CHAPTER 17
GOAL PROGRAMMING

Learning objectives
After completing this chapter, you should be able to
1. Identify the kinds of managerial problems that goal programming can address.
2. Describe how a goal programming model differs from other kinds of management science models.
3. Discuss the differences between the weighted goal programming approach and the preemptive goal programming approach.
4. Determine which of these approaches seems more appropriate for a given situation.
5. Formulate and apply a weighted goal programming model from a description of the problem.
6. Formulate and apply a preemptive goal programming model from a description of the problem.

In Chapters 2–8, you have seen various kinds of management science models that can address a wide variety of managerial problems. However, all these models share one common characteristic. They all have a single objective function that expresses the overall measure of performance for the problem. For example, the objective might be to maximize total profit.

Unfortunately, it is not always possible to encapsulate management’s objectives into one overall measure of performance in this way. The objectives might be so disparate that there is no common basis for measuring progress toward these objectives. In this kind of situation, management might instead set numeric goals for the various objectives and then seek a solution that makes as much progress as possible toward all these goals. This is the kind of problem that goal programming addresses.

The chapter begins with a case study that demonstrates how this kind of situation can arise. You will see three members of top management championing three very different goals for a project. The CEO agrees that all three goals are important and then mediates to assess their relative importance. You also will see how the head of the Management Science Department works with top management to elicit the kind of input needed to apply goal programming.

Goal programming provides two alternative ways of formulating problems with multiple goals. One, called weighted goal programming, assigns weights to the goals that measure their relative importance and then seeks a solution that minimizes the weighted sum of the deviations from the goals. This approach is described in Section 17.2 in the context of the case study. The second approach, called preemptive goal programming, requires deciding on the order of importance of the goals. It then focuses on one goal at a time in this order. This approach is described in Section 17.3. The dialogue for the case study in Section 17.1 also introduces both approaches, including when each one is more appropriate and how management should provide the needed input, in a practical setting.

17.1 A CASE STUDY: THE DEWRIGHT CO. GOAL-PROGRAMMING PROBLEM

“What’s the matter, honey? A rough day at the office?”

“Well, it really wasn’t all that bad,” Kathleen responds to her husband Scott. “Mainly just frustrating. Ever since I got this job as head of Dewright’s Management Science Department, I have emphasized making sure that everything we do is responding to management’s needs.
Understand what management’s objectives are for the decisions they need to make based on our studies. Then address those objectives rather than what we think the goals should be. I preach that all the time.”

“So what happened?”

“Well, we’ve just been handed an extremely important new project, a really juicy management science study. So I made the rounds today interviewing the key people in top management to clarify just what they wanted to get out of our study. What is the basic objective for the decisions they need to make? Usually this goes pretty smoothly, with a lot of consensus about what the overriding objective should be. But not today. First I was told that we should focus on such and such as the main goal of the study. Then the next person I interviewed said no, the key goal was something completely different. Then the next guy had an entirely new slant on it. I’ve never seen so much disagreement. Each one was only protecting his or her own interests instead of looking at the big picture of what is best for the company as a whole. So now we’re stymied. I’ve already selected the members of the team to work on this study. But we can’t really get started until we receive much clearer direction from management. And, of course, management needs the study completed quickly. One jokester said they would like our report the day before yesterday. I laughed politely, but I felt like kicking him. Don’t they realize that our output from the study can only be as good as their input!?! And that we can only act as quickly as they give us the direction we need!”

“Wow, no wonder you’re frustrated,” Scott responds. “It sounds like management really dropped the ball on this one.”

“Yes, they did. It was clear that they hadn’t talked to each other about this issue, even though they knew I would be interviewing all of them about this today. It’s management’s responsibility to thrash this out and come to a common understanding of what they want out of a management science study, and then give us clear direction. They really didn’t do their job this time!”

“So what’s the next step?”

“I’ve already called our CEO late this afternoon. Direction needs to come from the top. Actually, Gary was pretty sympathetic. He even volunteered that he thought his people had let me down this time.”

“So did he give you the direction you need?” Scott asks Kathleen.

“No, I really wasn’t asking for that at this point. He wasn’t involved with requesting this management science study, so I was hitting him cold. But he understood right away what had gone wrong. Even before I could suggest it, he said that the managers involved with this project should be brought together in a meeting to thrash out what the main goals should be. He even said he would chair the meeting himself. He also wants me and key members of my team there. He says it is very important that we have a clear understanding of management’s thinking on this issue. And I certainly agreed.”

“Great! So it sounds to me like all you have to do is attend the meeting and listen carefully. Let them do their homework and then come to a meeting of the minds. Press them if necessary to get the clarity you need. Then you’ll be off and running.”

**Background**

The Dewright Company is one of the largest producers of power tools in the United States. The company has had its ups and downs but has managed to maintain its position as one of the market leaders for over 20 years. This is largely due to superior products produced by a skilled and loyal work force, many of whom have been with the company for most of its existence. One of management’s priorities has been to maintain a relatively stable employment level to retain the high morale and loyalty of this work force.
The company has just gone through one of its leaner years. Sales were down slightly from the preceding year and earnings dipped as well (much to the discontent of the company’s stockholders). One consequence is that the company now has less capital available than usual with which to invest in new product development. Management also is concerned that some downsizing may be needed if sales don’t improve soon.

Fortunately, help is on the way. The company is preparing to replace its current product line with the next generation of products—specifically, three exciting new power tools with the latest state-of-the-art features, so they are expected to sell well for at least a year or two. Because of the limited amount of capital available, management needs to make some difficult choices about how much to invest in each of these products. Another concern is the effect of these decisions on the company’s ability to maintain a relatively stable employment level. A competitor is known to be developing similar new products, so decisions must be made quickly.

These kinds of considerations recently led the company’s president, Tasha Johnson, to call Kathleen Donaldson, head of the Management Science Department, to request an urgent management science study to analyze what the product mix should be. Tasha asked Kathleen to come see her for a briefing on management’s objectives in making the product-mix decisions. Tasha also suggested that Kathleen talk with Vijay Shah (vice president for manufacturing) and Hien Nguyen (the chief financial officer).

Kathleen has just completed these interviews, with the unsatisfactory results reported to her husband.

Gary Lang, the company’s CEO, now has arranged for the meeting to bring these parties together with Kathleen and key members of her team.

**The Management Science Team Meets with Top Management**

After some pleasantries, the meeting gets under way.

Gary Lang (CEO): I’ve called this meeting to clarify what we want to accomplish when we introduce these three new products. What are our main goals? I have some thoughts on that. But first I want to hear your thinking. Then we can work this out together. Once this is settled, planning can get under way to accomplish what we want accomplished. As you know, Tasha has asked Kathleen to personally head a management science team from her department to analyze how we can best meet our goals. Before the team can do that, they need some clear direction from us on just what our goals and priorities are. Kathleen, is that a fair summary of what you want to get out of this meeting?

Kathleen Donaldson (head of Management Science Department): Yes, it is, Gary. Thank you.

Gary: OK. Let’s make the rounds then and get your thinking. Tasha, let’s start with you.

Tasha Johnson (president): Well, as usual, I think we need to focus on the bottom line. If we do that, everything else will fall into place. After the year we just went through, we’ve got to get our earnings up. That’s certainly the message our board of directors has been giving us. You’ll recall the plan I presented at the last board meeting. To get our earnings headed up where we want them, we need to generate a total profit of at least $125 million from these three products until they’re replaced by the next generation of products. And I think that’s doable. I’ve already told Kathleen that I would like her team to find the mix of the three products that would maximize our profit. And to make sure that it is at least $125 million, as I promised the board of directors. I think that should be our main focus.

