CHAPTER 6
Fans and Blowers

Summary

Terminology

Refer Section 6.2 and Equations (6.1) to (6.4). Let Q is the capacity in m^3/s, γ_w is the specific weight of water (gauge fluid) in N/m^3 or kN/m^3.

- Fan total power $=\gamma_w Q h_{tw}$
- Fan static power $=\gamma_w Q h_{sw}$
- Fan total efficiency $=\frac{\text{Fan power (total)}}{\text{Fan input power}}\times 100\ \%$
- Fan static efficiency $=\frac{\text{Fan power (static)}}{\text{Fan input power}}\times 100\ \%$

Fan and System

Refer to Section 6.4

- Fan total pressure $=$ system (total) pressure loss
- Fan static pressure $+$ fan velocity pressure $=$ system pressure loss

Euler Equation of Specific Work

Refer Section 6.6.

Centrifugal Fan

Refer Equations (1.82), (1.98), (1.100), (6.5) and (6.7).

$$w = C_{b2}C_{w2} - C_{b1}C_{w1} = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2}$$
$$= \frac{(p_2 - p_1)}{\rho} + \frac{(C_2^2 - C_1^2)}{2}$$

For centrifugal machines, $C_{r1} \approx C_{r2}$

Axial Fan

For axial fan $C_{b1} = C_{b2} = C_b$. Refer Equations (1.82), (1.98), (1.100), (6.6) and (6.7).

$$w = C_b(C_{w2} - C_{w1}) = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} = \frac{(p_2 - p_1)}{\rho} + \frac{(C_2^2 - C_1^2)}{2}$$

Fan Efficiencies

- **Volumetric Efficiency**

$$\eta_v = \frac{Q}{Q + \Delta Q}$$

- **Vane Efficiency**

Circulatory flow leads to a reduction in head developed by fan/pump/compressor as discussed in Sections 1.7 and 6.10. Refer to Equations (1.106), (1.108) and (6.134). Theoretical impeller head or impeller input head is less than the Euler head which is expressed as,

$$H_{th} = H_e - H_{slip} = \frac{\sigma_s C_{b2}C_{w2} - C_{b1}C_{w1}}{g}$$

where, slip factor σ_s is expressed by Eq. (1.107) as $\sigma_s = \frac{C'_{w2}}{C_{w2}}$

Refer either Eq. (1.84) or Eq. (1.86). Euler head may be expressed as,

$H_e = \frac{C_{b2}C_{w2} - C_{b1}C_{w1}}{g}$

Vane efficiency is nothing but another way of expressing slip. Vane efficiency is used to take account of reduction in pressure rise due to circulatory motion (slip) and is defined as,

$\eta_{vane} = \frac{H_{th}}{H_e} = \frac{\sigma_s C_{b2}C_{w2} - C_{b1}C_{w1}}{C_{b2}C_{w2} - C_{b1}C_{w1}}$

If $C_{w1} = 0$, then,

$$\eta_{vane} = \frac{H_{th}}{H_e} = \sigma_s$$

Refer Figure 1.41, Section 2.3.1 and Equations (2.21), (2.24) taking casing losses to be zero $(h_c = 0,)$, hydraulic efficiency is,

$\eta_h = \frac{H}{H_{th}}$

If the casing losses are zero, $h_c = 0$, manometric efficiency referring to Eq. (2.22) is,

$\eta_{mano} = \frac{H}{H + h_c} = \frac{H}{H+0} = 1$

Multiplying Equations (4) and (5), we get,

$$\eta_{vane}\eta_h = \frac{H}{H_e}$$

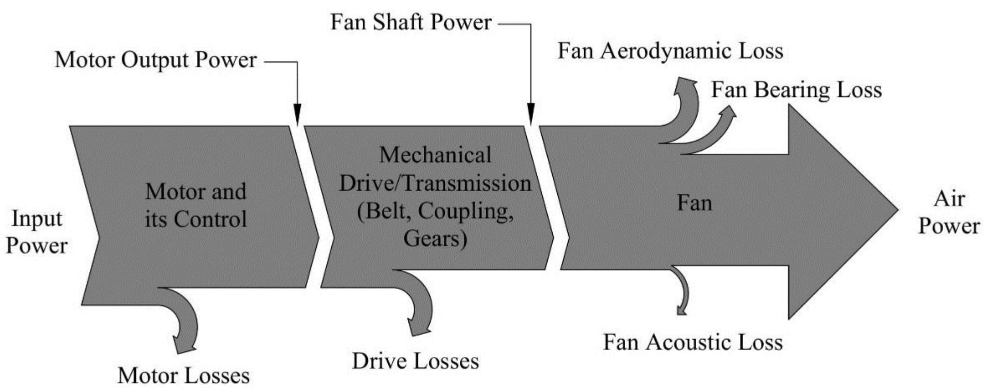

Figure SMS 6.1 *Energy Flow through a Fan System*

Figure SMS 6.1 shows the energy flow through a fan system. A typical fan system consists of a motor and its control, mechanical/transmission drive, and the fan. Power flows from the left to right with each component rejecting a portion of input energy as a result of inefficiencies. Efficiency of each component is the ratio of the output power to the input power. The fan imparts energy to the air stream by converting mechanical power at the fan shaft to air power at the outlet.

- **Fan Total Efficiency**

Total efficiency of the fan is the ratio of actual air power to fan shaft power. Neglecting leakage loss and slip in the impeller, fan shaft power is the Euler power to drive the fan.

$$\eta_{ftotal} = \frac{\text{Actual Air Power}}{\text{Fan Shaft Power}} = \frac{(\Delta p_o)_{stage-actual}}{\rho C_b (C_{w2} - C_{w1})}$$

Actual power input to the stage is,

$$P_{shaft} = \dot{m} C_b (C_{w2} - C_{w1}) = \frac{Q(\Delta p_0)_{stage-actual}}{\eta_{f\ total}}$$

- **Drive Efficiency**

$\eta_d = \frac{\text{Fan Shaft Power}}{\text{Motor Output Power}}$

Motor Efficiency

$$\eta_{motor} = \frac{\text{Motor Output Power}}{\text{Motor Input Power}}$$

If motor efficiency is 100 %, then,

Motor output power = Motor input power $= P_{motor}$

- **Overall Efficiency**

$$\eta_o = \frac{\text{Actual Air Power}}{\text{Motor Power}} = \frac{\text{Actual air power}}{\text{Fan shaft power}} \times \frac{\text{Fan shaft power}}{\text{Motor output power}} \times \frac{\text{Motor output power}}{\text{Motor input power}}$$

$$\eta_o = \eta_{f\ total}\eta_d\eta_{motor}$$

Axial Fan Stage Parameters

Refer Section 6.8.

Stage with Upstream Guide Blades

Refer Section 6.8.1 and Equations from (6.11) to (6.38).

- **Mass flow rate**

Velocity of flow at inlet and outlet may be taken to be the same, that is $C_{f1} = C_{f2} = C_f$.

$$\dot{m} = \rho\frac{\pi}{4}(D_t^2 - D_h^2)C_f$$

- **Stage Work**

$$w = (\Delta h_0)_{stage} = \frac{(\Delta p_0)_{stage-Euler}}{\rho} = C_b(C_{w2} - C_{w1}) = C_b^2\varphi(\tan\beta_1 - \tan\beta_2)$$

- **Stage Pressure Rise**

$$(\Delta p_o)_{stage-Euler} = \rho(\Delta h_o)_{stage} = \rho C_b(C_{w2} - C_{w1}) = \rho C_b^2\varphi(\tan\beta_1 - \tan\beta_2)$$

- **Stage Pressure Coefficient**

$$\Psi = \frac{(\Delta p_o)_{stage}}{0.5\rho C_b^2}$$

$$\psi_{ideal} = \frac{(\Delta p_o)_{stage-Euler}}{0.5\rho C_b^2} = 2\varphi(\tan\beta_1 - \tan\beta_2)$$

- **Degree of Reaction**

$$R = \frac{(\Delta p)_{rotor}}{(\Delta p_0)_{stage}} = \frac{[C_{w1}^2 - C_{w2}^2 - 2C_b(C_{w1} - C_{w2})]}{[2C_b(C_{w2} - C_{w1})]}$$

Stage without Guide Vanes

Refer Section 6.8.2 and Equations from (6.39) to (6.63).

- **Stage Work**

$$w = (\Delta h_0)_{stage} = \frac{(\Delta p_0)_{stage-Euler}}{\rho} = C_b(C_{w2} - C_{w1}) = C_b^2(1 - \varphi tan\beta_2)$$

- **Stage Pressure Rise**

$$(\Delta p_o)_{stage-Euler} = \rho(\Delta h_o)_{stage} = \rho C_b(C_{w2} - C_{w1}) = \rho C_b^2(1 - \varphi tan\beta_2)$$

$$(\Delta p)_{stage-Euler} = (\Delta p)_{rotor} = \rho\frac{C_b^2}{2}(1 - \varphi^2 tan^2\beta_2)$$

- **Stage Pressure Coefficient**

$$\Psi = \frac{(\Delta p_o)_{stage}}{0.5\rho C_b^2}$$

$$\psi_{ideal} = 2(1 - \varphi tan\beta_2)$$

$$\psi_{rotor} = (1 - \varphi^2 tan^2\beta_2)$$

- **Degree of Reaction**

$$R = \frac{(\Delta p)_{rotor}}{(\Delta p_0)_{stage}} = \frac{(1 + \varphi tan\beta_2)}{2}$$

Stage with Downstream Guide Blades

Refer Section 6.8.3 and Equations from (6.64) to (6.83).

- **Stage Work**

$$w = (\Delta h_0)_{stage} = \frac{(\Delta p_0)_{stage-Euler}}{\rho} = C_b(C_{w2} - C_{w1}) = C_b^2(1 - \varphi tan\beta_2)$$

- **Stage Pressure Rise**

$$(\Delta p_o)_{stage-Euler} = \rho(\Delta h_o)_{stage} = \rho C_b(C_{w2} - C_{w1}) = \rho C_b^2(1 - \varphi \tan\beta_2)$$

$$(\Delta p)_{rotor} = \rho\frac{C_b^2}{2}(1 - \varphi^2 tan^2\beta_2)$$

- **Stage Pressure Coefficient**

$$\Psi = \frac{(\Delta p_o)_{stage}}{0.5\rho C_b^2}$$

$$\psi_{ideal} = 2(1 - \varphi tan\beta_2)$$

- **Degree of Reaction**

$$R = \frac{(\Delta p)_{rotor}}{(\Delta p_0)_{stage}} = \frac{(1 + \varphi tan\beta_2)}{2}$$

Counter Rotating Stage

Refer Section 6.8.4 and Equations from (6.84) to (6.94).

- **Specific Work**

$$w = w_I + w_{II} = (\Delta h_o)_{stage} = \frac{(\Delta p_0)_{stage-Euler}}{\rho} = 2C_bC_{w2} = 2C_b^2(1 - \varphi\tan\beta_2)$$

- **Stage Pressure Rise**

$$(\Delta p_0)_{stage-Euler} = \rho(\Delta h_o)_{stage} = 2\rho C_bC_{w2} = 2\rho C_b^2(1 - \varphi\tan\beta_2)$$

- **Stage Pressure Coefficient**

$$\Psi = \frac{(\Delta p_o)_{stage}}{0.5\rho C_b^2}$$

$$\psi_{ideal} = 4(1 - \varphi\tan\beta_2)$$

Centrifugal Fan Stage Parameters

Refer Section 6.9 and Equations from (6.95) to (6.131).

