**Chapter 4**

**4-1** For a torsion bar, *kT*= *T/θ*= *Fl/θ*, and so *θ*= *Fl/kT.*For a cantilever, *kl*= *F/δ,δ*= *F/kl.* For the assembly, *k* = *F/y*, or, *y* = *F/k* = *lθ*+ *δ*

Thus



Solving for *k*



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**4-2** For a torsion bar, *kT*= *T/θ*= *Fl/θ*, and so *θ*= *Fl/kT.* For each cantilever, *kl*= *F/δl, δl*= *F/kl*, and*,δL*= *F/kL.* For the assembly, *k* = *F/y*, or, *y* = *F/k* = *lθ*+ *δl +δL*.

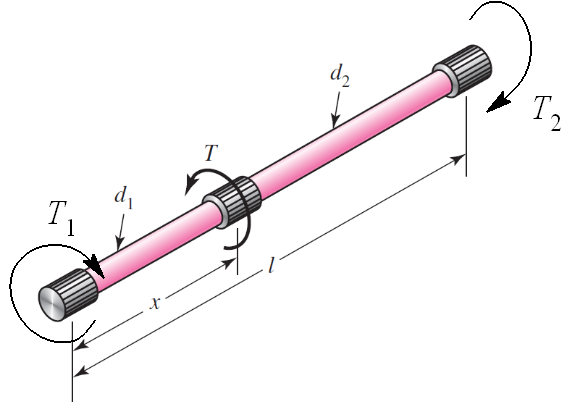
Thus



Solving for *k*



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**4-3** **(a**) For a torsion bar, *k =T/θ =GJ/l*.

Two springs in parallel, with *J =πdi* 4/32,

and *d*1 = *d*2 = *d*,



Deflection equation,



From statics, *T*1 + *T*2 = *T* = 0.26 Nm. Substitute Eq. (2)



Substitute into Eq. (2) resulting in 

(**b**) From Eq. (1), 

From Eq. (4), 

From Eq. (3), 

From either section, 

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**4-4** Deflection to be the same as Prob. 4-3 where *T*1 = 0.13N⋅m, *l*1 = *l* / 2 = 127mm, and *d*1 = 12.7mm

*θ*1 = *θ*2 = *θ*



Or, 



Equal stress, 

Divide Eq. (4) by the first two equations of Eq.(1) results in



Statics, *T*1 + *T*2 = 0.26 (6)

Substitute in Eqs. (2) and (3), with Eq. (5) gives



Solving for *d*1 and substituting it back into Eq. (5) gives

*d*1 = 0.00987m (9.87 mm), *d*2 = 0.0148m (14.8 mm) *Ans.*

From Eqs. (2) and (3),

*T*1 = 6.246(106)(0.00987)4 = 0.059N⋅m*Ans.*

*T*2 = 4.164(106)(0.0148)4 = 0.200N⋅m*Ans.*

Deflection of *T* is 

Spring constant is 

The stress in *d*1 is 

The stress in *d*1 is 

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**4-5** (**a**) Let the radii of the straight sections be *r*1 = *d*1 /2 and *r*2 = *d*2 /2. Let the angle of the taper be ** where tan ** = (*r*2 −*r*1)/2. Thus, the radius in the taper as a function of *x* is

*r* = *r*1 + *x* tan **, and the area is *A* = *π* (*r*1 + *x* tan **)2. The deflection of the tapered portion is



(**b**) For section 1,



For the tapered section,



For section 2,



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**4-6** (**a**) Let the radii of the straight sections be *r*1 = *d*1 /2 and *r*2 = *d*2 /2. Let the angle of the taper be ** where tan ** = (*r*2 −*r*1)/2. Thus, the radius in the taper as a function of *x* is

*r* = *r*1 + *x* tan **, and the polar second area moment is *J* = (*π* /2) (*r*1 + *x* tan **)4. The angular deflection of the tapered portion is



(**b**) The deflections, in degrees, are:

Section 1,



Tapered section,



Section 2,



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**4-7** The area and the elastic modulus remain constant. However, the force changes with respect to *x*. From Table A-5, the unit weight of steel is *γ* = 76.0kN/m3 and the elastic modulus is *E* = 207 GPa. Starting from the top of the cable (i.e. *x* = 0, at the top).