Gary: Thank you, Tasha. It’s certainly true that the board has been pressing us to substantially improve earnings. And they were encouraged by your plan to generate profits of at least $125 million from these products. Vijay, what is your take on this issue of where our focus should be?
Vijay Shah (vice president for manufacturing): Well, I’m certainly not going to argue against making profits. But there are different ways of accomplishing that. By and large, we’ve been a very profitable company for over 20 years. And despite our occasional off years like last year, I think we will continue to be a very successful company as long as we don’t forget what got us here. Our number one asset is our work force. They’re the best in the business and we all know it. Besides our strong leadership at the top, they’re our main reason for success. If we simply go scrambling after big profits in the short run to satisfy the board of directors for a little while, I think that’s going to mean some downsizing. That would ruin morale! And cause all kinds of disruption. I know a lot of companies have been doing it, often to their regret, but it would be a huge mistake in our case. Let’s not kill the goose that’s been laying our golden eggs. We have great morale and an exceptionally efficient work force largely because we’ve kept them together all these years. We’re going to have larger profits in the long run if we maintain a stable employment level and continue developing new products to keep them fully utilized. I told Kathleen that I thought her team should develop a plan for the current new products that would maintain our present employment level, and then profits would take care of themselves.

Tasha: But in this global economy, the companies that are surviving are those that downsize quickly when they need to in order to stay competitive.

Vijay: That would be shortsighted, especially in our situation.

Gary: Vijay, I do agree that we have a terrific work force and we should try to maintain it if possible. It is my hope that these three new products will enable us to do just that. We currently have 4,000 employees. We might even be able to increase that if everything falls into place. When you talk about the disadvantages of changing the employment level, how would you feel about an increase rather than a decrease?

Vijay: An increase wouldn’t be so bad. But it still would cause some problems, especially since the increase probably would be temporary as these products wind down. First, we would incur the expense and disruption of training these inexperienced workers. Then we would turn around and need to lay them off because we have so little attrition here. Any layoffs are not good for morale. I think we’re better off sticking pretty close to the 4,000 employees.

Gary: OK. Thanks, Vijay. Now I’m anxious to hear from Hien, especially after our financial downturn this past year.

Hien Nguyen (chief financial officer): Yes. You know well that we’re not in a good financial situation. We seldom have been as strapped for capital as we are right now.

Gary: Unfortunately, we’re going to need a lot of capital to launch these new products properly. And it is very important to the future of this company to have a good launch. I’m going to need to depend on you to work your usual magic to come up with at least the minimum amounts necessary to invest in the production facilities, marketing campaigns, and so forth, that we need for these products. How much do you think we can do?

Hien: I’ve been looking into that pretty carefully. I think we can scrape together something close to $55 million. However, I wouldn’t advise trying to go beyond that. If we get that overextended, I fear that our corporate bonds will be downgraded into the junk bond category. And then we would be paying through the nose in high interest rates for all our debt. So when Kathleen saw me recently, I advised her to stick with plans that would hold the capital investment down to no more than $55 million.

Gary: I hear you. OK, here is my conclusion so far. I think all three of you have raised very valid concerns. You each have enunciated a goal: Achieve a total profit from these products of at least $125 million, maintain the current employment level of 4,000 employees, and hold the capital investment down to no more than $55 million. These all are legitimate goals. I seriously doubt
that we can fully achieve all of them. However, rather than selecting just one of them, I think we need to try to come as close to meeting all three goals simultaneously as we can.

**Kathleen:** I have a question.

**Gary:** Shoot.

**Kathleen:** Do you see any way of combining all three goals into a single overriding objective—one objective that would encompass all three?

**Gary:** Such as?

**Kathleen:** Well, perhaps maximizing long-run profit. The problems associated with either changing the employment level or overextending our capital outlays affect our profit in the long run. Can we measure these effects on long-run profits and combine them with the direct profit from the new products?

**Gary:** Hmmm. An interesting idea. But no, I don’t think so. You’re really comparing apples and oranges. There are too many intangibles involved in the impact of missing either the second or third goal. I don’t see how you can develop any reasonable estimate of the long-run profit that would result from all this.

**Kathleen:** Yes, that was my reaction too. But yours is the one that counts. So it sounds like we should consider all three goals as separate goals, but then analyze them simultaneously.

**Gary:** Yes, I think so. Do you have a good way of doing this?

**Kathleen:** Well, I can think of two possibilities. But we need further guidance from all of you to determine the approach we should use.

**Gary:** Go ahead.

**Kathleen:** One possibility is to use a linear programming approach. You’ll recall that we’ve conducted several linear programming studies for you recently.

**Gary:** Yes.

**Kathleen:** This would involve maximizing the total profit from the new products, subject to constraints that the second and third goals are met. But this would mean requiring that the second and third goals are completely satisfied. Would that requirement seem reasonable to you?

**Tasha:** No, no, no! I think the first goal is the most important. We should make sure we meet it even if that means missing the second and third goals somewhat.

**Vijay:** But we also should permit missing the first goal somewhat to avoid missing the other goals by a large amount.

**Gary:** Well, there you have it. I agree that we shouldn’t require any of the goals to be completely satisfied if they can’t all be satisfied simultaneously.

**Kathleen:** OK, fine. So formulating a linear programming model would not be appropriate to meet your needs. But now it sounds to me like the second approach would be perfect for you.

**Gary:** What’s that?

**Kathleen:** It’s a management science technique called **goal programming**. It is designed to find the best way of striving toward several goals.

**Gary:** Yes, that sounds like just the ticket. So now you have what you need from us to start your study?

**Kathleen:** Not quite. I need to ask your indulgence for a few minutes to elicit a little more input—information we need to be able to use goal programming.
Gary: This is important. We’ll take as long as you need.

Kathleen: Thank you. What we need is your collective assessment of the relative importance of these three goals.

Gary: I would like to take a crack at that. I’ve been thinking hard about this during our discussion here. I must say that Tasha, Vijay, and Hien all have made strong cases. I think all three goals are important. However, I don’t think we have any choice but to put top priority on achieving our profit goal. That is the engine that drives everything else. And our board of directors has made it very clear that this needs to be our top priority. But I also resonate with what Vijay had to say about our work force being our number one asset. However, I would divide his goal of maintaining a stable employment level into two parts—avoiding a decrease in the employment level and avoiding an increase in the employment level. I think the negative impact of laying off some of our loyal long-time employees would be much more serious than that of hiring new people and perhaps laying them off in a year or two. Therefore, I would place a pretty strong second priority on avoiding layoffs, but not on avoiding new hiring. Then sorry, Hien, but I think we can only give third priority to the goal of holding our capital investment under $55 million. We mustn’t ignore your very legitimate concerns. However, we are in a hole that we need to dig out of, even if that means stretching our finances more than we normally would be willing to do. Then finally, I would put fourth priority on the second part of Vijay’s goal—avoiding an increase in our employment level since it might need to be temporary. What do the rest of you think? Does this seem reasonable?

Tasha: Definitely.

Vijay: I can live with it.

Hien: You’re the one that needs to set priorities. But we do need to be cautious about getting overextended financially.

Gary: I hear you. OK, Kathleen, what additional input do you need from us?

Kathleen: I need your advice on the following issue. Goal programming is a flexible technique that provides two different approaches to analyzing managerial problems. Which approach is appropriate for any particular problem depends on management’s assessment of how big the differences are in the importance of the goals. Are all the goals quite important with only modest differences in their importance? Or are there really big differences in their importance? In the first case, the approach is to literally consider all the goals simultaneously while recognizing the modest differences in their importance. We call this approach weighted goal programming because it places weights on the goals to reflect their relative importance. In the second case, because of the really big differences in the importance of the goals, the approach is to begin by focusing solely on the most important goal and going as far as possible toward achieving that goal. Then it turns to the second most important goal, and then to the third one, and so forth. This approach is called preemptive goal programming because its focus on each goal in turn is preempting any consideration yet of less important goals. My group would be comfortable with using either approach. However, the decision on which one to use really needs to depend on your input. How big do you think the differences are in the importance of the goals? Which approach seems more appropriate for this situation?

Gary: I could see us going in either direction. It seems to me that there are clear differences in the importance of the goals. However, they all are quite important. I wouldn’t say that there are very big differences in their importance. So my inclination would be to go with that first approach you mentioned.

Kathleen: Yes, the weighted goal programming approach. Since you say you could see us going in either direction, this apparently is not a clear-cut situation. Therefore, I think what we’ll do is
begin with the weighted goal programming approach, but then double-check our conclusions by applying the preemptive goal programming approach as well. We’ll include all our results in our report and then the four of you can put your heads together to make the decision on what the product mix should be for the three new products.