- **Mass Flow Rate**

$$\dot{m} = \rho\pi D_1B_1C_{f1} = \rho\pi D_2B_2C_{f2}$$

- **Specific Work**

$$w = (\Delta h_o)_{stage} = \frac{(\Delta p_0)_{stage-Euler}}{\rho} = C_{w2}C_{b2}$$

- **Stage Pressure Rise**

$$(\Delta p_o)_{stage-Euler} = \rho(\Delta h_o)_{stage} = \rho C_{w2}C_{b2} = \rho C_{b2}[C_{b2} - (Q/\pi D_2B_2)\cot\beta_2]$$

$$= \rho C_{b2}^2(1 - \varphi\cot\beta_2)$$

- **Stage Pressure Coefficient**

Forward and Backward Curved Blades

$$\psi_{ideal} = 2(1 - \varphi\cot\beta_2)$$

$\lambda = \psi\varphi = 2\varphi(1 - \varphi \cot\beta_2)$

Radial Blades

$\psi_{ideal} = 2$

$\lambda = 2\varphi$

- **Degree of Reaction**

$$R = \frac{(\Delta p)_{impeller}}{(\Delta p_0)_{stage}} = 1 - \frac{C_{w2}}{2C_{b2}}$$

Backward Curved Vanes ($\boldsymbol{\beta_2 < 90°}$)

$\frac{C_{w2}}{C_{b2}} < 1$, therefore, therefore $R < 1$

Radial Vanes ($\boldsymbol{\beta_2 = 90°}$)

$\frac{C_{w2}}{C_{b2}} = 1$, therefore $R = \frac{1}{2}$

Forward Curved Vanes ($\boldsymbol{\beta_2 > 90°}$)

$\frac{C_{w2}}{C_{b2}} > 1$, therefore, therefore $R < \frac{1}{2}$

Fan Laws

Refer Section 6.15 and Equations (6.145), (6.146) and (6.147).

- **Fan Pressure Law**

$$\frac{(\Delta p)}{N^2 D^2} = constant$$

- **Fan Air Flow Law**

$$\frac{Q}{ND^3} = constant$$

- **Fan Air Power Law**

$$\frac{P}{N^3 D^5} = constant$$

Application of Fan Laws with Variation of One Independent Variable

Variable Speed ($\boldsymbol{N}$)	**Variable Diameter ($\boldsymbol{D}$)**	**Variable Density ($\boldsymbol{\rho}$)**
$\Delta p \propto N^2$	$\Delta p \propto D^2$	$\Delta p \propto \rho$
$Q \propto N$	$Q \propto D^3$	$Q = constant$
$P \propto N^3$	$P \propto D^5$	$P \propto \rho$

Note

Problems (6.1), (6.3) and (6.5) *are dealt as compressor problems because of the data given. Readers should note that the effect of compressibility is considered and density is not taken as a constant. Thermodynamic relations are used to calculate efficiencies.*

6.1 The air at $100\ kPa$ and 27°C enters the impeller of a centrifugal blower without any whirl component. The speed of the blower is $6000\ rpm$ and external diameter of the impeller is $0.6\ m$. Total-to-total efficiency is 70 % and degree of reaction is 0.6. The radial component of velocity is $95\ m/s$ which remains constant throughout. Determine (a) exit blade angle, (b) power input, and (c) pressure at outlet.

Solution

Given: $p_{01} = p_1 = 100\ kPa, T_{01} = T_1 = 27°C = 300\ K, C_{w1} = 0, N = 6000\ rpm, D_2 = 0.6\ m, \eta_{tt} = 70\ \%, R = 0.6, C_{f1} = C_{f2} = 95\ m/s$

The tangential or peripheral velocity of the blades at the exit is,

$$C_{b2} = \frac{\pi D_2 N}{60} \quad (1)$$

$$C_{b2} = \frac{\pi \times 0.6 \times 6000}{60} \Rightarrow C_{b2} = 188.4956\ m/s \quad (2)$$

Refer Eq. (6.130), degree of reaction for a centrifugal fan is,

$$R = 1 - \frac{C_{w2}}{2C_{b2}} \quad (3)$$

$$0.6 = 1 - \frac{C_{w2}}{2 \times 188.4956} \Rightarrow C_{w2} = 150.7965\ m/s \quad (4)$$

(a) Exit Blade Angle

From Equations (2) and (4), $C_{w2} < C_{b2}$, therefore, impeller blades are backward curved ones for which velocity triangles at inlet and outlet are shown in Figure P 6.1. Since, air enters the impeller of centrifugal blower without any whirl component, therefore, $C_{w1} = 0$. From the velocity triangle at exit,

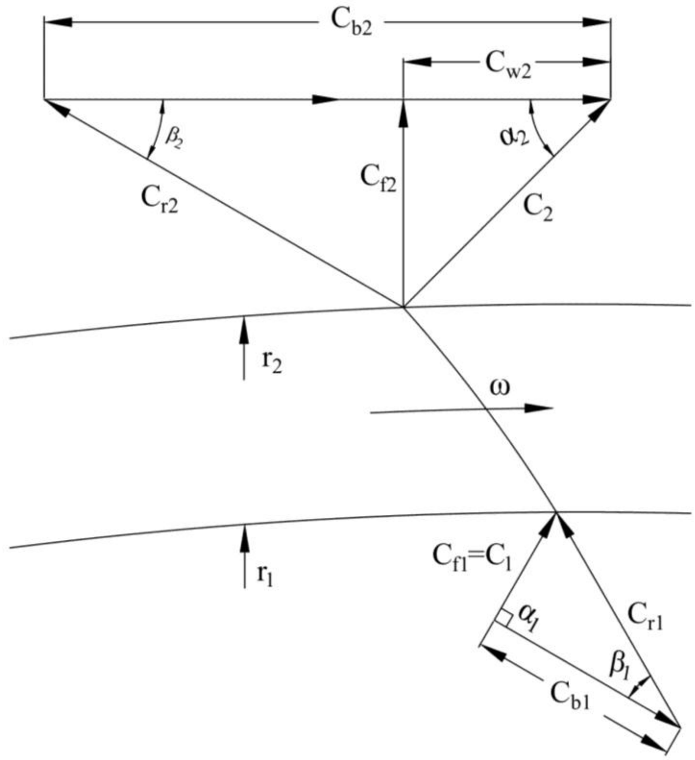

Figure P 6.1 *Velocity Triangles of Centrifugal Blower of Problem 6.1*

$$\tan\beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \quad (5)$$

$$\tan\beta_2 = \frac{95}{188.4956 - 150.7965} \Rightarrow \beta_2 = 68.3552° \quad (6)$$

(b) Power Input

Since the air enters radially in the impeller, therefore, $C_{w1} = 0$. Neglecting leakage loss and slip in the impeller, specific work input to the blower is Euler work which is expressed referring to Equations (1.82), (6.5) and/or (6.96) as,

$$w = C_{w2} C_{b2} \quad (7)$$

$$w = 150.7965 \times 188.4956 = 28424.4768\ J/kg = 28.4245\ kJ/kg \quad (8)$$

(c) Pressure at Outlet

Refer to Eq. (1.49) and assuming potential energy change to be zero $(Z_1 = Z_2)$, specific work is also expressed as,

$$w = \Delta h_0 = c_p (T_{02} - T_{01}) \quad (9)$$

$$28.4245 = 1.005(T_{02} - 300) \Rightarrow T_{02} = 328.2831\ K \quad (10)$$

Refer Eq. (2.37) of Section 2.4.1. Total-to-total efficiency is expressed as,

$$\eta_{tt} = \frac{T_{01} - T_{02s}}{T_{01} - T_{02}} \quad (11)$$

$$0.7 = \frac{300 - T_{02s}}{300 - 328.2831} \Rightarrow T_{02s} = 319.7982\ K \quad (12)$$

Referring Eq. (1.69) for an isentropic process,

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02s}}{T_{01}}\right)^{\frac{\gamma}{\gamma - 1}} \quad (13)$$

$$\frac{p_{02}}{100} = \left(\frac{319.7982}{300}\right)^{\frac{1.4}{1.4 - 1}} \Rightarrow p_{02} = 125.0667\ kPa \quad (14)$$

6.2 Air at a flow rate of 3 kg/s enters in a centrifugal fan at 100 kPa, 20°C from still atmosphere. The fan running at 580 rpm raises the static pressure by 2 kPa. The power input is 44 kW. If the speed is changed to 490 rpm and inlet conditions to 98 kPa, 200°C, find (a) static pressure at exit, (b) the mass flow rate, and (c) input power.

Solution

Subscript 1 represents one set operating conditions whereas subscript 2 represents the other set of operating conditions of the same fan.

Given: $\dot{m}_1 = 3\ kg/s, p_{01} = p_1 = 100\ kPa, T_{01} = T_1 = 20°C = 293\ K, N_1 = 580\ rpm, \Delta p_1 = 2\ kPa, P_1 = 44\ kW, N_2 = 490\ rpm, p_{02} = p_2 = 98\ kPa, T_{02} = T_2 = 200\ °C = 473\ K$

Assuming characteristic gas constant (R) for air,

$$R = 0.2874\ kJ/kg - K \quad (1)$$

From thermodynamics, applying characteristic gas equation at suction of the fan under operating condition 1,

$$p_1 V_1 = m R T_1 \Rightarrow p_1 = \rho_1 R T_1 \quad (2)$$

$$100 = \rho_1 \times 0.2874 \times 293 \Rightarrow \rho_1 = 1.1875\ kg/m^3 \quad (3)$$

Applying gas equation at the inlet of the fan under second operating conditions,

$$\rho_2 = \frac{(p_{\text{suction}})_2}{R(T_{\text{suction}})_2} = \frac{98}{0.2874 \times 473} = 0.7209\ kg/m^3 \quad (4)$$

Refer to fan laws discussed in Section 6.15 which can be applied to geometrically similar fans. The same laws can also be applied to a fan operating under homologous operating conditions.

(a) Static Pressure at Exit

Referring to Eq. (6.148) of Section 6.15.4,

$$\frac{(\Delta p)_1}{(\Delta p)_2} = \frac{N_1^2 D_1^2 \rho_1}{N_2^2 D_2^2 \rho_2} \quad (5)$$

Since the same fan $(D_1 = D_2)$ is operating under different set of operating conditions, therefore,

$$\Delta p_2 = \Delta p_1 \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{N_2}{N_1}\right)^2 \quad (6)$$

$$\Delta p_2 = 2\left(\frac{0.7209}{1.1875}\right)\left(\frac{490}{580}\right)^2 = 0.8666\ kPa \quad (7)$$

Static pressure at exit $=(p_{exit})_2 = (p_{suction})_2 + \Delta p_2$ (8)

$(p_{exit})_2 = 98 + 0.8666 = 98.8666\ kPa$ (9)

(b) Mass Flow Rate

From fluid mechanics we know that mass flow rate through a section of a control volume is given by,

$$\dot{m}_1 = \rho_1 Q_1 \quad (10)$$

$$3 = 1.1875 \times Q_1 \Rightarrow Q_1 = 2.5263\ kg/m^3 \quad (11)$$

Referring to Eq. (6.149) of Section 6.15.4,

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 \quad (12)$$

$$\frac{2.5263}{Q_2} = \left(\frac{580}{490}\right) \times 1^3 \Rightarrow Q_2 = 2.1343\ m^3/s \quad (13)$$

$$\dot{m}_2 = \rho_2 Q_2 \Rightarrow \dot{m}_2 = 0.7209 \times 2.1343 = 1.5386\ kg/s \quad (14)$$

(c) Input Power

Referring to Eq. (6.150) of Section 6.15.4,

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 \quad (15)$$

$$\frac{44}{P_2} = \left(\frac{1.1875}{0.7209}\right) \times \left(\frac{580}{490}\right)^3 \times (1)^5 \Rightarrow P_2 = 16.1064\ kW \quad (16)$$

The problem can also be solved by equating the energy, flow and power coefficients referring to Equations (1.84), (2.15), (2.16) and (2.18) keeping in mind that effect of density variation has to be considered due to change in operating conditions.

6.3 An overhead fan with a vertical shaft is fitted to extract air through a small cooling tower from its top. The speed of the fan is $500\ rpm$. The tip and hub diameters of the fan blades are $2\ m$ and $0.5\ m$, respectively. The blade loading is uniform through its length, i.e. specific work is same at any section of the blade. The exit blade angle at tip is $12°$ with the tangential or peripheral blade velocity direction and the flow velocity remains constant throughout the rotor at $10.5\ m/s$. The state of the air at inlet is $1\ bar, 15°C$, total-to-total efficiency is $85\ \%$ and mechanical efficiency is $90\ \%$. Determine (a) inlet blade angle of the tip, (b) inlet and outlet blade angles at the hub, (c) total pressure rise, (d) mass flow rate, and (e) the input power.