*F* = *γ*(*A*)(*l−x*)



From the weight at the bottom of the cable,





The percentage of total elongation due to the cable’s own weight



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**4-8** Σ*Fy* = 0 = *R* 1−*F*⇒ *R* 1 = *F*

Σ*MA* = 0 =*M*1−*Fa*⇒*M*1 =*Fa*

*VAB* = *F, MAB =F* (*x − a* ), *VBC* =*MBC =* 0

Section *AB*:

 (1)

*θAB* = 0 at *x* = 0 ⇒*C*1 = 0

 (2)

*yAB* = 0 at *x* = 0 ⇒*C*2 = 0, and



Section *BC*:



From Eq. (1), at *x* = *a* (with *C*1 = 0), = *C*3. Thus,



 (3)

From Eq. (2), at *x* = *a* (with *C*2 = 0), . Thus, from Eq. (3)

 Substitute into Eq. (3), obtaining



The maximum deflection occurs at *x= l*,



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**4-9** Σ*MC* = 0 =*F* (*l* /2) −*R*1*l*⇒*R*1 =*F* /2

Σ*Fy* = 0 = *F* /2 + *R* 2−*F*⇒ *R* 2 = *F* /2

Break at 0 ≤*x*≤*l* /2:

*VAB = R* 1 = *F* /2, *MAB* = *R* 1*x* =*Fx* /2

Break at *l* /2 ≤*x*≤*l* :

*VBC = R* 1−*F* = −*R* 2 = −*F* /2, *MBC* = *R* 1*x*−*F* ( *x*−*l* / 2) =*F*(*l − x*) /2

Section *AB*:



From symmetry, *θAB* = 0 at *x = l* /2 ⇒. Thus,

 (1)



*yAB* = 0 at *x* = 0 ⇒*C*2 = 0, and,

 (2)

*yBC*is not given, because with symmetry, Eq. (2) canbe used in this region. The maximum deflection occurs at *x =l* /2,



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**4-10** From Table A-6, for each angle, *I*1-1 = 207 cm4. Thus, *I* = 2(207) (104) = 4.14(106) mm4

From Table A-9, use beam 2 with *F* = 2500 N, *a* = 2000 mm, and *l* = 3000 mm; and beam 3 with*w* = 1 N/mm and *l* = 3000 mm.





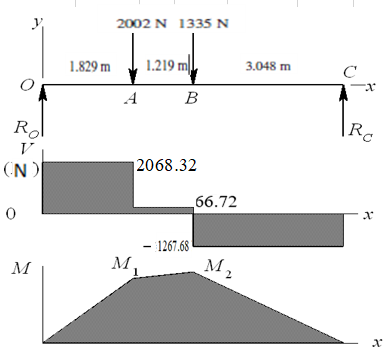


= − 2500(2000) − [1(30002)/2]= −9.5(106) N⋅mm

From Table A-6, from centroid to upper surface is *y* = 29 mm. From centroid to bottom surface is *y* =29.0 − 100=− 71 mm. The maximum stress is compressive at the bottom of the beam at the wall. This stress is



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**4-11**



*M*1 = 2068.32(1.829) = 3.78(103) N⋅m

*M*2 = 3.78(103) +66.72(1.219)

= 3.86(103) N⋅m



For deflections, use beams 5 and 6 of Table A-9

Select two 127 x 64 channels from Table A-7, *I* = 2(482.5) = 965cm4, *Z* =2(75.99) = 151.98cm3

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**4-12** 

From Table A-9 by superposition of beams 6 and 7, at *x = a* = 381mm, with *b* = 609.6mm and *l* = 990.6mm