**Gary:** I like it. Full steam ahead. Are you all set now? Does this give you everything you need?

**Kathleen:** Nearly. This has been extremely helpful. However, we still need your input on one more thing in order to implement the weighted preemptive programming approach. Let me explain a little more about how this approach works. It assigns penalties to not achieving goals. The more you miss a goal, the larger the penalty. The top priority goals get the largest penalties for missing them and the lowest priority goals get the smallest penalties. Then goal programming finds the set of decisions—in this case the production rates for the three products—that minimizes the total number of penalty points incurred by missing goals.

**Gary:** Sounds like a good approach.

**Kathleen:** Yes. But what this means is that we need to assign penalty weights that measure the relative seriousness of missing the respective goals. Now we could try to assign the penalty weights based on the discussion here and the priorities you have set. But that really isn’t our place. These penalty weights need to reflect your assessment, not ours, of the relative seriousness of missing these goals.

**Gary:** I agree. How do we go about that?

**Kathleen:** Well, the first step is that we assign any old number as the penalty weight for missing one of the goals, just to establish a standard of comparison. Then you would scale this penalty weight up or down for each of the other goals, depending on whether you think the seriousness of missing that goal is larger or smaller than for missing the first goal. OK, since our top priority is the goal of achieving a total profit from these new products of at least $125 million, let’s assign a penalty weight of 5 for each $1 million you undershoot this goal. In other words, if the estimated total profit resulting from the selected profit mix is $124 million, 5 penalty points would be assessed. If the estimated total profit is $115 million, undershooting the goal by $10 million, then 50 penalty points would be assessed. Ten times five is 50.

**Gary:** I get it.

**Kathleen:** OK. With this penalty weight of 5 as a standard of comparison, now we’re ready for the hard questions. Going down your priorities, what should the penalty weight be for each 100 employees we undershoot the goal of maintaining the current employment level at 4,000 employees? For each $1 million we miss the goal of holding the capital investment down to no more than $55 million? For each 100 employees we overshoot the goal of sticking with 4,000 employees?

**Gary:** Hmm. Good questions. Hmm. Well, I think I would go 5, 4, 3, 2. Five for the first goal, and then 4, 3, 2 for your three questions.

**Kathleen:** Great. Understood. That gives us exactly what we need. We can launch into our study immediately now.

**Gary:** Very good. You understand that we’ll need your report quite soon.

**Kathleen:** Yes, I think we can finish in a month. I’ll tell you what our biggest job is going to be. Gathering data. We’re going to need to get good data on the effect of each product’s production rate toward meeting each of the three goals. How much profit will each product generate? How much employment level? How much capital investment is needed? We’ll have to get a lot of help from various staff people.

**Gary:** I’ll see to it that everybody makes this their top priority.
Kathleen: Then we can do it. I don’t think any of us will see much of our families for the next month, but I’ll make sure that we get it done in time.

Gary: Good for you, Kathleen. Thank you so much. And let us know whenever you need more input or help from any of us.

Kathleen: I will. Thank you.

The meeting concludes, except for a somewhat heated private conversation between Tasha and Vijay.

**The Conclusions from This Meeting**

To summarize, here are the key conclusions from this meeting.

The management science team led by Kathleen Donaldson will conduct a study to be completed within the next month. The study will focus on determining the mix of the company’s three new products that would best meet management’s goals. The specific decisions to be made are the production rates for the three products.

In addressing these decisions, management wants primary consideration given to three factors: total profit, stability in the work force, and the level of capital investment needed to launch these products. In particular, management has established the following goals.

**Goal 1**: Achieve a total profit (net present value) from these products of at least $125 million.

**Goal 2**: Maintain the current employment level of 4,000 employees.

**Goal 3**: Hold the capital investment down to no more than $55 million.

However, management realizes that it probably will not be possible to attain all these goals simultaneously, and so they evaluated the relative importance of the goals. All are important, but by small margins their order of importance is

**Order of Importance**: Goal 1, part of Goal 2 (avoid decreasing the employment level), Goal 3, and the other part of Goal 2 (avoid increasing the employment level).

To further quantify this ordering, **penalty weights** were assigned to indicate the relative seriousness of missing these goals. Discussions between management and Kathleen led to the choice of the penalty weights shown in Table 17.1.
Table 17.1  Penalty Weights That Measure the Relative Seriousness of Missing the Goals for the Dewright Co. Problem

<table>
<thead>
<tr>
<th>Goal</th>
<th>Factor</th>
<th>Penalty Weight for Missing Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Total profit</td>
<td>5 (per $1 million under the goal)</td>
</tr>
<tr>
<td>2</td>
<td>Employment level</td>
<td>4 (per 100 employees under the goal)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 (per 100 employees over the goal)</td>
</tr>
<tr>
<td>3</td>
<td>Capital investment</td>
<td>3 (per $1 million over the goal)</td>
</tr>
</tbody>
</table>

Relevant Data
What the total profit, employment level, and capital investment level will be depends on the production rates (number of units produced per day) of the three products. Each product’s contribution to each of these three quantities is proportional to the rate of production of the product. Therefore, for each of the three products, the management science team focuses on estimating the contribution to each of these quantities per unit rate of production of the product. Much of these data are not readily available, so the team has to do considerable digging with much help from knowledgeable staff. Based on the limited information it can uncover, the team then makes the best estimates that it can of each product’s contribution to each of the three quantities.

These estimated contributions per unit rate of production are shown in Table 17.2, where the contributions are in the units indicated (in parentheses) in the first column. Thus, for example, producing one unit per day of product 1 would contribute $12 million toward total profit, 500 employees to the employment level, and $5 million to the capital investment level.

Table 17.2  Contributions to the Goals per Unit Rate of Production of Each Product for the Dewright Co. Problem

<table>
<thead>
<tr>
<th>Factor</th>
<th>Unit Contribution of Product</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Total profit (millions of dollars)</td>
<td>12</td>
</tr>
<tr>
<td>Employment level (hundreds of employees)</td>
<td>5</td>
</tr>
<tr>
<td>Capital investment (millions of dollars)</td>
<td>5</td>
</tr>
</tbody>
</table>

The story of how the team uses these data to complete its study continues in the next section, after we introduce the general subject of goal programming.

REVIEW QUESTIONS
1. What is the problem that Dewright’s management science team has been asked to address?
2. What are the three goals that management has established for addressing this problem?
3. What is to be minimized when using a weighted goal-programming approach?
17.2 WEIGHTED GOAL PROGRAMMING

One common characteristic of all the different kinds of management science models introduced in Chapters 2–8 (including linear programming, integer programming, and nonlinear programming) is that they have a single objective function. This implies that all the managerial objectives for the problem being studied can be encompassed within a single overriding objective, such as maximizing total profit or minimizing total cost. However, this is not always possible, as you have just seen in the Dewright case study.

When managing a for-profit organization, the managerial objectives might well include some of the following:

1. Maintain stable profits.
2. Increase market share.
3. Diversify the product line.
5. Improve worker morale.
7. Increase company prestige.

These objectives are so different in nature that it really is not realistic to combine them into a single overriding objective. Instead, analysis of the problem of concern requires individual consideration of the separate objectives.

Weighted goal programming provides a way of striving toward several such objectives simultaneously. The basic approach is to establish a specific numeric goal for each of the objectives and then to seek a solution that balances how close this solution comes to each of these goals. Penalty weights are assigned to the objectives to measure the relative seriousness of missing their numeric goals. An objective function is formulated for each of the objectives. The overall objective is to minimize the weighted sum of deviations of these objective functions from their respective goals. Assuming that all the individual objective functions and the constraints of the problem fit the format for linear programming, the overall problem then can be formulated as a linear programming problem.

Now let us see how Dewright’s management science team does this for the case study introduced in the preceding section.