Solution

Given: $N = 500\ rpm, D_t = 2\ m, D_h = 0.5\ m, \beta_{2t} = 12°, C_{f1} = C_{f2} = 10.5\ m/s,$

$p_{01} = p_1 = 1\ bar, T_{01} = T_1 = 15°C = 288\ K, \eta_{tt} = 85\ \% = 0.85, \eta_m = 90\ \%$

Axial flow induced draft fan is used at the top of the cooling tower. Refer to Eq. (1.95). The tangential or peripheral blade velocities at the tip and hub respectively are,

$$C_{b1t} = C_{b2t} = C_{bt} = \frac{\pi D_t N}{60} = \frac{\pi \times 2 \times 500}{60} = 52.3599\ m/s \quad (1)$$

$$C_{b1h} = C_{b2h} = C_{bh} = \frac{\pi D_h N}{60} = \frac{\pi \times 0.5 \times 500}{60} = 13.09\ m/s \quad (2)$$

As neither upstream nor downstream guide vanes are given, therefore, axial fan stage may be considered without guide vanes. The velocity triangles at inlet and outlet are shown in Figure P 6.3 assuming the flow is radial at inlet $(C_{w1} = 0)$.

(a) Inlet Blade Angle at Tip

From the inlet velocity triangle,

$\tan\beta_{1t} = \frac{C_{bt}}{C_{f1}} = \frac{52.3599}{10.5} \Rightarrow \beta_{1t} = 78.6606°$ (3)

The blade angle at tip inlet calculated above is with the meridional (flow) direction. However, if relative flow angle at tip inlet i. e. inlet blade angle at tip is measured with the tangential or peripheral blade velocity (C_{bt}), then, $\beta_{1t} = 11.3394°$.

(b) Inlet and Outlet Blade Angles at Hub

From the inlet velocity triangle,

$\tan\beta_{1h} = \frac{C_{bh}}{C_{f1}} = \frac{13.09}{10.5} \Rightarrow \beta_{1h} = 51.2656°$ (4)

Inlet blade angle at hub measured with the tangential or peripheral blade velocity (C_{bt}) would then be, $\beta_{1h} = 38.7344°$.

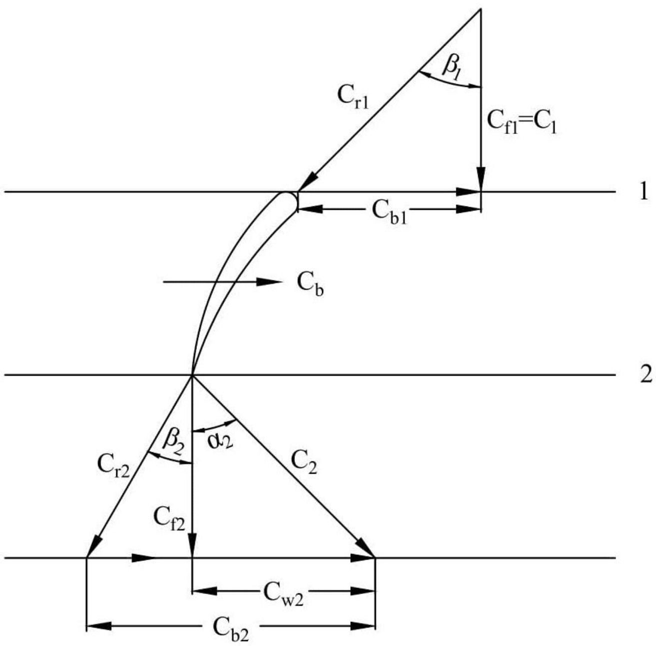

Figure P 6.3 *Velocity Diagrams at Inlet and Outlet of Fan Blades of Problem 6.3*

Since the exit blade angle at tip with the tangential or peripheral blade velocity (C_{bt}) is 12°, therefore, $\beta_{2t} = 78°$ with the meridional or flow velocity.

From outlet velocity triangle,

$\tan\beta_{2t} = \frac{C_{bt} - C_{w2t}}{C_{f2}} \Rightarrow \tan 78 = \frac{52.3599 - C_{w2t}}{10.5} \Rightarrow C_{w2t} = 2.9613\ m/s$ (5)

Refer to either Eq. (1.82) or (6.5) assuming the flow is radial at inlet $(C_{w1} = 0)$ or Eq. (6.6), specific work at the tip is,

$w_t = C_{bt}C_{w2t}$ (6)

$w_t = 52.3599 \times 2.9613 = 155.0534\ J/kg$ (7)

Since the blade loading is uniform through its length i.e. specific work is same at any section of the blade, therefore,

$w = w_t = w_h \Rightarrow C_{bt}C_{w2t} = C_{bh}C_{w2h}$ (8)

$52.3599 \times 2.9613 = 13.09 \times C_{w2h} \Rightarrow C_{w2h} = 11.8452\ m/s$ (9)

From outlet velocity triangle,

$\tan\beta_{2h} = \frac{C_{bh} - C_{w2h}}{C_{f2}} \Rightarrow \tan\beta_{2h} = \frac{13.09 - 11.8452}{10.5} \Rightarrow \beta_{2h} = 6.761°$ (10)

Outlet blade angle at hub measured with the tangential or peripheral blade velocity (C_{bh}) would then be, $\beta_{2h} = 90 - 6.761 = 83.239°$.

(c) Total Pressure Rise

Refer Eq. (2.37) and Eq. (1.84). Total-to-total efficiency is expressed as,

$$\eta_{tt} = \eta_{sc} = \frac{h_{01} - h_{02s}}{h_{01} - h_{02}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{c_p(T_{02s} - T_{01})}{w} \quad (11)$$

$$0.85 = \frac{1005\,(T_{02s} - 288)}{155.0534} \Rightarrow T_{02s} = 288.1311\ K \quad (12)$$

Referring Eq. (1.69) for an isentropic process,

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \quad (13)$$

$$\frac{p_{02}}{1} = \left(\frac{288.1311}{288}\right)^{\frac{1.4}{1.4-1}} \Rightarrow p_{02} = 1.002\ bar \quad (14)$$

Therefore, rise in total pressure,

$\Delta p_0 = p_{02} - p_{01} = 1.002 - 1 = 0.002\ bar = 20.3874\ mm$ of water (15)

(d) Mass Flow Rate

Refer to Eq. (1.20), flow area is given by,

$$A = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(2^2 - 0.5^2) = 2.9452\ m^2 \quad (16)$$

From fluid mechanics, we know that discharge or volume flow rate is given by,

$$Q = AC_f = 2.9452 \times 10.5 = 30.9246\ m^3/s$$

From thermodynamics, applying characteristic gas equation at inlet of the fan

$$p_1V_1 = mRT_1 \Rightarrow p_1 = \rho_1 RT_1 \quad (17)$$

$$100 = \rho_1 \times 0.2874 \times 288 \Rightarrow \rho_1 = 1.2082\ kg/m^3 \quad (18)$$

From fluid mechanics we know that mass flow rate through a section of a control volume is given by,

$$\dot{m} = \rho_1 Q \quad (19)$$

$$\dot{m} = 1.2082 \times 30.9246 = 37.3631\ m^3/s \quad (20)$$

(e) Input Power

Considering leakage loss and slip in the impeller to be zero. Then, fluid power supplied to the impeller is Euler power $(\dot{m}w)$. Refer Eq. (2.20). Mechanical efficiency is expressed as,

$$\eta_m = \frac{\text{Fluid power supplied to the impeller}}{\text{Power input to the shaft (Fan Shaft Power)}} = \frac{\dot{m}w}{P_{\text{shaft}}} \quad (21)$$

$$0.90 = \frac{37.3631 \times 155}{P_{\text{shaft}}} \Rightarrow P_{\text{shaft}} = 6434.7561\ W = 6.4348\ kW \quad (22)$$

6.4 Characteristic curves are available for a fan running at $850\ rpm$ and passing air of inlet density $1.2\ kg/m^3$. Readings from the curves indicate that at an airflow of $150\,m^3/s$, the fan pressure is $2.2\ kPa$ gauge and the shaft power is $440\ kW$. Assuming that the efficiency remains unchanged, calculate the corresponding points if the fan is running at $1100\ rpm$ in air of density $1.1\ kg/m^3$.

Solution

Given: $N_1 = 850\ rpm, \rho_1 = 1.2\ kg/m^3, Q_1 = 150\ m^3/s, \Delta p_1 = 2.2\ kPa, P_1 = 440\ kW,$
$N_2 = 1100\ rpm, \rho_2 = 1.1\ kg/m^3$

(a) Static Pressure at Exit

Referring to Eq. (6.148) of Section 6.15.4,

$$\frac{(\Delta p)_1}{(\Delta p)_2} = \frac{N_1^2 D_1^2 \rho_1}{N_2^2 D_2^2 \rho_2} \quad (1)$$

Since the same fan is operating under different set of operating conditions, therefore, $D_1 = D_2$. Hence,

$$\Delta p_2 = \Delta p_1 \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{N_2}{N_1}\right)^2 \tag{2}$$

$$\Delta p_2 = 2.2\left(\frac{1.1}{1.2}\right)\left(\frac{1100}{850}\right)^2 = 3.3774\ kPa\ gauge \tag{3}$$

(b) Air Flow Rate

Referring to Eq. (6.149) of Section 6.15.4,

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 \tag{4}$$

$$\frac{150}{Q_2} = \left(\frac{850}{1100}\right) \times 1^3 \Rightarrow Q_2 = 194.1177\ m^3/s \tag{5}$$

(c) Shaft Power

Referring to Eq. (6.150) of Section 6.15.4,

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 \tag{6}$$

$$\frac{440}{P_2} = \left(\frac{1.2}{1.1}\right) \times \left(\frac{850}{1100}\right)^3 \times (1)^5 \Rightarrow P_2 = 874.1489\ kW \tag{7}$$

The problem can also be solved by equating the energy, flow and power coefficients referring to Equations (1.84), (2.15), (2.16) and (2.18) keeping in mind that effect of density variation has to be considered due to change in operating conditions.

6.5 Air flow rate at the inlet of a fan is $300\,m^3/s$. The fan develops a pressure of 2.5 kPa. The barometric pressure at the fan inlet is 97 kPa. The motor consumes an electrical power of $1100\,kW$. If the combined motor/transmission efficiency is 95 %, determine the isentropic efficiency of the impeller and, also, of the total unit.

Solution

Given: $Q_1 = 300\ m^3/s, \Delta p = 2.5\ kPa, p_1 = p_a = 97\ kPa, P_{motor} = 1100\ kW, \eta_{motor} = 95\ \%$

Pressure at the exit of the fan,

$$p_2 = p_1 + \Delta p \Rightarrow p_2 = 97 + 2.5 = 99.5\ kPa \tag{1}$$

Actual power input to the fan (fan shaft power) may be found as,

$$\eta_{motor} = \frac{P_{shaft}}{P_{motor}} \Rightarrow P_{shaft} = \eta_{motor}\ P_{motor} \tag{2}$$

$$P_{shaft} = 0.95 \times 1100 = 1045\ kW \tag{3}$$

From thermodynamics, we know that isentropic process obeys the relation $pv^\gamma = constant$. We know that specific work for a flow process may be obtained by $w = \int_1^2 -vdp$ which for an isentropic process is given by,

$$w_s = \frac{\gamma}{\gamma-1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{4}$$

From fluid mechanics, mass flow rate of air is,

$$\dot{m} = \rho_1 Q_1 = \frac{Q_1}{v_1} \tag{5}$$

where, v_1 is the specific volume at the inlet. Therefore, power input to the fan when compression is isentropic i.e. isentropic fan power is,

$$P_s = \dot{m} w_s = \frac{\gamma}{\gamma-1} p_1 Q_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{6}$$

$$P_s = \frac{1.4}{1.4-1} \times 97 \times 300 \times \left[\left(\frac{99.5}{97}\right)^{\frac{1.4-1}{1.4}} - 1\right] = 743.1964\ kW \tag{7}$$