At *x = l* /2 = 495.3 mm



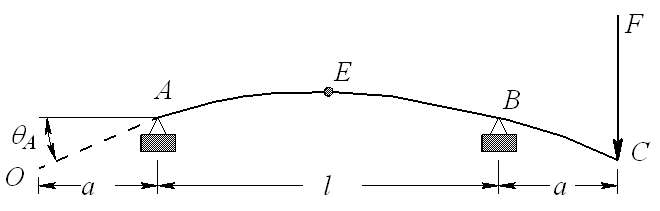




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**4-13** ****

From Table A-9-10, beam 10





At *x* = 0, 





With both loads,





At midspan,



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**4-14** 

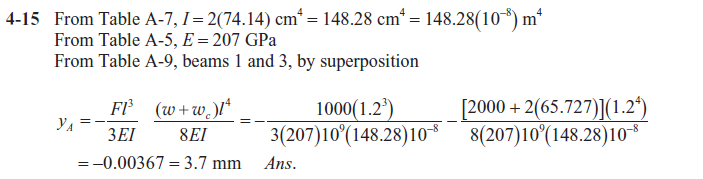
From Table A-5, *E* = 71.7GPa

From Table A-9, beams 1 and 2, by superposition





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**4-16** 

From Table A-5,

From Table A-9, beams 5 and 9, with *FC = FA= F*,by superposition







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**4-17** From Table A-9, beams 8 (region *BC* for this beam with*a* = 0) and 10 (with *a* = *a*), by superposition









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**4-18** **Note to the instructor:** Beams with discontinuous loading are better solved using singularity functions. This eliminates matching the slopes and displacements at the discontinuity as is done in this solution.

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Substituting (1) into (2) yields. Substituting this back into (2) gives

. Thus,







This result is sufficient for *yBC*. However, this can be shown to be equivalent to



by expanding this or by solving the problem using singularity functions.

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**4-19** The beam can be broken up into a uniform load *w* downward from points *A* to *C* and a uniform load *w*upward from points *A* to *B*. Using the results of Prob. 4-18, with *b* = *a* for *A* to *C* and *a* = *a* for *A* to *B*, results in

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**4-20** **Note to the instructor:** See the note in the solution for Problem 4-18.



For region *BC*, isolate right-hand element of length (*l + a − x*)









*yAB* = 0 at *x* = 0 ⇒*C*2 = 0 ∴

*yAB* = 0 at *x* = *l*⇒∴





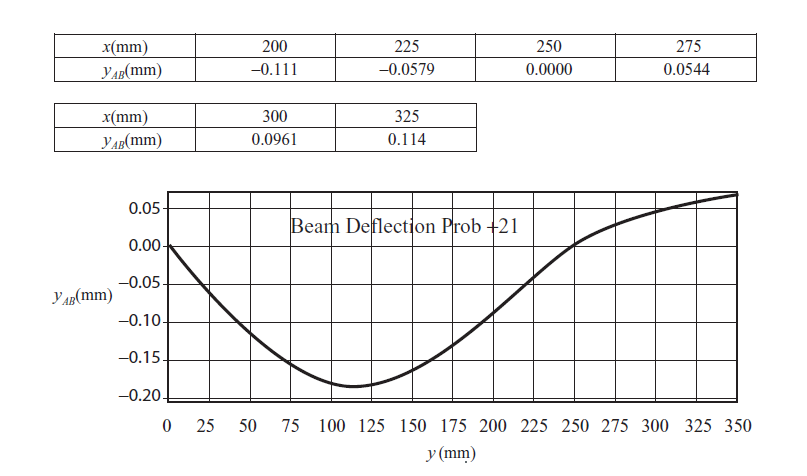
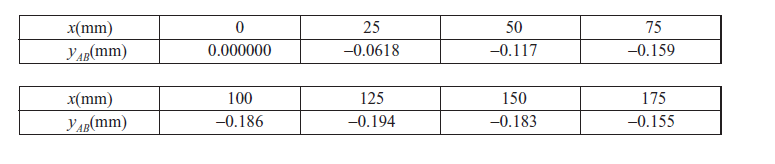
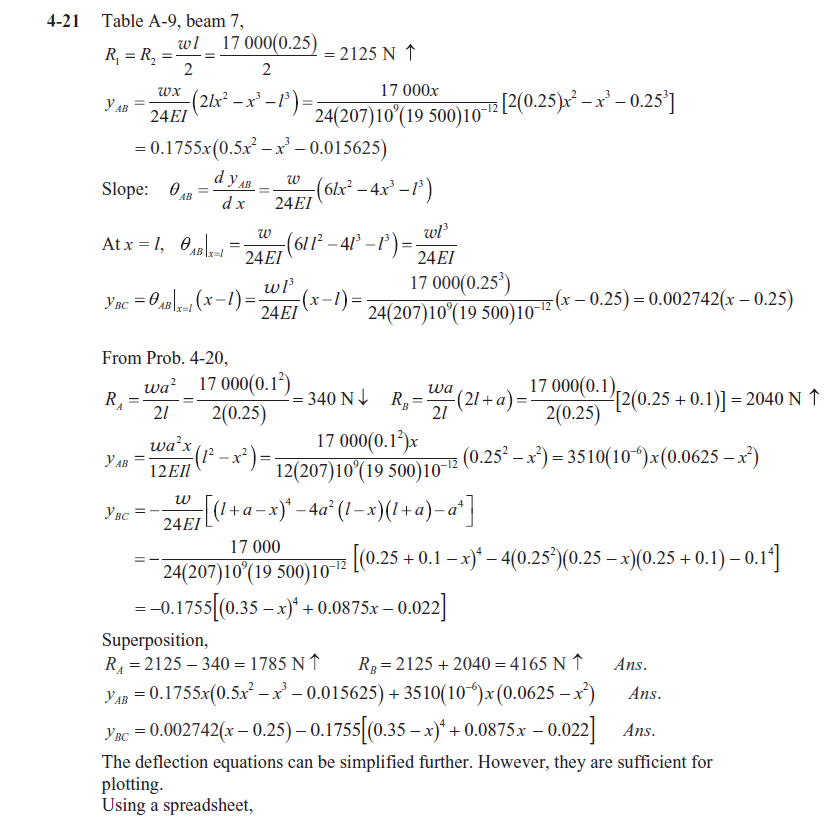
*yBC* = 0 at *x* = *l*⇒ (1)