Formulation of a Weighted Goal-Programming Model for the Dewright Co. Problem

The decisions that need to be made for the Dewright Co. problem are the production rates for the three new products that will be introduced soon. Therefore, the decision variables are

\[
P_1 = \text{Number of units of product 1 to produce per day}
\]

\[
P_2 = \text{Number of units of product 2 to produce per day}
\]

\[
P_3 = \text{Number of units of product 3 to produce per day}
\]

Using the unit contributions given in Table 17.2, the three goals can be expressed in terms of these decision variables as

- **Goal 1:** \[12P_1 + 9P_2 + 15P_3 \geq 125\] (Total profit goal)
- **Goal 2:** \[5P_1 + 3P_2 + 4P_3 = 40\] (Employment level goal)
- **Goal 3:** \[5P_1 + 7P_2 + 8P_3 \leq 55\] (Capital investment goal)
These mathematical expressions for the goals look like linear programming constraints. However, they cannot be used as constraints in a mathematical model because constraints definitely must be satisfied whereas Dewright management already has concluded that it probably will not be possible to attain all these goals simultaneously. For a goal-programming model, the overall objective instead is to come as close as possible to satisfying all these goals simultaneously.

More precisely, using the penalty weights given in Table 17.1, let

\[ W = \text{Weighted sum of deviations from the goals} \]
\[ = \text{Number of penalty points incurred by missing the goals} \]

For each goal that is missed, the number of penalty points incurred is the penalty weight \textit{times} the deviation from the goal. Therefore, the overall objective then is to choose the values of \( P_1, P_2, \) and \( P_3 \) so as to

\[
\text{Minimize } \quad W = 5 \text{ (amount under goal 1)} + 2 \text{ (amount over goal 2)} + 4 \text{ (amount under goal 2)} + 3 \text{ (amount over goal 3)}
\]

where no penalty points are incurred for being over goal 1 (exceeding the target for total profit is fine) or for being under goal 3 (underexpending the capital investment budget is satisfactory).

Figure 17.1 shows one way of formulating the spreadsheet model for this problem. Three of the changing cells UnitsProduced (C12:E12) display the values of the decision variables \( P_1, P_2, \) and \( P_3 \). Given these values, the equations entered into LevelAchieved (F6:F8) provide the levels achieved toward meeting the goals expressed in columns B, G, and H.

The most subtle part of this formulation involves columns, J, K, M, and O. Deviations (J6:K8) are additional changing cells that display the decisions on the amounts over and amounts under the respective goals. WeightedSumOfDeviations (M13) is the objective cell giving the value of \( W \), where the Solver Parameters box specifies that the objective is to minimize this value. Using the expression for \( W \) given earlier, the equation entered into this objective cell is the \texttt{SUMPRODUCT} of the data cells PenaltyWeights (J13:K15) and the changing cells Deviations (J6:K8).

The Solver options selected at the bottom of Figure 17.1 specify that this model now has been formulated in a way that fits linear programming (which enables solving the model) and that all the changing cells need to be nonnegative. There are no constraints involving the output cells LevelAchieved (F6:F8) since these cell values are not required to fully satisfy the goals specified in columns B, G, and H. However, the output cells Balance (M6:M8) do need to satisfy the constraints specified by the Solver Parameters box, Balance (M6:M8) = Goal (H6:H8).
Figure 17.1 A spreadsheet model for the Dewright Co. weighted goal-programming problem formulated as a linear programming problem, where the changing cells UnitsProduced (C12:E12) show the optimal production rates and the changing cells Deviations (J6:K8) show the optimal amounts over and under the goals. The objective cell WeightedSumOfDeviations (M13) gives the resulting weighted sum of deviations from the goals.

These constraints play the key role of ensuring that the changing cells Deviations (J6:K8) will equal the amounts by which LevelAchieved (F6:F8) deviate from the goals. To see why, note that the equations entered into Balance (M6:M8) are

\[
\text{Balance} = \text{LevelAchieved} - \text{AmountOver} + \text{AmountUnder}
\]

Therefore, the constraints that Balance (M6:M8) = Goal (H6:H8) require that

\[
\text{Goal} = \text{LevelAchieved} - \text{AmountOver} + \text{AmountUnder}
\]

To minimize the objective cell, the smallest nonnegative values of AmountOver (J6:J8) and AmountUnder (K6:K8) that satisfy these equations will need to be chosen. Consequently, in any

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dewright Co. Goal Programming (Weighted)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Contribution per Unit Produced</th>
<th>Level</th>
<th>Amount</th>
<th>Amount</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Goal 1 (Profit)</td>
<td>12</td>
<td>9</td>
<td>15</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>125</td>
<td>0</td>
<td>0</td>
<td>125</td>
</tr>
<tr>
<td>7</td>
<td>Goal 2 (Employment)</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>18.3333</td>
</tr>
<tr>
<td></td>
<td></td>
<td>333333</td>
<td>0</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>Goal 3 (Investment)</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>55</td>
<td>0</td>
<td>0</td>
<td>55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Penalty</th>
<th>Over</th>
<th>Under</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Product 1</td>
<td>5</td>
<td>0</td>
<td>J13:K15</td>
</tr>
<tr>
<td></td>
<td>Product 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Product 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Units Produced</td>
<td>8.3333</td>
<td>0</td>
<td>16.6667</td>
</tr>
<tr>
<td>13</td>
<td>PenaltyWeights</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>Employment</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>Investment</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Solver Parameters

Set Objective Cell: WeightedSumOfDeviations

To: Min

By Changing Variable Cells:

UnitsProduced, Deviations

Subject to the Constraints:

Balance = Goal

Solver Options:

Make Variables Nonnegative

Solving Method: Simplex LP
row where LevelAchieved deviates from Goal, either AmountOver or AmountUnder will equal this deviation (which one depends on whether it is a positive or negative deviation) and the other one will have a value of 0.

Running Solver yields the optimal solution for all the changing cells shown in Figure 17.1. Since the units being used in the spreadsheet are those given in the first column of Table 17.2, this solution thereby provides the following:

- Production rate for product 1 = 8 1/3 units per day
- Production rate for product 2 = 0
- Production rate for product 3 = 1 2/3 units per day
- Total profit = $125 million
- Employment level = 4,833 employees
- Capital investment = $55 million

The only deviation from management’s goals is the one considered least serious (exceeding the employment level goal of 4,000 employees).

**REVIEW QUESTIONS**

1. What is the one common characteristic of the management science models introduced in previous chapters that is not possessed by goal-programming problems?
2. What is the basic approach of goal programming?
3. What is represented by the objective function in a goal-programming model?
4. What is shown by the changing cells (other than those displaying the values of the decision variables for the original problem) in the spreadsheet model for a goal-programming problem?
5. To enable solving a goal-programming problem, it can be formulated to fit what kind of spreadsheet model?

**17.3 PREEMPTIVE GOAL PROGRAMMING**

Goal programming provides two distinct approaches for dealing with managerial problems where multiple goals need to be considered. The approach described in the preceding section, weighted goal programming, is designed for problems where all the goals are quite important with only modest differences in importance that can be measured by assigning weights to these goals. We now turn to the other approach, preemptive goal programming.

This second approach is used when there are such major differences in the importance of the goals that it is not feasible to assign meaningful weights to these goals to measure their relative importance. Therefore, the goals are instead listed in the order of their importance. Preemptive goal programming then begins by focusing solely on meeting the most important goal as closely as possible, and next doing the same for the second most important goal, and so on through the rest of the goals in order. Thus, while this approach is focusing on one of the goals, it is preempting any consideration yet of less important goals.

One advantage of this approach is that it is less difficult for management to assess the order of importance of its goals than to assign weights to the goals to measure their relative importance. It is not easy to get a handle on what the penalty weights should be for deviations from the goals when these goals are as disparate as those for the Dewright Co. problem. Assessing the order of importance is a much more concrete task. Another advantage is that the process of focusing on one goal at a time in the order of its importance is intuitive and readily
understood. Management has more confidence in the output of a model when it has confidence in both the validity of its input and the process involved in obtaining the output.

There are situations where the features of both approaches perhaps should be combined to analyze the problem. This occurs when the goals can be divided into groups where the goals within each group are of comparable importance but there are great differences between the groups in their level of importance. In this kind of situation, weighted goal programming can be used within each group in turn while preemptive goal programming is being applied to deal with each group in order of importance. However, we will not delve further into this more advanced subject.

The spreadsheet models employed by weighted goal programming and preemptive goal programming for the same problem are quite similar. The major difference is in their objective cells. Rather than using the weighted sum of deviations from the goals as the objective cell to be minimized, preemptive goal programming begins by minimizing only the deviation from the most important goal. When this is completed, the second step is to add a constraint that the minimal deviation achieved must continue to be met while switching to minimizing the deviation from the second most important goal. Next, a second constraint is added that the minimal deviation achieved for this second most important goal must continue to be met while switching to minimizing the deviation from the third most important goal. This process continues until all the goals have been considered.