Isentropic efficiency of the fan,

$$\eta_s = \frac{\text{Isentroipc Fan Power}}{\text{Fan Shaft Power}} = \frac{P_s}{P_{shaft}} \quad (8)$$

$$\eta_s = \frac{743.1964}{1045} = 0.711193 = 71.1193\ \% \quad (9)$$

Overall efficiency of the fan,

$$\eta_o = \frac{\text{Isentropic Fan Power}}{\text{Power Input to Motor (Motor power)}} \quad (10)$$

$$\eta_o = \frac{743.1964}{1100} = 0.675633 = 67.5633\ \% \quad (11)$$

Overall efficiency of the fan may also be found as,

$$\eta_o = \frac{\text{Isentropic Fan Power}}{\text{Power Input to Motor (Motor Power)}}$$

$$\eta_o = \frac{\text{Isentropic Fan Power}}{\text{Fan Shaft Power}} \times \frac{\text{Fan Shaft Power}}{\text{Motor Power}} = \eta_s\, \eta_{motor} \quad (12)$$

$$\eta_o = 0.711193 \times 0.95 = 0.675633 = 67.5633\ \% \quad (13)$$

Alternate Method

From Eq. (6),

$$P_s = \dot{m} w_s = \frac{\gamma}{\gamma - 1} p_1 Q_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = \frac{\Delta p}{\Delta p} \frac{\gamma}{\gamma - 1} p_1 Q_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$P_s = \Delta p Q_1 \frac{\gamma}{\gamma-1} \frac{p_1}{\Delta p} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = K \Delta p Q_1 \quad (14)$$

where, K known as compressibility coefficient is,

$$K = \frac{\gamma}{\gamma-1} \frac{p_1}{\Delta p} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = \frac{\gamma}{\gamma-1} \left[\frac{\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{p_2}{p_1} - 1} \right] \quad (15)$$

If compressibility had been ignored treating air as an incompressible fluid, $K = 1$, the isentropic power input to the fan i.e. isentropic fan power is,

$$P_s = \Delta p Q = 2.5 \times 300 = 750\ kW \quad (16)$$

$$\eta_s = \frac{\text{Isentroipc Fan Power}}{\text{Fan Shaft Power}} = \frac{P_s}{P_{shaft}} \quad (17)$$

$$\eta_s = \eta_{f\ total} = \frac{750}{1045} = 0.717703 = 71.7703\ \% \quad (18)$$

The value of the isentropic fan efficiency given by Eq. (18) involves an error of 0.9155 % from the true value.

Overall efficiency of the fan,

$$\eta_o = \frac{\text{Isentropic Fan Power}}{\text{Power Input to Motor (Motor Power)}} \quad (19)$$

$$\eta_o = \frac{750}{1100} = 0.681818 = 68.1818\ \% \quad (20)$$

6.6 A centrifugal fan is used for air circulation in a water cooling tower. The fan running at $1200\ rpm$ delivers $16\, m^3/s$ of air against a stagnation pressure difference of $2.8\ kPa$. This operating condition represents the design point at which there are no shock losses and no whirl at inlet. The fan sucks air of density $1.2\ kg/m^3$ from the atmosphere. The air enters the impeller radially (with no pre-whirl) at the design point. The inlet and exit diameters of the impeller are $0.9\ m$ and $1.35\ m$ respectively. The velocity of flow is constant through the impeller at $20\ m/s$.

Take $\eta_v = 96\ \%, \eta_{vane} = 88\ \%, \eta_h = 85\ \%, \eta_m = 95\ \%$. Determine (a) vane angles at inlet and at exit, (b) impeller widths at inlet and exit, and (c) fan power consumption.

Solution

Given: $N = 1200\ rpm, Q = 16\ m^3/s, \Delta p_0 = 2.8\ kPa, C_{w1} = 0, D_1 = 0.9\ m, D_2 = 1.35\ m,$
$C_{f1} = C_{f2} = 20\ m/s, \eta_v = 96\ \%, \eta_{vane} = 88\ \%, \eta_h = 85\ \%, \eta_m = 95\ \%$

Tangential or peripheral velocities of the blades at the inlet and outlet of the fan are found by referring Eq. (1.95) as,

$$C_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.9 \times 1200}{60} = 56.5487\ m/s \quad (1)$$

$$C_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.35 \times 1200}{60} = 84.823\ m/s \quad (2)$$

The velocity triangles at inlet and outlet of fan are shown in Figure P 6.6.

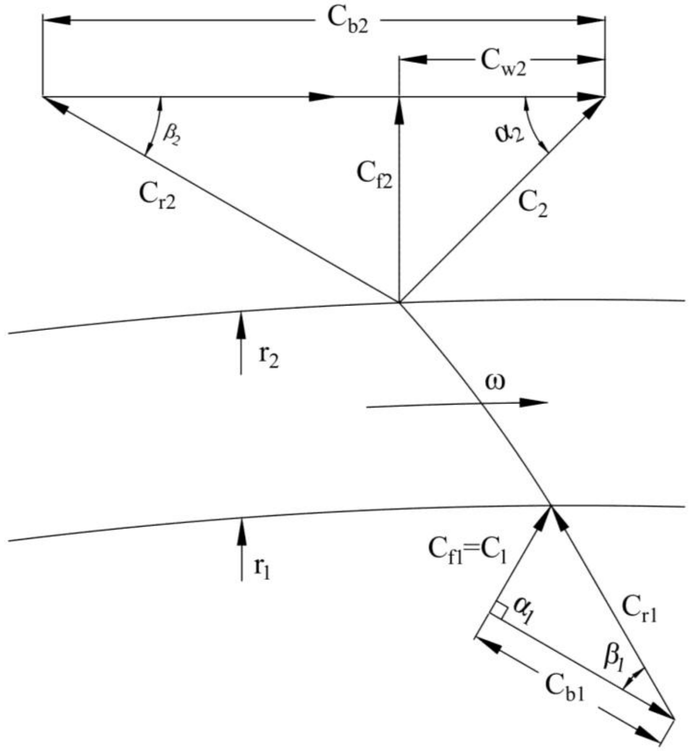

Figure P 6.6 *Velocity Triangles of Centrifugal Blower of Problem 6.6*

(a) Vane Angles at Inlet and Outlet

From the velocity triangle at inlet as shown in Figure P 6.6,

$$\tan\beta_1 = \frac{C_{f1}}{C_{b1}} \Rightarrow \tan\beta_1 = \frac{20}{56.5487} \Rightarrow \beta_1 = 19.4775° \quad (3)$$

Actual head developed by the pump or imparted to the air,

$$H = \frac{\Delta p_{0-actual}}{\rho g} = \frac{2.8 \times 1000}{1.2 \times 9.81} = 237.8526\ m \quad (4)$$

Circulatory flow lead to a reduction in head developed by fan/pump/compressor as discussed in Sections 1.7 and 6.10 and Equations (1.106), (1.108) and (6.134). Therefore, theoretical impeller head or impeller input head is less than the Euler head and vane efficiency is used to take account of slip and is defined as,

$$\eta_{vane} = \frac{H_{th}}{H_e} \quad (5)$$

Refer Figure 1.41, Section 2.3.1 and Equations (2.21), (2.24) taking casing losses to be zero $(h_c = 0,)$, hydraulic efficiency is,

$\eta_h = \frac{H}{H_{th}}$ (6)

Since casing losses are zero, $h_c = 0$, therefore, manometric efficiency referring to Eq. (2.22) is,

$\eta_{mano} = \frac{H}{H+h_c} = \frac{H}{H+0} = 1$ (7)

Multiplying Equations (4) and (5), we get,

$\eta_{vane}\eta_h = \frac{H}{H_e} \Rightarrow 0.88 \times 0.85 = \frac{237.8526}{H_e} \Rightarrow H_e = 317.9848\ m$ (8)

Considering the fan to be an incompressible machine, refer to Eq. (1.84) or Eq. (1.85),

$w = \Delta h_0 = gH_e = (C_{w1}C_{b1} - C_{w2}C_{b2})$ (9)

Since there are no shock losses and no whirl at inlet, therefore, $C_{w1} = 0$ and neglecting the negative sign in the RHS of Eq. (7) as it represents the input work. Hence,

$gH_e = C_{w2}C_{b2} \Rightarrow 9.81 \times 317.9848 = C_{w2} \times 84.823 \Rightarrow C_{w2} = 36.7758\ m/s$ (10)

From the velocity triangle at inlet as shown in Figure P 6.6,

$\tan\beta_2 = \frac{C_{f2}}{C_{b2}-C_{w2}} \Rightarrow \tan\beta_2 = \frac{20}{84.823-36.7758} \Rightarrow \beta_2 = 22.6°$ (11)

(b) Impeller Widths at Inlet and Exit

Refer Eq. (2.23). Volumetric efficiency of the fan is expressed as,

$\eta_v = \frac{\text{Flow rate through fan outlet (fan discharge)}}{\text{Flow rate (discharge) through impeller}}$ (12)

$0.96 = \frac{16}{\text{Flow rate (discharge) through impeller}}$

$\text{Discharge through impeller} = 16.6667\ m^3/s$ (13)

The discharge through the impeller is expressed referring to Eq. (6.95) as,

$\text{Discharge through impeller} = \pi D_1 B_1 C_{f1} = \pi D_2 B_2 C_{f2}$ (14)

$16.6667 = \pi \times 0.9 \times B_1 \times 20 = \pi \times 1.35 \times B_2 \times 20$

$B_1 = 0.294732\ m = 294.732\ mm, B_2 = 0.196488\ m = 196.488\ mm$ (15)

(c) Fan Power Consumption

Refer to Eq. (2.25). Overall efficiency of the fan is,

$\eta_o = \eta_m\eta_h\eta_{mano}\eta_v = \frac{\rho gQH}{P_{\text{shaft}}}$ (16)

$0.95 \times 0.85 \times 1 \times 0.96 = \frac{1.2 \times 9.81 \times 16 \times 237.8526}{P_{\text{shaft}}}$

$P_{\text{shaft}} = 57791.5543\ W = 57.7916\ kW$ (17)

6.7 A centrifugal fan equipped with inlet guide vanes is running at a speed of $750\ rpm$. The fan is designed to deliver air of density $1.2\ kg/m^3$ at a rate of $4.25\ m^3/s$. The flow has no pre-rotation at inlet. The diameter of impeller at inlet and outlet is $0.525\ m$ and $0.75\ m$ respectively. The width of the blades at inlet is $172\ mm$ while that at outlet is $100\ mm$. The blade angle at exit is $70°$. The pressure recovery in the volute casing is 40 % of the actual velocity head at the impeller exit and the leakage is negligibly small. The blade, hydraulic and mechanical efficiencies are $88\ \%, 85\ \%$ and $96\ \%$ respectively. Determine (a) the actual velocity and pressure at the fan discharge section, and (b) fan brake power.