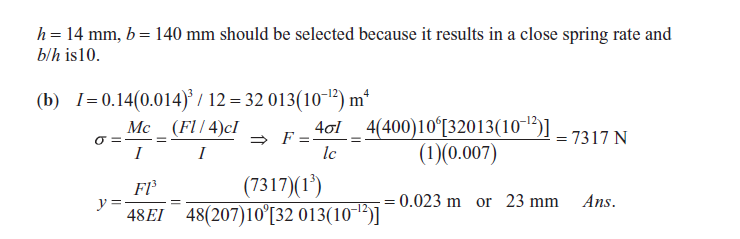
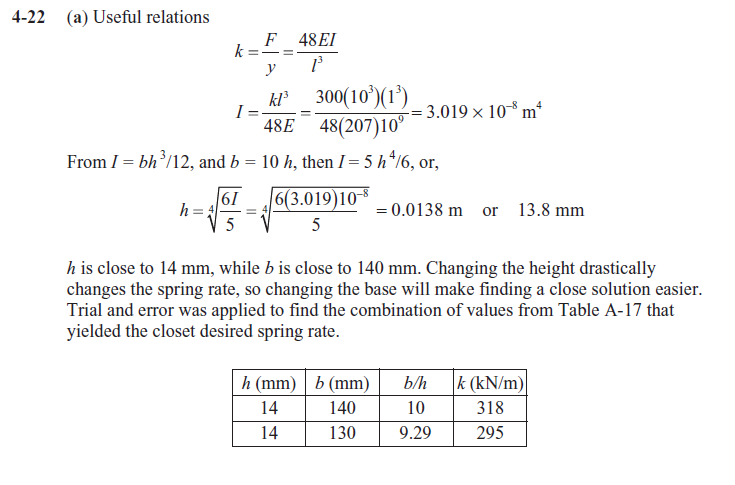
*θAB* = *θBC* at *x* = *l*⇒

Substitute *C*3 into Eq. (1) gives . Substitute back into *yBC*



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**4-23** From the solutions to Prob. 3-79,  

From Table A-9, beam 6,





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**4-24** From the solutions to Prob. 3-80, 