Now let us continue the Dewright Co. case study to see how this approach is applied in that context.

**The Application of Preemptive Goal Programming to the Dewright Co. Problem**

Dewright’s Management Science Department already has applied a weighted goal-programming model to the problem described in Section 17.1 and obtained the results shown in Figure 17.1. However, recall that when the head of this department (Kathleen Donaldson) and key members of her team met with top management to discuss this problem, it was agreed that the preemptive goal-programming approach would be applied as well.

During this meeting, the company’s CEO (Gary Lang) had clearly identified the order of importance of the goals, as summarized below.

**Priority 1:** Strive to achieve a total profit (net present value) from the three new products of at least $125 million (Goal 1).

**Priority 2:** Strive to avoid decreasing the employment level below 4,000 employees (the under part of Goal 2).

**Priority 3:** Strive to hold the capital investment down to no more than $55 million (Goal 3).

**Priority 4:** Strive to avoid increasing the employment level above 4,000 employees (the over part of Goal 2).

Thus, this is the order in which Kathleen and her team will address the four goals.

Figure 17.2 shows the spreadsheet model for focusing on the Priority 1 goal. This model is identical to the weighted goal-programming model in Figure 17.1 except for deleting Penalty-Weights (J13:K15) and changing the objective cell. Because the objective at this point is to minimize the amount under Goal 1 (a profit of at least $125 million), the objective cell now is Under-Goal1 (K6). This again is a linear programming model and so is easily solved.

The changing cells in Figure 17.2 show the solution obtained after running Solver. This actually is only one of numerous optimal solutions that will yield a value of 0 for the objective cell, so the next steps will focus on identifying which of these solutions will do the best job in striving toward the lower priority goals as well.
The second step is to focus on the Priority 2 goal of minimizing UnderGoal2 (K7). However, this cell already has a value of 0 in Figure 17.2, so this goal already is fully achieved along with the Priority 1 goal by the solution shown in this figure (and many other solutions).

Thus, we can immediately go on to the third step of focusing on the Priority 3 goal (do not exceed a capital investment of $55 million). Figure 17.3 shows the revised spreadsheet model for this step. The objective cell now is OverGoal3 (J8). The other key revision is the addition of the constraints, UnderGoal1 (K6) = 0 and UnderGoal2 (K7) = 0, in the Solver Parameters box (and also displayed in cells J11:K12). The changing cells in this figure show the optimal solution obtained after running Solver. Once again, a value of 0 has been obtained in the new objective cell.

The fourth and final goal is to focus on the Priority 4 goal of minimizing OverGoal2 (J7), as shown in Figure 17.4. The constraint, OverGoal3 (J8) = 0, now has been added to the constraints that were introduced at the preceding step, UnderGoal1 (K6) = 0 and UnderGoal2 (K7) = 0. Running Solver provides the optimal solution displayed in the changing cells along with a value of 8.3333 in the objective cell.

Note that this solution is the same as the one in Figure 17.3. Thus, the revised model in Figure 17.4 did not succeed in making any more progress in striving toward the Priority 4 goal. However, this solution has fully achieved the top three priority goals, so this is an excellent outcome.

Also note that the solution in Figures 17.3 and 17.4 is identical to the one in Figure 17.1 that was obtained by using weighted goal programming. This provides additional assurance that this is the solution that best meets management’s needs.
A spreadsheet model formulated as a linear programming model for the first step of the Dewright Co. preemptive goal-programming problem. Since Priority 1 is to minimize the deviation under Goal 1, the objective cell is UnderGoal1 (K6) for this step. The changing cells UnitsProduced (C12:E12) show the resulting production rates and the other changing cells Deviations (J6:K8) show the resulting amounts over and under the goals after running Solver. Since Priority 2 is to minimize UnderGoal2 (K7), which already has a value of 0, the procedure next will bypass step 2 and go directly to step 3.
Figure 17.3 The revision of the spreadsheet model in Figure 17.2 needed to perform step 3 of the preemptive goal-programming procedure. Since Priority 3 is to minimize the deviation over Goal 3, the objective cell is OverGoal3 (J8) for this step. Constraints that UnderGoal1 (K6) = 0 and UnderGoal (K7) = 0 also have been added to the model. The changing cells show the results after running Solver.
The revision of the spreadsheet model in Figure 17.3 needed to perform step 4 of the preemptive goal-programming procedure. Since Priority 4 is to minimize the deviation over Goal 2, the objective cell is OverGoal2 (J7) for this step. One more constraint, OverGoal3 (J8) = 0, has been added to the model. Since this is the final step, the changing cells show the optimal solution obtained for Dewright’s preemptive goal-programming problem by running Solver.

Epilogue to the Dewright Co. Case Study

Exactly one month after their first meeting, Gary Lang called the same group together again to hear the report of the management science team. The night before, Kathleen Donaldson had made sure that all the parties received the team’s written report via courier service. Based on both the above analysis and that in the preceding section, the report recommended that the company focus most of its efforts on producing and marketing large quantities of Product 1 (P1 = 8 1/3), while providing some diversification with a much smaller output of Product 3 (P3 = 1 2/3). Another recommendation was that any production of Product 2 be postponed indefinitely (P2 = 0), but that further development work be done on this product to see if it could be made sufficiently attractive for release with the next generation of products. The report then highlighted the fact that this plan would enable meeting all of management’s more important goals.
Everybody had read the written report with this wonderful news before entering the meeting. This completely changed the mood from the usual one that Kathleen encountered when presenting an oral report and recommendations to Dewright management. Gone were the usual probing and skeptical questioning of the presentation. (In the privacy of her home with her husband Scott, Kathleen referred to these sessions as her inquisitions.) Also missing was the zealous guarding of territory by some Dewright managers that Kathleen had observed in the past. (Kathleen marveled to Scott afterward that she actually spotted Tasha and Vijay smiling at each other for the first time in months.) Vijay did suggest, with nods all around, that some of the new employees be brought in as “temps” (temporary workers) and that the development of the next generation of new products be accelerated a little to try to avoid any future layoffs of permanent employees. Otherwise, the presentation was virtually uninterrupted. Following a quick pro forma vote to approve the plan recommended by the management science team, Gary had champagne brought in. He then offered a toast to the very fine work done by Kathleen and her team.

Thus began a very good year for the Dewright Company. However, some very rocky times—and managerial changes—awaited the company further down the road. Shortly before the downturn, Kathleen left Dewright to head up her own management science consulting firm. Her firm is doing very well.

### REVIEW QUESTIONS
1. When should preemptive goal programming be used instead of weighted goal programming?
2. How does the preemptive goal-programming approach differ from the weighted goal programming approach?
3. What is the major difference between the spreadsheet models employed by weighted goal programming and preemptive goal programming for the same problem?
4. After considering the first goal, what additional constraint needs to be added during each step of the preemptive goal-programming approach when attention is turned to the next goal?
5. How many of the top-priority goals for the Dewright Co. problem did preemptive goal programming succeed in fully achieving?

### 17.4 SUMMARY

Most management science models make the basic assumption that a single objective function is available that encompasses the overriding objective of management for the problem. However, management sometimes will instead have a variety of rather different objectives that require separate consideration. As illustrated by the Dewright Co. case study, goal programming provides some ways of striving toward several such objectives simultaneously.

One basic approach, called weighted goal programming, is to establish a specific numeric goal for each of the objectives and then to seek a solution that balances how close it comes to each of these goals. By introducing some new variables (changing cells) that represent the amounts over or under the respective goals, this approach leads to formulating a model where the objective is to minimize the weighted sum of the deviations from the goals.

The other basic approach, called preemptive goal programming, begins by listing the goals in the order of their importance. It then focuses on one goal at a time in this order. While focusing on a particular goal, the model uses the objective of minimizing the deviation from that goal. The model also includes constraints that require that there be no reduction in the progress toward the goals previously considered.

With either approach, the current model often can be formulated to be a linear programming model, in which case it can be solved very readily. Thus, goal programming often
provides a practical way of striving toward various managerial goals simultaneously while giving higher priority to the more important goals.