Solution

Given: $N = 750\ rpm, \rho = 1.2\ kg/m^3, Q = 4.25\ m^3/s, C_{w1} = 0, D_1 = 0.525\ m, D_2 = 0.75\ m,$ $B_1 = 172\ mm, B_2 = 100\ mm, \beta_2 = 70°, \eta_{vane} = 88\ \%, \eta_h = 85\ \%, \eta_m = 96\ \%, \Delta Q = 0,$

pressure recovery in casing = 40 % of actual exit velocity head

Tangential or peripheral velocities of the blades at the inlet and outlet of the fan are found by referring Eq. (1.95) as,

$C_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.525 \times 750}{60} = 20.6167\ m/s$ (1)

$C_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.75 \times 750}{60} = 29.4524\ m/s$ (2)

Since the leakage is negligibly small, therefore, $\eta_v = 1$, and discharge through fan impeller is same as that from the fan outlet. The discharge through the impeller is expressed referring to Eq. (6.95) as,

$\text{Discharge through impeller} = Q = \pi D_2 B_2 C_{f2}$ (3)

$4.25 = \pi \times 0.75 \times 0.1 \times C_{f2} \Rightarrow C_{f2} = 18.0376\ m/s$ (4)

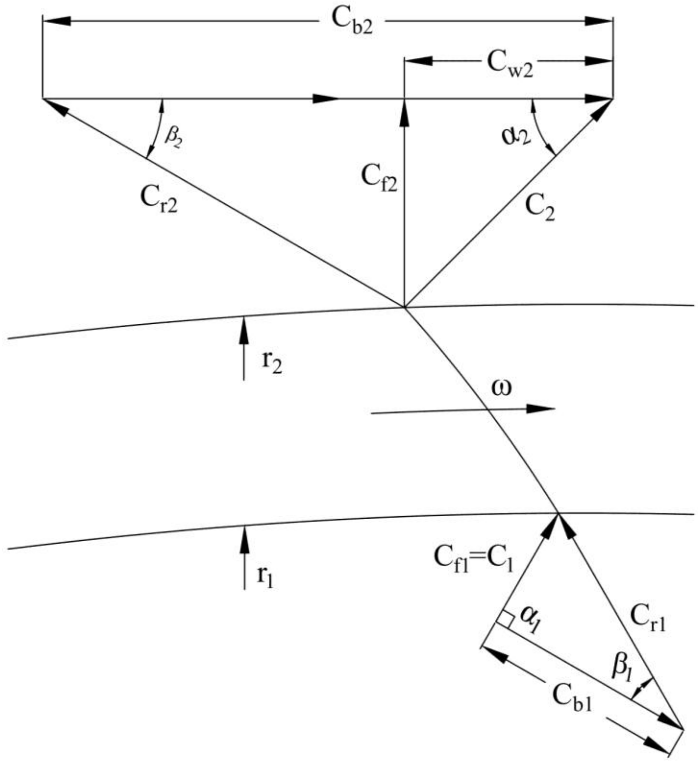

Figure P 6.7 (a) *Velocity Triangles of Centrifugal Blower of Problem 6.7 without Slip*

The velocity triangles at inlet and outlet of fan are shown in Figure P 6.7. From the outlet velocity triangle,

$C_{w2} = C_{b2} - C_{f2}\cot\beta_2 \Rightarrow C_{w2} = 29.4524 - 18.0376\cot 70 = 22.8873\ m/s$ (5)

Considering the fan to be an incompressible machine, refer to Eq. (1.84) or Eq. (1.85),

$w = \Delta h_0 = gH_e = (C_{w1}C_{b1} - C_{w2}C_{b2})$ (6)

Since there is no pre-rotation (no whirl) at inlet, therefore, $C_{w1} = 0$ and neglecting the negative sign in the RHS of Eq. (7) as it represents the input work. Hence,

$gH_e = C_{w2}C_{b2} \Rightarrow 9.81 \times H_e = 22.8873 \times 29.4524 \Rightarrow H_e = 68.7142\ m$ (7)

Circulatory flow lead to a reduction in head developed by fan/pump/compressor as discussed in Sections 1.7 and 6.10 and Equations (1.106), (1.108) and (6.134). Therefore, theoretical impeller head or impeller input head is less than the Euler head and vane efficiency is used to take account of slip and is defined as,

$\eta_{vane} = \frac{H_{th}}{H_e}$ (8)

$0.88 = \frac{H_{th}}{68.7142} \Rightarrow H_{th} = 60.4685\ m$ (9)

Refer Figure 1.41, Section 2.3.1 and Equations (2.21), (2.24) taking casing losses to be zero, hydraulic efficiency is,

$\eta_h = \frac{H}{H_{th}}$ (10)

$0.85 = \frac{H}{60.4685} \Rightarrow H = 51.3982\ m$ (11)

(a) Actual Velocity and Pressure at Fan Discharge Section

Refer to Eq. (1.107), slip factor is,

$\sigma_s = \frac{C'_{w2}}{C_{w2}}$ (12)

Theoretical impeller head or impeller input head is expressed by using Eq. (12) above in either Eq. (1.108) or (6.134) as,

$H_{th} = \frac{\sigma_s C_{b2} C_{w2} - C_{b1} C_{w1}}{g} = \frac{C'_{w2} C_{b2}}{g}$ (13)

since $C_{w1} = 0$.

$60.4685 = \frac{C'_{w2} \times 29.4524}{9.81} \Rightarrow C'_{w2} = 20.1408\ m/s$ (14)

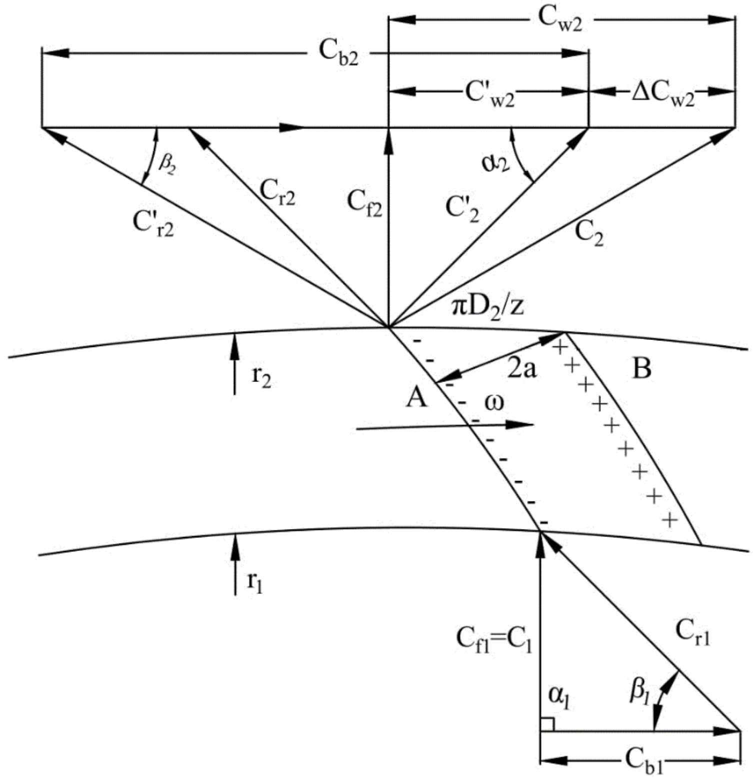

Figure P 6.7 (b) *Velocity Triangles of Centrifugal Blower of Problem 6.7 with Slip in Outlet Velocity Triangle*

From velocity triangle in Figure 6.7 (b), actual absolute velocity at impeller exit with slip,

$C'_2 = \sqrt{(C'_{w2})^2 + C_{f2}^2}$ (15)

$C'_2 = \sqrt{20.1408^2 + 18.0376^2} = 27.0371\ m/s$ (16)

Let C_3 is the air velocity at the fan exit section (casing outlet). Since the pressure recovery in the volute casing is 40 % of the actual velocity head at the impeller exit, therefore,

$\frac{C_3^2}{2g} = 0.6\frac{(C_2')^2}{2g} \Rightarrow C_3 = \sqrt{0.6}\,C_2' \Rightarrow C_3 = \sqrt{0.6} \times 27.0371 = 20.9429\ m/s$ (17)

Let p_3 be the pressure at the fan exit section. Applying the Bernoulli's equation between the fan exit section (casing outlet) and ambient, we get,

$\frac{p_3}{\rho g} + \frac{C_3^2}{2g} = \frac{p_a}{\rho g} + H \Rightarrow (p_3 - p_a) = \rho g\left(H - \frac{C_3^2}{2g}\right)$ (18)

where, p_a is the atmospheric pressure.

$(p_3 - p_a) = 1.2 \times 9.81 \times \left(51.3982 - \frac{20.9429^2}{2\times 9.81}\right)$

$(p_3 - p_a) = 341.8966\ Pa\ (gauge) = 0.3418966\ kPa\ (gauge)$ (19)

(b) Fan Brake Power

Assuming the casing losses to be zero, $h_c = 0$, then manometric efficiency referring to Eq. (2.22) is,

$\eta_{mano} = \frac{H}{H+h_c} = \frac{H}{H+0} = 1$ (20)

As leakage is negligibly small, therefore,

$\eta_v = 1$ (21)

Refer to Eq. (2.25). Overall efficiency of the fan is,

$\eta_o = \eta_m \eta_h \eta_{mano} \eta_v = \frac{\rho g Q H}{P_{\text{shaft}}}$ (22)

$0.96 \times 0.85 \times 1 \times 1 = \frac{1.2 \times 9.81 \times 4.25 \times 51.3982}{P_{\text{shaft}}}$

$P_{\text{shaft}} = 3151.3521\ W = 3.15136\ kW$ (23)

6.8 Assume that the fan given in Example 6.7 is now equipped with upstream guide vanes for flow rate control. The flow rate is required to be reduced to $3.2\ m^3/s$ by using upstream guide vanes to impose pre-whirl. What should be the flow angle at inlet? In this case, the hydraulic efficiency is reduced to 80 % due to the increase in shock losses. What will be the new brake power?

Solution

Given: $Q' = 3.2\ m^3/s, \eta_h = 80\ \% = 0.8,$ rest data same as problem 6.8.

Let us first find the blade angle at inlet of the fan from the data of problem 6.7. Since the leakage is negligibly small, therefore, discharge through fan impeller is same as that fan outlet and volumetric efficiency, $\eta_v = 1$. The discharge through the impeller is expressed referring to Eq. (6.95) as,

Discharge through impeller $= Q = \pi D_1 B_1 C_{f1}$ (1)

$4.25 = \pi \times 0.525 \times 0.172 \times C_{f1} \Rightarrow C_{f1} = 14.9814\ m/s$ (2)

From the inlet velocity triangle of Problem 6.7,

$\tan\beta_1 = \frac{C_{f1}}{C_{b1}} \Rightarrow \tan\beta_1 = \frac{14.9814}{20.6167} \Rightarrow \beta_1 = 36°$ (3)

Since the fan given in Example 6.7 is now equipped with upstream guide vanes to be reduce the discharge to $Q' = 3.2\ m^3/s$ in order to impose pre-whirl. Therefore,

$Q' = \pi D_1 B_1 C_{f1}' \Rightarrow 3.2 = \pi \times 0.525 \times 0.172 \times C_{f1}' \Rightarrow C_{f1}' = 11.28\ m/s$ (4)

Flow Angle at Inlet

The velocity triangles for the fan of this problem are shown in Figure 6.8. The blade angle for the fan of Problem 6.8 will be the same as that of problem 6.7. From the velocity triangle at inlet,

$C_{w1} = C_{b1} - C_{f1}'\cot\beta_1 \Rightarrow C_{w1} = 20.6167 - 11.28\cot 36 = 5.0911\ m/s$ (5)

$\tan\alpha_1 = \frac{C'_{f1}}{C_{w1}} \Rightarrow \tan\alpha_1 = \frac{11.28}{5.0911} \Rightarrow \alpha_1 = 65.7085°$ (6)

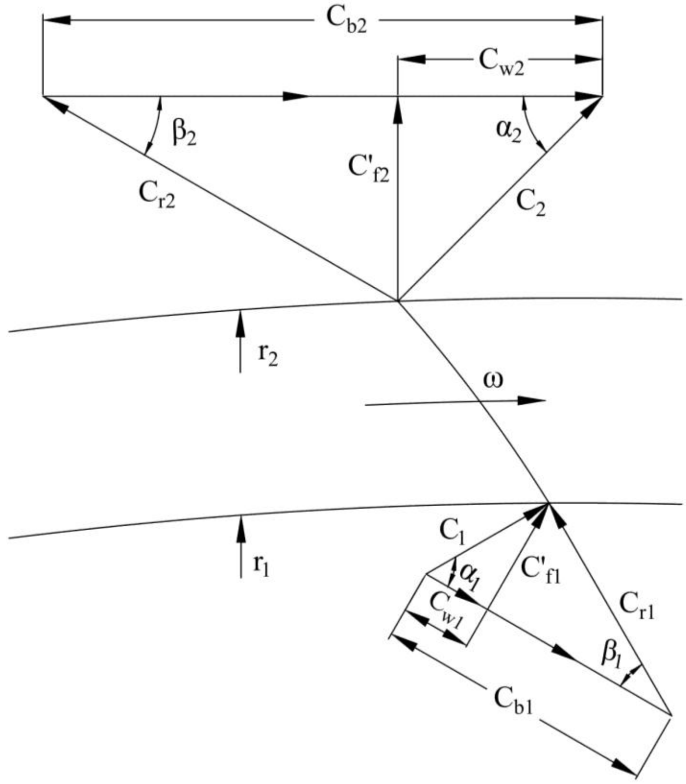

Figure P 6.8 *Velocity Triangles of Centrifugal Blower of Problem 6.8*

Brake Power

Referring to Eq. (6.95),

$\text{Discharge through impeller} = Q' = \pi D_2 B_2 C'_{f2}$ (7)

