional vector of generalized twist
t	the 6-dimensional twist vector of the end-effector
t.	the 6-dimensional twist vector associated with the $i^{\text{th}}$ body
t.	the 6-dimensional twist-rate vector of the $i^{th}$ body
$U^{i}$	Potential energy
W	the $6n \times 6n$ generalized matrix of the angular velocities
$\mathbf{W}_{i}$	the $6 \times 6$ angular velocity matrix for the <i>i</i> <sup>th</sup> body
<b>W</b> <sub>i</sub>	the 6-dimensional vector of wrench acting on the $i^{th}$ body
W	the 6n-dimensional vectors of generalized wrench
$\mathbf{w}^{E}, \mathbf{w}^{C}$	the 6-dimensional vectors of external and constraint wrenches, respectively
$\widetilde{\mathbf{w}}_{i}^{E}$	the 6-dimensional vector of external wrench for the $i^{th}$ composite body
x, y, and z	unit vectors along axes, X, Y and Z, respectively. Similar notations for other
	unit vectors.
1	the identity matrix of compatible dimension
U 0	the matrix of compatible dimension whose all elements are zeros
0	the vector of compatible dimension whose all elements are zeros
[•] <sup>1</sup>	transpose of the argument [•]