**Glossary**

**goal programming** A technique designed to find the best way of striving toward several goals. (Section 17.1)

**penalty weights** Values assigned to the goals of a weighted goal-programming problem that measure the relative seriousness of missing these goals. (Sections 17.1 and 17.2)

**preemptive goal programming** A type of goal programming that focuses on one goal at a time in order of importance while preempting any consideration yet of less important goals. (Sections 17.1 and 17.3)

**weighted goal programming** A type of goal programming that assigns penalty weights to the various goals and then seeks a solution that minimizes the weighted sum of the deviations from the goals. (Sections 17.1 and 17.2)

**Learning Aids for This Chapter in Your MS Courseware**

Chapter 17 Excel Files:

- Dewright, Weighted Goal Programming
- Dewright Preemptive Goal Programming (three spreadsheets)

An Excel Add-in:

- Risk Solver Platform for Education

**Problems**

To the left of each of the following problems (or their parts), we have inserted an E* whenever Excel should be used (unless your instructor gives you contrary instructions). An asterisk on the problem number indicates that at least a partial answer is given at the end of the problems.

17.1.* One of management’s goals in a goal-programming problem is to maintain the company’s employment level next year at its current level of 60 full-time equivalents (60 FTEs). Each FTE under this goal is considered three times as serious as each FTE over the goal. Suppose that the *amount over* appears in cell K7 of the spreadsheet model and the *amount under* appears in cell L7. (Both cells are changing cells.) What is the relationship between the coefficients of K7 and L7 in the equation entered into the objective cell?
17.2. Management of the Albert Franko Co. has established goals for the market share it wants each of the company’s two new products to capture in their respective markets. Specifically, management wants product 1 to capture at least 15 percent of its market and product 2 to capture at least 10 percent of its market. Three advertising campaigns are being planned to try to achieve these market shares. One is targeted directly on the first product. The second targets the second product. The third is intended to enhance the general reputation of the company and its products. Letting $x_1$, $x_2$, and $x_3$ be the amount of money allocated (in millions of dollars) to these respective campaigns, the resulting market share (expressed as a percentage) for the two products are estimated to be

\[
\text{Market share for product 1} = 0.5x_1 + 0.2x_3 \\
\text{Market share for product 2} = 0.3x_2 + 0.2x_3
\]

A total of $55 million is available for the three advertising campaigns, but management wants at least $10 million devoted to the third campaign. If both market share goals cannot be achieved, management considers each 1 percent decrease in the market share from the goal to be equally serious for the two products. In this light, management wants to know how to most effectively allocate the available money to the three campaigns.

a. Describe why this problem is a weighted goal-programming problem by giving quantitative expressions for the goals and the overall objective.

E* b. Formulate and solve this problem as a linear programming model on a spreadsheet.

c. Interpret this solution to management in its language.
17.3.* The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the work force, and achieving an increase in the company’s earnings next year from the $75 million achieved this year. In particular, using the units given in the following table, they want to

Maximize \[ M = P - 6C - 3D \]

where

- \( M \) = Overall measure of performance combining the three factors
- \( P \) = Total (discounted) profit over the life of the new products
- \( C \) = Change (in either direction) in the current level of employment
- \( D \) = Decrease (if any) in next year’s earnings from the current year’s level

The amount of any increase in earnings does not enter into \( M \), because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Goal</th>
<th>(Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total profit</td>
<td>20</td>
<td>15</td>
<td>25</td>
<td>Maximize</td>
<td>(millions of dollars)</td>
</tr>
<tr>
<td>Employment level</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>50</td>
<td>(hundreds of employees)</td>
</tr>
<tr>
<td>Earnings next year</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>( \geq 75 )</td>
<td>(millions of dollars)</td>
</tr>
</tbody>
</table>

E* a. Formulate and solve a spreadsheet model for this problem.

b. Interpret this solution to management in its language.

17.4. Reconsider the Dewright Co. case study as presented in Sections 17.1 and 17.2. After further reflection about the optimal solution obtained by using weighted goal programming, management now is asking some what-if questions.

a. Gary Lang wonders what would happen if the penalty weights in the rightmost column of Table 17.1 were to be changed to 7, 4, 1, and 3, respectively. Would you expect the optimal solution to change? Why?

E* b. Tasha Johnson is wondering what would happen if the total profit goal were to be increased to wanting at least $140 million (without any change in the original penalty weights). Solve the revised model with this change.

E* c. Solve the revised model if both Gary’s and Tasha’s changes are made.
17.5. Montega is a developing country that has 15,000,000 acres of publicly controlled agricultural land in active use. Its government currently is planning a way to divide this land among three basic crops (labeled 1, 2, and 3) next year. A certain percentage of each of these crops is exported to obtain badly needed foreign capital (dollars), and the rest of each of these crops is used to feed the populace. Raising these crops also provides employment for a significant proportion of the population. Therefore, the main factors to be considered in allocating the land to these crops are (1) the amount of foreign capital generated, (2) the number of citizens fed, and (3) the number of citizens employed in raising these crops. The following table shows how much each 1,000 acres of each crop contributes toward these factors, and the last column gives the goal established by the government for each of these factors.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Contribution per 1,000 Acres of Crop</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign capital</td>
<td>$3,000</td>
<td>$70 million</td>
</tr>
<tr>
<td>Citizens fed</td>
<td>$5,000</td>
<td>1,750,000</td>
</tr>
<tr>
<td>Citizens employed</td>
<td>$4,000</td>
<td>200,000</td>
</tr>
</tbody>
</table>

In evaluating the relative seriousness of not achieving these goals, the government has concluded that the following deviations from the goals should be considered *equally undesirable*:

1. Each $100 under the foreign-capital goal,
2. Each person under the citizens-fed goal, and
3. Each deviation of one (in either direction) from the citizens-employed goal.

a. Describe why this problem is a weighted goal-programming problem by giving quantitative expressions for the goals and the overall objective.

b. Formulate and solve this problem as a linear programming model on a spreadsheet.

c. Interpret this solution to management in its language.

E* 17.6. Reconsider the scenario described in Problem 17.5. The unemployment rate in Montega is rising and the shortage of foreign capital is becoming a more serious problem. Therefore, the Montega government now has decided that it needs to place higher priority on increasing employment and increasing foreign capital than on its other goals. Specifically, it has established the following order of priorities for its goals.

- **Priority 1:** Citizens employed \( \geq 200,000 \)
- **Priority 2:** Foreign capital \( \geq $70 million \)
- **Priority 3:** Citizens fed \( \geq 1,750,000 \)
- **Priority 4:** Citizens employed \( \leq 200,000 \)

Use preemptive goal programming to determine how the government should allocate the publicly controlled agricultural land to the three basic crops.
E* 17.7. The city council of Aberdeen must determine the tax policy for the city for the coming year. Four types of taxes are used to raise money:

- Property tax
- Sales tax (a surcharge on the state sales tax)
- Entertainment tax
- Utility tax (on city-owned utilities)

The city consists of three groups of people: low income, middle income, and high income. The amount of revenue (in thousands of dollars) raised from each group by setting a particular tax at a 1 percent level is given in the following table. (For example, a 3 percent sales tax will raise $1.2 million from low-income people.)

<table>
<thead>
<tr>
<th>Income Group</th>
<th>Property Tax</th>
<th>Sales Tax</th>
<th>Entertainment Tax</th>
<th>Utility Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-income</td>
<td>600</td>
<td>400</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Middle-income</td>
<td>800</td>
<td>350</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>High-income</td>
<td>1,200</td>
<td>250</td>
<td>120</td>
<td>80</td>
</tr>
</tbody>
</table>

The city council has decided that the tax policy must satisfy the following restrictions.

- The tax burden on middle-income people cannot exceed $2.5 million.
- The tax burden on high-income people cannot exceed $2.3 million.
- The total revenue raised must exceed the current level of $6 million.
- The sales tax must be between 1 percent and 3 percent.