$3.2 = \pi \times 0.75 \times 0.1 \times C'_{f2} \Rightarrow C'_{f2} = 13.5812\ m/s$ (8)

From the outlet velocity triangle,

$C_{w2} = C_{b2} - C'_{f2}\cot\beta_2 \Rightarrow C_{w2} = 29.4524 - 13.5812\cot 70 = 24.5093\ m/s$ (9)

Considering the fan to be an incompressible machine, refer to Eq. (1.84) or Eq. (1.85),

$w = \Delta h_0 = gH_e = (C_{w1}C_{b1} - C_{w2}C_{b2})$ (10)

Neglecting the negative sign in the RHS of Eq. (7) as it represents the input work. Hence,

$gH_e = (C_{w2}C_{b2} - C_{w1}C_{b1}) \Rightarrow 9.81 \times H_e = 24.5093 \times 29.4524 - 5.0911 \times 20.6167$

$H_e = 62.8844\ m$ (11)

$\eta_{vane} = \frac{H_{th}}{H_e}$ (12)

$0.88 = \frac{H_{th}}{62.8844} \Rightarrow H_{th} = 55.3383\ m$ (13)

Refer Figure 1.41, Section 2.3.1 and Equations (2.21), (2.24) taking casing losses to be zero, hydraulic efficiency is,

$\eta_h = \frac{H}{H_{th}}$ (14)

$0.85 = \frac{H}{55.3383} \Rightarrow H = 47.0376\ m$ (15)

Assuming the casing losses to be zero, $h_c = 0$, then manometric efficiency referring to Eq. (2.22) is,

$$\eta_{mano} = \frac{H}{H+h_c} = \frac{H}{H+0} = 1 \quad (16)$$

As leakage is negligibly small, therefore,

$$\eta_v = 1 \quad (17)$$

Refer to Eq. (2.25). Overall efficiency of the fan is,

$$\eta_o = \eta_m \eta_h \eta_{mano} \eta_v = \frac{\rho g Q H}{P_{shaft}} \quad (18)$$

$$0.96 \times 0.85 \times 1 \times 1 = \frac{1.2 \times 9.81 \times 3.2 \times 47.0376}{P_{shaft}}$$

$$P_{shaft} = 2171.477\, W = 2.1715\, kW \quad (19)$$

6.9 The tip and hub diameters of the rotor of an axial fan stage are $0.6\ m$ and $0.3\ m$ respectively. The speed of the fan is $960\ rpm$ and the blade angle at exit is $10°$ with meridional direction. The flow conditions at the inlet of the fan are $1.02\ bar$ and 43 °C. The flow coefficient is 0.245 and power required by the fan is $1\ kW$. Find (a) blade angle at inlet, (b) discharge, (c) stage pressure rise, (d) overall efficiency, (e) degree of reaction, and (f) specific speed.

Solution

Given: $D_t = 0.6\ m, D_h = 0.3\ m, N = 960\ rpm, \beta_2 = 10°, p_{01} = p_1 = 1.02\ bar = 102\ kPa, T_{01} = T_1 = 43 + 273 = 316\ K, \varphi = 0.245, P_{shaft} = 1\ kW$

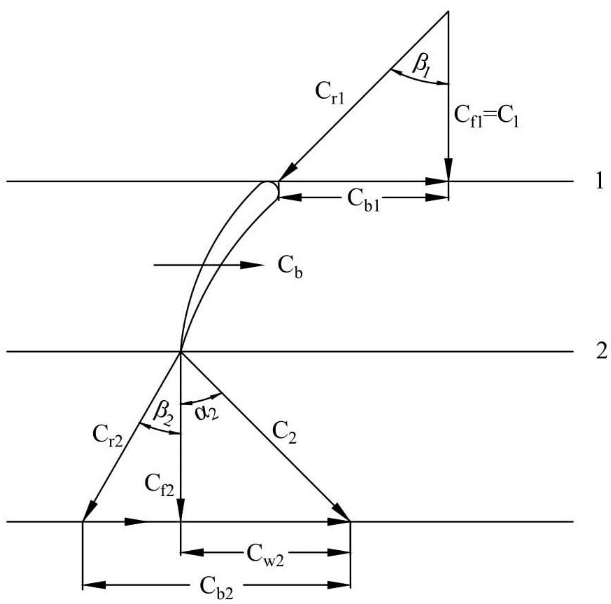

Figure P 6.9 *Velocity Triangles for an Axial Fan Stage of Problem 6.9*

Mean diameter of the blades,

$$D = \frac{D_t + D_h}{2} = \frac{0.6+0.3}{2} = 0.45\ m \quad (1)$$

Mean blade velocity referring to Section 1.5.2 and Eq. (1.95) is,

$$C_b = \frac{\pi D N}{60} \quad (2)$$

$$C_b = \frac{\pi \times 0.45 \times 960}{60} = 22.6195\ m/s \quad (3)$$

Assuming the flow velocity at inlet and outlet of the fan blades to be constant. Therefore,

$$C_{f1} = C_{f2} = C_f \tag{4}$$

Refer either Eq. (2.17) or (6.44). Flow coefficient is,

$$\varphi = \frac{C_f}{C_b} \tag{5}$$

$$0.245 = \frac{C_f}{22.6195} \Rightarrow C_f = 5.5418\ m/s \tag{6}$$

(a) Blade Angle at Inlet

Velocity triangles at inlet and outlet of an axial stage consisting of only rotor is shown in Figure P 6.9. From the velocity triangle at inlet,

$$\tan\beta_1 = \frac{C_b}{C_f} \Rightarrow \tan\beta_1 = \frac{22.6195}{5.5418} \Rightarrow \beta_1 = 76.2337° \tag{7}$$

(b) Discharge

Refer to Eq. (1.20), flow area is given by,

$$A = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(0.6^2 - 0.3^2) = 0.2121\ m^2 \tag{8}$$

From fluid mechanics, we know that discharge or volume flow rate is given by,

$$Q = AC_f = 0.2121 \times 5.5418 = 1.1754\ m^3/s \tag{9}$$

(c) Stage Pressure rise

From thermodynamics, applying characteristic gas equation at inlet of the fan

$$p_1V_1 = mRT_1 \Rightarrow p_1 = \rho_1 RT_1 \tag{10}$$

$$102 = \rho_1 \times 0.2874 \times 316 \Rightarrow \rho_1 = 1.1231\ kg/m^3 \tag{11}$$

Considering the fan to be an incompressible machine, therefore, $\rho_1 = \rho_2 = \rho$. Refer Eq. (6.40). The specific work is,

$$w = C_b(C_{w2} - C_{w1}) = \Delta h_0 = \frac{(\Delta p_0)_{\text{stage-Euler}}}{\rho}$$

For no guide vanes assuming axial entry, $C_{w1} = 0$,

$$w = C_bC_{w2} = \Delta h_0 = \frac{(\Delta p_0)_{\text{stage-Euler}}}{\rho} \tag{12}$$

From the outlet velocity triangle,

$$C_{w2} = C_b - C_f\tan\beta_2 = C_b\left(1 - \frac{C_f}{C_b}\tan\beta_2\right) = C_b(1 - \varphi\tan\beta_2) \tag{13}$$

Substituting value of C_{w2} from Eq. (13) into Eq. (12),

$$(\Delta p_0)_{\text{stage-Euler}} = \rho C_b^2(1 - \varphi\tan\beta_2) \tag{14}$$

$$(\Delta p_0)_{\text{stage-Euler}} = 1.1231 \times 22.6195^2(1 - 0.245\tan 10)$$

$$(\Delta p_0)_{\text{stage-Euler}} = 549.8010\ Pa = \frac{549.8010}{1000 \times 9.81} \times 1000 = 56.045\ mm \text{ of water gauge} \tag{15}$$

(d) Overall Efficiency

Neglecting leakage loss and slip in the impeller, therefore, the actual air power will be equal to Euler power as calculated below,

$$P = \dot{m}w = \rho QC_{w2}C_b = \rho Q\Delta h_0 = (\Delta p_0)_{\text{stage-Euler}}\ Q \tag{16}$$

$$P = 549.8010 \times 1.1754 = 646.2361\ W = 0.64624\ kW \tag{17}$$

Assuming the drive and motor efficiencies to be 100 %, then, the overall efficiency of the fan is,

$$\eta_o = \frac{\text{Actual Air Power}}{\text{Motor Power}} \tag{18}$$

$$\eta_o = \frac{\text{Ideal Air Power}}{\text{Fan Shaft Power}} = \frac{0.64624}{1} = 0.64624 = 64.624\ \% \tag{19}$$

(e) Degree of Reaction

Refer to Eq. (6.53). Ideal static pressure rise in the stage or rotor is,

$(\Delta p)_{stage} = (\Delta p)_{rotor} = \rho \frac{C_b^2}{2}(1 - \varphi^2 tan^2\beta_2)$ (20)

$(\Delta p)_{stage} = (\Delta p)_{rotor} = 1.1231 \times \frac{22.6195^2}{2}(1 - 0.245^2 tan^2 10) = 286.7763\ Pa$ (21)

Refer to Eq. (6.31). Degree of reaction is,

$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p_0)_{\text{stage}}}$ (22)

$R = \frac{286.7763}{549.8010} = 0.5216$ (23)

The degree of reaction may also be found by Eq. (6.60) which is deduced from Eq. (6.31).

(f) Specific Speed

From earlier studies of fluid mechanics,

$(\Delta p_0)_{\text{stage}} = \rho g H$ (24)

$549.8010 = 1.1231 \times 9.81 \times H \Rightarrow H = 49.9020\ m$ (25)

Refer Eq. (2.63). Shape number or dimensionless specific speed of the fan is,

$N_{sh} = \frac{N\sqrt{Q}}{(gH)^{3/4}}$ (26)

$N_{sh} = \frac{960 \times \sqrt{1.1754}}{(9.81 \times 49.9020)^{3/4}} = 0.1667\ rev = 2\pi \times 0.1667 = 1.0474\ rad$ (27)

6.10 Calculate all the parameters if the fan of Problem 6.9 is provided with downstream guide vanes. What is the guide blade angle at inlet?

Solution

Given: $D_t = 0.6\ m, D_h = 0.3\ m, N = 960\ rpm, \beta_2 = 10°, p_{01} = p_1 = 1.02\ bar = 102\ kPa,$
$T_{01} = T_1 = 43 + 273 = 316\ K, \varphi = 0.245, P_{shaft} = 1\ kW$

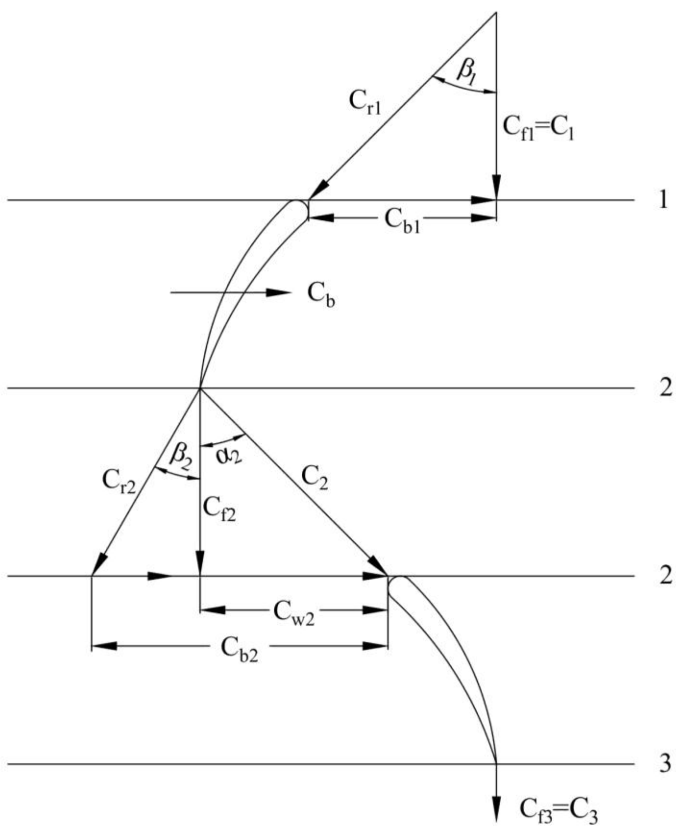

Figure P 6.10 *Velocity Triangles for an Axial Fan Stage of Problem 6.10*

The velocity triangles of a fan with downstream guide vanes is shown in Figure P 6.10. The stage pressure rise, rotor blade angles, discharge, power input, overall efficiency, degree of reaction and specific speed are the same as of Problem 6.9. The axial velocity of the flow is assumed to be constant throughout the stage. Therefore, the static pressure rise in the stage is same as that of stagnation pressure rise in the stage.

$(\Delta p)_{\text{stage}} = (\Delta p_0)_{\text{stage}} = 549.8010\ Pa = 56.045\ mm$ of water gauge (1)

$(\Delta p)_{rotor} = \rho \frac{C_b^2}{2}(1 - \varphi^2 tan^2 \beta_2)$ (2)

$(\Delta p)_{rotor} = 1.1231 \times \frac{22.6195^2}{2}(1 - 0.245^2 tan^2 10) = 286.7763\ Pa$ (3)

Refer to Eq. (6.31). Degree of reaction is,

$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p_0)_{\text{stage}}}$ (4)

$R = \frac{286.7763}{549.8010} = 0.5216$ (5)

The exit angle of the downstream guide vanes with the meridional velocity,

$\alpha_3 = 0$ (6)

From the outlet velocity triangle,

$C_{w2} = C_b - C_f \tan \beta_2 = C_b \left(1 - \frac{C_f}{C_b} \tan \beta_2\right) = C_b(1 - \varphi \tan \beta_2)$ (7)

$C_{w2} = 22.6195(1 - 0.245 \tan 10) = 21.6423\ m/s$ (8)

The inlet guide blade angle is α_2 with the meridional direction,

$\tan \alpha_2 = \frac{C_{w2}}{C_{f2}} \Rightarrow \tan \alpha_2 = \frac{21.6423}{5.5418} \Rightarrow \alpha_2 = 75.6373°$ (9)

6.11 The upstream guide blades are provided in the fan of Problem 6.9 for negative swirl. The blade angle at inlet is 86°. Calculate (a) the stage pressure rise and the static pressure rise in the rotor, (b) stage pressure coefficient, (c) degree of reaction, (d) exit air angle of the rotor blades and upstream guide blades, and (e) power input if the overall efficiency of the drive is 64.7%.

Solution

Given: $D_t = 0.6\ m, D_h = 0.3\ m, N = 960\ rpm, \beta_2 = 10°, p_{01} = p_1 = 1.02\ bar, T_{01} = T_1 = 43 + 273 = 316\ K, \varphi = 0.245, \beta_1 = 86°, \eta_o = 64.7\ \% = 0.647$

The velocity triangles of a fan with upstream guide vanes is shown in Figure P 6.11. The upstream guide vanes are used to eliminate swirl at the rotor exit. The upstream guide vanes accelerate the flow and supply the rotor with a flow having negative swirl ($-C_{w1}$). The action of the rotor cancels or removes the swirl ($C_{w2} = 0$). The axial velocity of the flow is assumed to be constant throughout the stage. Therefore,

$C_0 = C_{f0} = C_{f1} = C_{f2} = C_f = C_2$ (1)

Refer to Eq. (6.6). Specific stage work is,

$w = C_b(C_{w2} - C_{w1})$ (2)

From the velocity triangle at the inlet of rotor,

$C_{w1} + C_b = C_{f1} \tan \beta_1 \Rightarrow C_{w1} = C_{f1} \tan \beta_1 - C_b = C_f \tan \beta_1 - C_b = C_b \left(\frac{C_f}{C_b} \tan \beta_1 - 1\right)$

Referring to Eq. (2.17),

$\frac{C_f}{C_b} = \varphi.$ (3)

Therefore,

$C_{w1} = C_b(\varphi \tan \beta_1 - 1)$ (4)

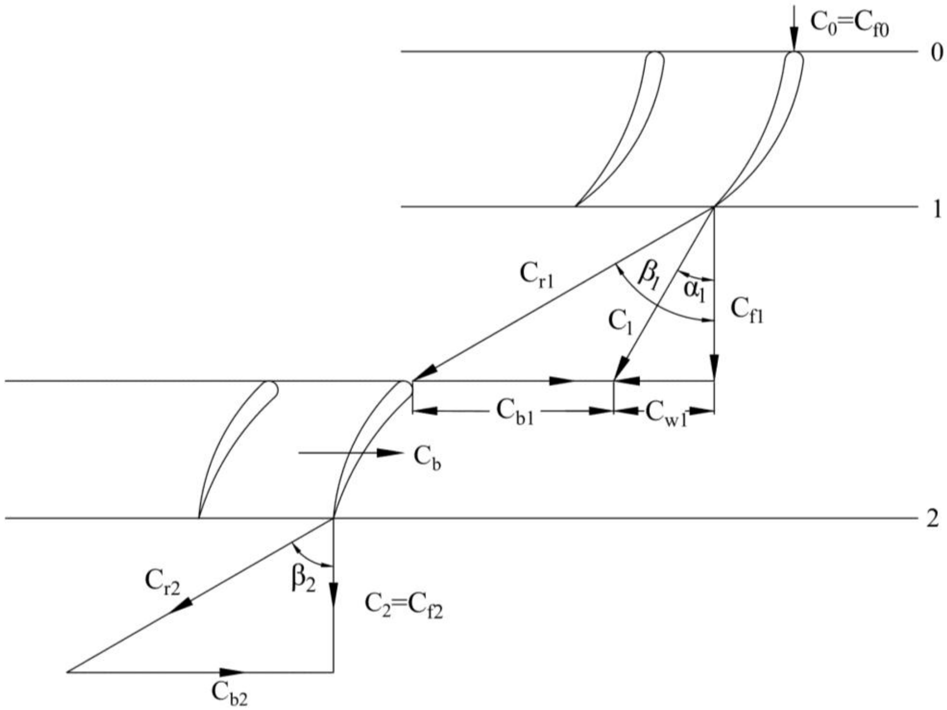

Figure P 6.11 *Velocity Triangles for an Axial Fan Stage of Problem 6.11*

Referring to Eq. (6.6), specific stage work considering there is negative swirl at inlet,

$w = C_b[C_{w2} - (-C_{w1})] = C_b(0 + C_{w1})$

$w = C_b C_{w1} = C_b^2(\varphi \tan\beta_1 - 1)$ (5)

$w = 22.6195^2(0.245 \tan 86 - 1) = 1280.9787\ J/kg$ (6)

(a) Stage Pressure Rise and Static Pressure Rise in Rotor

Referring to Eq. (1.84), ideal stage pressure change for an incompressible machine,

$(\Delta p_0)_{\text{stage-Euler}} = \rho w = \rho \Delta h_0$ (7)

$(\Delta p_0)_{\text{stage-Euler}} = 1.1231 \times 1280.9787 = 1438.6672\ Pa = 146.6531$ mm of water (8)

The velocities at the entry and exit of the stage are same $\left(C_0 = C_{f0} = C_{f1} = C_{f2} = C_f = C_2\right)$. Therefore, the ideal static pressure rise in the stage is same as that of ideal stagnation pressure rise in the stage.

$(\Delta p)_{\text{stage}} = (\Delta p_0)_{\text{stage-Euler}} = 1438.6672\ Pa = 146.6531$ mm of water (9)

(b) Stage Pressure Coefficient

Stage pressure coefficient,

$\psi = \frac{(\Delta p_0)_{\text{stage}}}{0.5\rho\, C_b^2}$ (10)

$\psi = \frac{1438.6672}{0.5 \times 1.1231 \times 22.6195^2} = 5.0073$ (11)

(c) Degree of Reaction

Referring to Section (1.6) and Eq. (1.98) keeping in mind that for an axial machine $C_{b1} = C_{b2}$, $(C_{r2}^2 - C_{r1}^2)/2$, the energy transfer due to reaction effect results in equivalent pressure increase in the rotor. Therefore,

Energy transfer due to reaction effect $= \frac{1}{2}(C_{r1}^2 - C_{r2}^2)$ (12)

From the velocity triangles in Figure 6.11,

$C_{r1}^2 = C_{f1}^2 + (C_b + C_{w1})^2 = C_f^2 + (C_b + C_{w1})^2$

$C_{r2}^2 = C_{f2}^2 + C_b^2 = C_f^2 + C_b^2$

$C_{r1}^2 - C_{r2}^2 = 2C_b C_{w1} + C_{w1}^2$ (13)

Energy transfer in rotor $= \frac{1}{2}(2C_b C_{w2} + C_{w1}^2)$ (14)

For an incompressible machine,

Energy transfer in rotor $= \Delta h = \frac{(\Delta p)_{\text{rotor}}}{\rho} = \frac{1}{2}(2C_b C_{w1} + C_{w1}^2)$ (15)

$\therefore (\Delta p)_{\text{rotor}} = (p_2 - p_1) = \frac{1}{2}\rho(2C_b C_{w1} + C_{w1}^2)$ (16)

Referring to Eq. (1.112). Degree of reaction is,

$$R = \frac{\text{Energy transfer due to reaction effect or pressure change in rotor}}{\text{Total energy transfer or stagnation pressure change of the stage}} = \frac{\frac{(\Delta p)_{\text{rotor}}}{\rho}}{w} = \frac{\Delta p_{\text{rotor}}}{\rho w} = \frac{\Delta p_{\text{rotor}}}{(\Delta p_0)_{\text{stage}}} \quad (17)$$

$$R = \frac{\frac{1}{2}\rho(2C_b C_{w1} + C_{w1}^2)}{\rho C_b C_{w1}} = 1 + \frac{1}{2}\frac{C_{w1}}{C_b} \quad (18)$$

Substituting value of C_{w1} from Eq. (4),

$$R = 1 + \frac{1}{2}\frac{C_{w1}}{C_b} = \frac{1}{2}(1 + \varphi \tan\beta_1) \quad (19)$$

$$R = \frac{1}{2}(1 + 0.245 \times \tan 86) = 2.25184 = 225.184\ \% \quad (20)$$

(d) Exit Air Angles of Rotor Blades and Upstream Guide Blades

Exit air angle of rotor blades is the relative flow angle or blade angle β_2. From the outlet velocity triangle,

$$\tan\beta_2 = \frac{C_b}{C_f} = \frac{1}{\varphi} = \frac{1}{0.245} \Rightarrow \beta_2 = 76.2337° \quad (21)$$

The upstream guide blade exit angle is the absolute air angle to the inlet of rotor. From the velocity triangle at inlet of the rotor,

$$\tan\alpha_1 = \frac{C_{w1}}{C_f} = \frac{C_b(\varphi\tan\beta_1 - 1)}{C_f} = \frac{(\varphi\tan\beta_1 - 1)}{\varphi} = \tan\beta_1 - \frac{1}{\varphi} = \tan 86 - \frac{1}{0.245}$$

$$\alpha_1 = 84.411° \quad (22)$$

(e) Power Input

From Eq. (9) of problem 6.9, discharge or volume flow rate is,

$$Q = AC_f = 0.2121 \times 5.5418 = 1.1754\ m^3/s \quad (23)$$

From studies of fluid mechanics, mass flow rate is,

$$\dot{m} = \rho Q \quad (24)$$

$$\dot{m} = 1.1231 \times 1.1754 = 1.3201\ kg/s \quad (25)$$

Neglecting leakage loss and slip in the impeller, then, actual air power is equal to the ideal power (Euler power) as given below,

$$P = \dot{m}w = \rho Q C_{w1} C_b = \rho Q \Delta h_0 = (\Delta p_0)_{\text{stage}} Q \quad (26)$$

$$P = 1438.6672 \times 1.1754 = 1691.0095\ W = 1.691\ kW \quad (27)$$

Assuming the drive and motor efficiencies to be 100 %, then, overall efficiency of the fan is,

$$\eta_o = \frac{\text{Actual Air Power}}{\text{Motor Power}} \quad (28)$$

$$\eta_o = \frac{\text{Ideal Air Power}}{\text{Fan Shaft Power}} \Rightarrow 0.647 = \frac{1.691}{P_{shaft}} \Rightarrow P_{shaft} = 2.614\ kW \quad (29)$$

6.12 The impeller and upstream guide vanes of Example 6.9 are symmetrical and degree of reaction is 0.5. The blade angles at inlet and outlet are 30° and 10° respectively. Determine (a) the stage pressure rise, (b) pressure coefficient, and (c) power required if the fan efficiency is 80 % and drive efficiency is 88 %.