Given these restrictions, the city council has set the following three goals (listed in order of priority):

- Goal 1: Limit the tax burden on low-income people to no more than $2 million.
- Goal 2: Set the property tax rate at no less than 1 percent.
- If their tax burden becomes too high, 20 percent of the low-income people, 20 percent of the middle-income people, and 40 percent of the high-income people may consider moving. This will start to happen if the total tax burden of this subset of the population exceeds $1.5 million. Goal 3 is thus to limit the total tax burden on this group of people to no more than $1.5 million.

a. Use preemptive goal programming to determine how the various tax rates should be set.

b. Use weighted goal programming to determine how the various tax rates should be set when using the following penalty weights: 1 per $1,000 in excess of goal 1; 90 per 1 percent short of goal 2; and 1 per $1,000 in excess of goal 3.

E* 17.8. Reconsider the scenario described in Problem 17.2. Management of the Albert Franko Co. now has decided that it should give higher priority to the goal of having product 2 capture at least 10 percent of its market than to the goal of having product 1 capture at least 15 percent of its market. Use preemptive goal programming to determine how to most effectively allocate the available money to the three advertising campaigns.
Reconsider the Dewright Co. case study introduced in Section 17.1. Vijay Shah (vice president for manufacturing) still feels that the top-priority goal should be retaining the employment level at 4,000 employees (avoiding a deviation in either direction) and that satisfactory profits will then follow. Therefore, he places second priority on the goal of holding the capital investment down to no more than $55 million and places only third priority on the goal of achieving a total profit of at least $125 million. Apply preemptive goal programming to the Dewright problem using this revised order of priorities.

The admissions committee for the Whartvard Business School will be making its decisions regarding which applicants to admit to its MBA program for the coming year. In addition to considering each applicant on his or her own merit, the committee also needs to take three policy guidelines into account. One guideline is that, although a relatively low GMAT total score should not disqualify an applicant if other factors are very positive, the average GMAT total score for the entire MBA class should be reasonably high. (About 85 percent of all individuals taking the GMAT receive a total score below 650, but Whartvard is such a selective school that it considers anything below 650 to be a low score.) A second guideline is that the number of men and number of women in the MBA class should not be too badly out of balance. The third guideline is that the class should include a substantial number of students who are at least 30 years old, since they bring considerable work experience and maturity into the mix.

The committee now has divided both the male applicants and female applicants into three categories according to whether they have high, medium, or low GMAT total scores. The following table shows the number of applicants whose age is under 30 and at least 30 in each category.

<table>
<thead>
<tr>
<th>Category</th>
<th>Average GMAT Total Score</th>
<th>Number Whose Age Is under 30</th>
<th>Number Whose Age Is at Least 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>High men</td>
<td>720</td>
<td>120</td>
<td>32</td>
</tr>
<tr>
<td>High women</td>
<td>720</td>
<td>28</td>
<td>4</td>
</tr>
<tr>
<td>Medium men</td>
<td>670</td>
<td>104</td>
<td>56</td>
</tr>
<tr>
<td>Medium women</td>
<td>670</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>Low men</td>
<td>620</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Low women</td>
<td>620</td>
<td>32</td>
<td>48</td>
</tr>
</tbody>
</table>

The admissions committee has set four goals for this entering MBA class, in the following order of priority:

- Goal 1: The entering class should include at least 240 students.
- Goal 2: The entering class should have an average GMAT total score of at least 690.
- Goal 3: The entering class should consist of at least 35 percent women.
- Goal 4: At least 120 members of the entering class should be at least 30 years old.

Based on past experience, 60 percent of all applicants who are admitted will accept admission.

Use preemptive goal programming to determine approximately how many applicants to admit from each category.
Partial Answers to Selected Problems

17.1. The coefficient for L7 is three times as large as the coefficient for K7.

17.3. a. Produce 15 units of product 3.
Case 17-1  A Cure for Cuba

Fulgencio Batista led Cuba with a cold heart and iron fist—greedily stealing from poor citizens, capriciously ruling the Cuban population that looked to him for guidance, and violently murdering the innocent critics of his politics. In 1958, tired of watching his fellow Cubans suffer from corruption and tyranny, Fidel Castro led a guerrilla attack against the Batista regime and wrested power from Batista in January 1959. Cubans, along with members of the international community, believed that political and economic freedom had finally triumphed on the island. The next two years showed, however, that Castro was leading a Communist dictatorship—killing his political opponents and nationalizing all privately held assets. The United States responded to Castro’s leadership in 1961 by invoking a trade embargo against Cuba. The embargo forbade any country from selling Cuban products in the United States and forbade businesses from selling American products to Cuba. Cubans did not feel the true impact of the embargo until 1989 when the Soviet economy collapsed. Prior to the disintegration of the Soviet Union, Cuba had received an average of $5 billion in annual economic assistance from the Soviet Union. With the disappearance of the economy that Cuba had almost exclusively depended upon for trade, Cubans had few avenues from which to purchase food, clothes, and medicine. The avenues narrowed even further when the United States passed the Torricelli Act in 1992 that forbade American subsidiaries in third world countries from doing business with Cuba that had been worth a total of $700 million annually.

Since 1989, the Cuban economy has certainly felt the impact from decades of frozen trade. Today poverty ravages the island of Cuba. Families do not have money to purchase bare necessities, such as food, milk, and clothing. Children die from malnutrition or exposure. Disease infects the island because medicine is unavailable. Optical neuritis, tuberculosis, pneumonia, and influenza run rampant among the population.

Few Americans hold sympathy for Cuba, but Robert Baker, director of Helping Hand, leads a handful of tender souls on Capitol Hill who cannot bear to see politics destroy so many human lives. His organization distributes humanitarian aid annually to needy countries around the world. Mr. Baker recognizes the dire situation in Cuba, and he wants to allocate aid to Cuba for the coming year.

Mr. Baker wants to send numerous aid packages to Cuban citizens. Three different types of packages are available. The basic package contains only food, such as grain and powdered milk. Each basic package costs $300, weighs 120 pounds, and aids 30 people. The advanced package contains food and clothing, such as blankets and fabrics. Each advanced package costs $350, weighs 180 pounds, and aids 35 people. The supreme package contains food, clothing, and medicine. Each supreme package costs $720, weighs 220 pounds, and aids 54 people.

Mr. Baker has several goals he wants to achieve when deciding upon the number and types of aid packages to allocate to Cuba. First, he wants to aid at least 20 percent of Cuba’s 11 million citizens. Second, because disease runs rampant among the Cuban population, he wants at least 3,000 of the aid packages sent to Cuba to be the supreme packages. Third, because he knows many other nations also require humanitarian aid, he wants to keep the cost of aiding Cuba below $20 million.

Mr. Baker places different levels of importance on his three goals. He believes the most important goal is keeping costs down since low costs mean that his organization is able to aid a larger number of needy nations. He decides to penalize his plan by one point for every $1 million above his $20 million goal. He believes the second most important goal is ensuring that at least 3,000 of the aid packages sent to Cuba are supreme packages since he does not want to see an epidemic develop and completely destroy the Cuban population. He decides to penalize his plan by one point for every 1,000 packages below his goal of 3,000 packages. Finally, he believes the least important goal is reaching at least 20 percent of the population since he would rather give a smaller number of individuals all they need to thrive instead of a larger number of individuals.
only some of what they need to thrive. He therefore decides to penalize his plan by seven points for every 100,000 people below his 20 percent goal.

Mr. Baker realizes that he has certain limitations on the aid packages that he delivers to Cuba. Each type of package is approximately the same size, and because only a limited number of cargo flights from the United States are allowed into Cuba, he is only able to send a maximum of 40,000 packages. Along with a size limitation, he also encounters a weight restriction. He cannot ship more than six million pounds of cargo. Finally, he has a safety restriction. When sending medicine, he needs to ensure that the Cubans know how to use the medicine properly. Therefore, for every 100 supreme packages, Mr. Baker must send one doctor to Cuba at a cost of $33,000 per doctor.

a. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba?

b. Mr. Baker reevaluates the levels of importance he places on each of the three goals. To sell his efforts to potential donors, he must show that his program is effective. Donors generally judge the effectiveness of a program on the number of people reached by aid packages. Mr. Baker therefore decides that he must put more importance on the goal of reaching at least 20 percent of the population. He decides to penalize his plan by 10 points for every half a percentage point below his 20 percent goal. The penalties for his other two goals remain the same. Under this scenario, how many basic, advanced, and supreme packages should Mr. Baker send to Cuba? How sensitive is the plan to changes in the penalty weights?

c. Mr. Baker realizes that sending more doctors along with the supreme packages will improve the proper use and distribution of the packages’ contents, which in turn will increase the effectiveness of the program. He therefore decides to send one doctor with every 75 supreme packages. The penalties for the goals remain the same as in part b. Under this scenario, how many basic, advanced, and supreme packages should Mr. Baker send to Cuba?

d. The aid budget is cut, and Mr. Baker learns that he definitely cannot allocate more than $20 million in aid to Cuba. Due to the budget cut, Mr. Baker decides to stay with his original policy of sending one doctor with every 100 supreme packages. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba, assuming that the penalties for not meeting the other two goals remain the same as in part a?

e. Now that the aid budget has been cut, Mr. Baker feels that the levels of importance of his three goals differ so much that it is difficult to assign meaningful penalty weights to deviations from these goals. Therefore, he decides that it would be more appropriate to apply a preemptive goal-programming approach (which will ensure that his budget goal is fully met if possible), while retaining his original policy of sending one doctor with every 100 supreme packages. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba according to this approach?

Case 17-2  Remembering September 11

Adeline Jonasson lost two close friends in the collapse of the World Trade Center on September 11, 2001. Both had been vibrant young women who left grieving husbands and children behind. What terrible losses. Even now, many years later, not a day goes by that she doesn’t think of these friends and feel the anger yet again over those senseless deaths. She still feels a real sense of mission to do something about it. What a relief it had been to be offered a top managerial position in the Transportation Security Administration. After being told that the job would
involve heading a task force on airport security, Adeline had not hesitated a moment in accepting the position. She had greatly enjoyed her career as a management science consultant in the airline industry. It was very satisfying to help several airline companies save many millions of dollars. However, she now felt a greater calling. She would be able to use her expertise in management science to help save lives. There was no way to bring her friends back, but at least she could do everything possible to prevent this from happening again.

Adeline is indeed in the right spot to carry out her mission. Shortly after the tragic events of September 11, 2001, the United States Congress enacted emergency legislation to give the Department of Transportation primary responsibility for providing security at over 400 major U.S. airports. The Transportation Security Administration was then created within the Department of Transportation to carry out this responsibility. One assignment given to Adeline’s task force is to investigate what advanced security technology should be developed and used at airport checkpoints to maximize the effectiveness with which passengers can be screened within budget constraints.

Even prior to 2001, airline passengers had become familiar with the two basic types of systems used to check each passenger at a security checkpoint. One is a portal that can detect concealed weapons as the passenger walks through. The other is a screening system that scans the passenger’s carry-on luggage. Various proposals have been made for advanced security technology that would improve these two systems. Adeline’s task force now needs to make recommendations on which direction to go for the next generation of these systems.

The task force has been told that the functional requirement for the new portal system is that it must be able to detect even one ounce of explosives and hazardous liquids as well as metallic weapons being concealed by a passenger. The technology needed to do this includes quadrupole resonance (closely related to magnetic resonance technology used by the medical industry) and magnetic sensors. There are various ways to design the portal with this technology that would satisfactorily meet the functional requirement. However, the designs would differ greatly in the frequency with which false alarms would occur as well as in the purchase cost and maintenance cost for the portal. The frequency of false alarms is a key consideration since it substantially affects the efficiency with which the passengers can be processed. Even more importantly, a high frequency of false alarms greatly decreases the alertness of the security personnel for detecting the relatively rare terrorists who are actually concealing destructive devices.

The most basic version of the portal system that satisfactorily meets the functional requirement has an estimated purchase price of $90,000 and, on the average, would incur an annual maintenance cost of $15,000. The drawback of this version is that it would generate a false alarm for approximately 10 percent of the passengers. This false alarm rate can be reduced by using more expensive versions of the system. Each additional $15,000 in the cost of the portal system would lower the false alarm rate 1 percent and also would increase the annual maintenance cost by $1,500. The most expensive version would cost $210,000, so it would have a false alarm rate of only 2 percent of the customers as well as an annual maintenance cost of $27,000.

Regarding the new screening system for carry-on luggage, the functional requirement is that it must clearly reveal suspicious objects as small as the smallest Swiss army knife. The technology needed to do this combines X-ray imaging, a thermal neutron scanner, and computer tomography imaging (which compares the density and other physical properties of any suspicious objects with known high-risk materials). It is estimated that the most basic version that satisfactorily meets this functional requirement would cost $60,000 plus an annual maintenance cost of $9,000. As with the most basic portal system, the drawback of this version is that it doesn’t sufficiently discriminate between suspicious objects that actually are destructive devices and those that are harmless. Thus, this version would generate false alarms for approximately 6 percent of the customers. In addition to wasting time and delaying passengers, such a high false
alarm rate would make it very difficult for the screening operator to pay sufficient attention when
the far more unusual true alarms occur. However, more expensive versions of the screening
system would be considerably more discriminating. In particular, each additional $30,000 in the
cost of the system would enable a reduction of 1 percent in the false alarm rate, while also
increasing the annual maintenance cost by $1,200. Thus, the most expensive version, costing
$150,000, would decrease the false alarm rate to 3 percent and incur an annual maintenance cost
of $12,600.

The task force has been given two budgetary guidelines.

**First budgetary guideline:** Plan on a total expenditure of $250,000 for both the portal
system and the screening system for carry-on luggage at each security checkpoint.

**Second budgetary guideline:** Plan on holding down the average total maintenance costs
for the two systems at each security checkpoint to no more than $30,000.

These budgetary guidelines prohibit using the most expensive versions of both the portal
system and the screening system for carry-on baggage. Therefore, the task force needs to
determine which financially feasible combination of versions for the two systems will maximize
the effectiveness with which passengers can be screened. Doing this requires first obtaining input
from the top management of the Transportation Security Administration regarding what the
measure of effectiveness should be and then what management’s goals and priorities are for
achieving substantial effectiveness and meeting the budgetary guidelines.

Fortunately, Adeline already has had extensive discussions with top management to
obtain its guidance on these matters. These discussions led to the adoption of a clear policy that
was approved all the way up to the Secretary of Transportation (who also informed the
chairpersons of the congressional oversight committees of this action). The policy establishes the
following order of priorities.

**Priority 1:** The functional requirement for each of the two new systems must be met.
(This is satisfied by all the versions under consideration by the task force.)

**Priority 2:** The total false alarm rate for both systems should not exceed 0.1 per
passenger.

**Priority 3:** Meet the first budgetary guideline.

**Priority 4:** Meet the second budgetary guideline.

Now that it has obtained all the needed managerial input, the task force is ready to begin its
analysis.

1. Identify the two decisions to be made and define a decision variable for each one.
2. Describe why this problem is a preemptive goal-programming problem by giving
quantitative expressions for each of the goals in terms of the decision variables
defined in part 1.
3. Draw a single two-dimensional graph where the two axes correspond to the decision
variables defined in part 1. Consider each of the goals in order of priority and use the
quantitative expression obtained in part 2 for this goal to draw a plot on the graph
that displays the values of the decision variables that fully satisfy this goal. After
completing this for all the goals, use the graph to determine the optimal solution for
this preemptive goal-programming problem.

4. Use preemptive goal programming to formulate and solve this problem on a
spreadsheet.
e. If it is possible to fully satisfy all the goals except the lowest priority goal, one can quickly solve a preemptive goal-programming problem by formulating and solving a linear programming model that includes all the goals except the last one as constraints and then uses the objective function to strive toward the lowest priority goal. Formulate and solve such a linear programming model for this problem on a spreadsheet. What would be the interpretation for the preemptive goal-programming problem if this linear programming model had no feasible solutions?

f. Perform some what-if analysis by determining how far the total false alarm rate per passenger can be reduced (perhaps even below the goal) by ignoring the second budgetary guideline but fully meeting the first one.

g. What additional what-if analysis do you feel should be performed in order to provide top management with the information needed to make a sound judgment decision about the best trade-off between (1) the total false alarm rate per passenger, (2) the total expenditure for the two new security systems per security checkpoint, and (3) the total annual maintenance cost for these two systems per security checkpoint?