Solution

Given: $D_t = 0.6\ m, D_h = 0.3\ m, N = 960\ rpm, p_{01} = p_1 = 1.02\ bar, T_{01} = T_1 = 43 + 273 = 316\ K, \varphi = 0.245, R = 0.5, \beta_1 = 30°, \beta_2 = 10°, \eta_{fan\ \text{total}} = 80\ \% = 0.8, \eta_d = 88\ \% = 0.88$

The velocity triangles for an axial fan stage with upstream guide vanes are shown in Figure P 6.12.

(a) Stage Pressure Rise

Considering the fan to be an incompressible machine, refer to Eq. (1.84) or Eq. (1.85) or Eq. (6.23), ideal specific stage work (Euler work) is,

$$w = (\Delta h_o)_{stage} = C_b(C_{w2} - C_{w1}) = \frac{(\Delta p_0)_{\text{stage-Euler}}}{\rho} = C_b^2\varphi(\tan\beta_1 - \tan\beta_2) \quad (1)$$

$$w = 22.6195^2 \times 0.245 \times (\tan 30 - \tan 10) = 50.2692\ J/kg \quad (2)$$

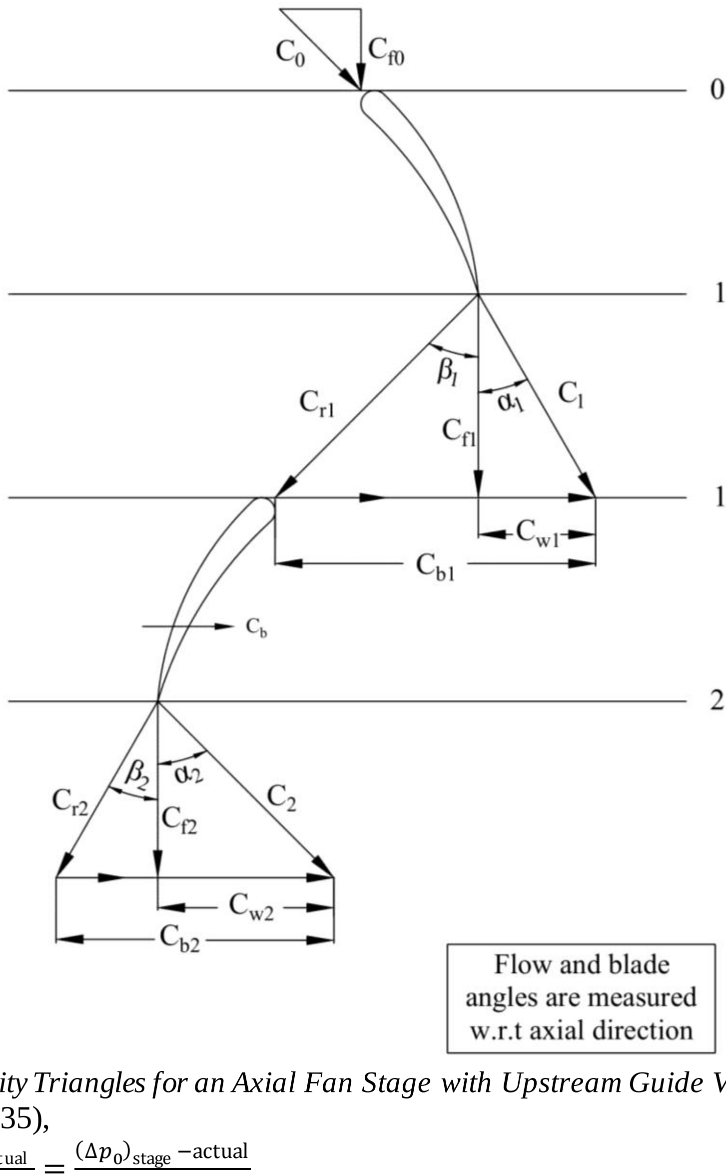

Figure P 6.12 *Velocity Triangles for an Axial Fan Stage with Upstream Guide Vanes of Problem 6.12*

Referring to Eq. (6.35),

$$\eta_{f\ \text{total}} = \frac{(\Delta p_0)_{\text{stage-actual}}}{\rho C_b (C_{w2} - C_{w1})} = \frac{(\Delta p_0)_{\text{stage}} - \text{actual}}{\rho w} \quad (3)$$

$$0.8 = \frac{(\Delta p_0)_{\text{stage-actual}}}{1.1231 \times 50.2692} \Rightarrow (\Delta p_0)_{\text{stage-actual}} = 45.1659\ Pa = 4.6041\ mm\ of\ water \quad (4)$$

From Eq. (1), ideal or Euler pressure rise in the stage is,

$$(\Delta p_0)_{\text{stage-Euler}} = \rho w = 1.1231 \times 50.2692 = 56.4573\ Pa = 5.7551\ mm\ of\ water \quad (5)$$

(b) Pressure Coefficient

Refer to Eq. (6.29). Stage pressure coefficient,

$$\psi = \frac{(\Delta p_0)_{stage}}{0.5\rho C_b^2} \quad (6)$$

Actual pressure coefficient

$$\psi_{actual} = \frac{(\Delta p_0)_{\text{stage}} - \text{actual}}{0.5\rho C_b^2} = \frac{45.1659}{0.5 \times 1.1231 \times 22.6195^2} = 0.1572 \quad (7)$$

Ideal Pressure coefficient is,

$$\psi_{ideal/Euler} = \frac{(\Delta p_0)_{\text{stage-Euler}}}{0.5\rho C_b^2} = \frac{56.4573}{0.5 \times 1.1231 \times 22.6195^2} = 0.1965 \quad (8)$$

Ideal Pressure coefficient may also be found alternately by Eq. (6.30). Eq. (6.30) is deduced by finding $(\Delta p_0)_{\text{stage-Euler}}$ by substituting the values of C_{w1} and C_{w2} from velocity triangles into Eq. (1) and then using Eq. (6.29).

$$\psi_{ideal/Euler} = 2\varphi(\tan\beta_1 - \tan\beta_2) \quad (9)$$

$$\psi_{ideal/Euler} = 2 \times 0.245 \times (\tan 30 - \tan 10) = 0.1965 \quad (10)$$

(c) Power Required

Refer to Eq. (6.37). Assuming the motor efficiency to be 100 %, overall efficiency of the fan is,

$$\eta_o = \eta_d \eta_{fan\ \text{total}} \quad (11)$$

$$\eta_o = 0.88 \times 0.80 = 0.704 \quad (12)$$

Overall efficiency of the fan drive is,

$$\eta_o = \frac{\text{Actual Air Power}}{\text{Motor Input Power}} = \frac{(\Delta p_0)_{\text{stage-actual}}\, Q}{P_{motor}} \quad (13)$$

$$0.704 = \frac{45.1659 \times 1.1754}{P_{motor}} \Rightarrow P_{motor} = 75.4091\ W = 0.0754091\ kW \quad (14)$$

Alternate Method

First Method

Fan total efficiency is,

$$\eta_{f-total} = \frac{(\Delta p_0)_{\text{stage-actual}}\, Q}{P_{shaft}} \quad (15)$$

$$0.8 = \frac{45.1659 \times 1.1754}{P_{shaft}} \Rightarrow P_{shaft} = 66.36\ W = 0.06636\ kW \quad (16)$$

$$\eta_d = \frac{\text{Fan Shaft Power}}{\text{Motor Output Power}} \quad (17)$$

$$0.88 = \frac{0.06636}{\text{Motor Output Power}} \Rightarrow \text{Motor Output Power} = 0.0754091\ kW \quad (18)$$

Assuming the motor efficiency to be 100 %, then,

$$P_{motor} = \text{Motor input power} = \text{Motor output power} = 0.0754091\ kW \quad (19)$$

Second Method

Neglecting leakage loss and slip in the impeller, therefore, fan shaft power is the Euler power to drive the fan. The drive efficiency is,

$$\eta_d = \frac{\text{Fan Shaft Power}}{\text{Motor Output Power}} \quad (20)$$

Assuming the motor efficiency to be 100 %, then,

$$\text{Motor output power} = \text{Motor input power} = P_{motor} \quad (21)$$

$$\eta_d = \frac{\text{Fan Shaft Power}}{\text{Motor Output Power}} = \frac{\text{Euler Power}}{\text{Motor Input Power}} = \frac{(\Delta p_0)_{stage-Euler} Q}{P_{motor}} = \frac{\dot{m}w}{P_{motor}}$$

$$0.88 = \frac{1.3202 \times 50.2692}{P_{motor}} \Rightarrow P_{motor} = 75.4152\ W = 0.07542\ kW \quad (22)$$

6.13 The diameter of an open air propeller fan is $0.5m$. The velocities on the upstream and downstream of the fan are $5\ m/s$ and $25\ m/s$ respectively. The overall efficiency of the fan is $40\ \%$ and ambient conditions are $1.02\ bar$ and $37\ °C$. Determine (a) mass flow rate, (b) total pressure developed by the fan, and (c) power input.

Solution

Given: $D = 0.5\ m, C_0 = 5\ m/s, C_2 = 25\ m/s, \eta_o = 40\ \% = 0.4, p_{01} = p_1 = 1.02\ bar = 102\ kPa, T_{01} = T_1 = 273 + 37 = 310\ K$

From thermodynamics, applying characteristic gas equation at inlet of the fan

$$p_1 V_1 = mRT_1 \Rightarrow p_1 = \rho_1 R T_1 \quad (1)$$

$$102 = \rho_1 \times 0.2874 \times 310 \Rightarrow \rho_1 = 1.145\ kg/m^3 \quad (2)$$

(a) Mass Flow Rate

Refer to Eq. (1.136), the velocity of flow through the propeller disc is,

$$C_1 = \frac{1}{2}(C_2 + C_0) \quad (3)$$

$$C_1 = \frac{1}{2} \times (25 + 5) = 15\ m/s \quad (4)$$

From fluid mechanics, we know that discharge or volume flow rate is given by,

$$Q_1 = AC_1 = \frac{\pi}{4} D^2 C_1 = \frac{\pi}{4} \times 0.5^2 \times 15 = 2.9452\ m^3/s \quad (5)$$

From studies of fluid mechanics, mass flow rate is,

$$\dot{m} = \rho_1 Q_1 \quad (6)$$

$$\dot{m} = 1.145 \times 2.9452 = 3.3723\ kg/s \quad (7)$$

(b) Total Pressure Developed by Fan

Referring to Eq. (1.49) taking $Z_1 = Z_2$ in conjunction with Eq. (1.141), ideal specific work input to the propeller,

$$w = \Delta h_0 = \frac{1}{2}(C_2^2 - C_0^2) \quad (8)$$

$$w = \Delta h_0 = \frac{1}{2}(25^2 - 5^2) = 300\ J/kg \quad (9)$$

Referring to Eq. (1.84), for an incompressible flow machine such as fan, ideal specific work input is,

$$w = \Delta h_0 = \frac{\Delta p_{0-Euler}}{\rho} \quad (10)$$

$$300 = \frac{\Delta p_{0-Euler}}{1.145} \Rightarrow \Delta p_{0-Euler} = 343.5\ Pa = 35.0153\ mm\ of\ water \quad (11)$$

(c) Power Input

Neglecting leakage loss and slip in the impeller, then, actual air power is equal to the ideal power as given below,

$$P = \dot{m}w = 3.3723 \times 300 = 1011.69\ W = 1.0117\ kW \quad (12)$$

Assuming the drive and motor efficiencies to be $100\ \%$, then, overall efficiency of the fan drive is,

$$\eta_o = \frac{\text{Actual Air Power}}{\text{Motor Power}} = \frac{\text{Ideal Air Power}}{\text{Fan Shaft Power}} \quad (13)$$

$$0.4 = \frac{1.0117}{P_{\text{shaft}}} \Rightarrow P_{\text{shaft}} = 2.5293\ kW \quad (14)$$