The load in between the supports supplies an angle to the overhanging end of the beam. That angle is found by taking the derivative of the deflection from that load. From Table A-9, beams 6 (subscript 1) and 10 (subscript 2),

 (1)



Equation (1) is thus



The slope at *A*, relative to the *z* axis is



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**4-25** From the solutions to Prob. 3-81, 



From Table A-9, beam 6,





The displacement magnitude is 



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The slope magnitude is 

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4-26** From the solutions to Prob. 3-82, 







The displacement magnitude is 





The slope magnitude is 

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**4-27** From the solutions to Prob. 3-83, 



From Table A-9, beams 6 (subscript 1) and 10 (subscript 2)





The displacement magnitude is 





The slope magnitude is 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-28** From the solutions to Prob. 3-84, *FB* = 22.8 (103) N



From Table A-9, beam 6,





The displacement magnitude is 





The slope magnitude is 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-29** From the solutions to Prob. 3-79, *T*1 = 267N and *T*2 = 1780N, and Prob. 4-23, *I* = 4.98(10-8)m4.From Table A-9, beam 6,





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**4-30** From the solutions to Prob. 3-80, *T*1 = 2 880 N and *T*2 = 432 N, and Prob. 4-24, *I* = 39.76 (103) mm4. From Table A-9, beams 6 and 10





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**4-31** From the solutions to Prob. 3-81,*T*1 = 1744Nand *T*2 = 262N , and Prob. 4-25, *I* = 2.04(10-8) m4. From Table A-9, beam 6





The slope magnitude is 





The slope magnitude is 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-32** From the solutions to Prob. 3-82, *T*1 =250 N and *T*1 =37.5 N, and Prob. 4-26, *I* = 7 854 mm4. From Table A-9, beam 6





The slope magnitude is 





The slope magnitude is 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-33** From the solutions to Prob. 3-83, *FB* = 3336N, and Prob. 4-27, *I* = 4.98(10-8)m4. From Table A-9, beams 6 and 10





The slope magnitude is 





The slope magnitude is 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-34** From the solutions to Prob. 3-84, *FB* = 22.8 kN, and Prob. 4-28, *I* = 306.8 (103) mm4. From Table A-9, beam 6





The slope magnitude is 





The slope magnitude is 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-35** The required new slope in radians is *θ*new= 0.06(*π* /180) = 0.00105 rad.

In Prob. 4-29, *I* = 4.98(10-8) m4, and it was found that the greater angle occurs at the bearing at *O* where (*θO*)*y*= − 0.00468 rad.

Since*θ* is inversely proportional to *I*,

*θ* new*I*new = *θ* old*I*old⇒*I*new = *π*/64 = *θ* old*I*old /*θ* new

or, 

The absolute sign is used as the old slope may be negative.



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**4-36** The required new slope in radians is *θ*new = 0.06(*π* /180) = 0.00105 rad.

In Prob. 4-30, *I* = 39.76 (103) mm4, and it was found that the greater angle occurs at the bearing at *C* where (*θC*)*y*= − 0.0191 rad.

See the solution to Prob. 4-35 for the development of the equation





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**4-37** The required new slope in radians is *θ*new = 0.06(*π* /180) = 0.00105 rad.

In Prob. 4-31, *I* = 2.04(10-8) m4, and the maximum slope is *ΘC* = 0.0104 rad.

See the solution to Prob. 4-35 for the development of the equation





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**4-38** The required new slope in radians is *θ*new = 0.06(*π* /180) = 0.00105 rad.

In Prob. 4-32, *I* = 7 854mm4, and the maximum slope is *ΘO* = 0.00750 rad.

See the solution to Prob. 4-35 for the development of the equation





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**4-39** The required new slope in radians is *θ*new = 0.06(*π* /180) = 0.00105 rad.

In Prob. 4-33, *I* = 4.98(10-8)m4, and the maximum slope *Θ* = 0.0222 rad.

See the solution to Prob. 4-35 for the development of the equation





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**4-40** The required new slope in radians is *θ*new = 0.06(*π* /180) = 0.00105 rad.

In Prob. 4-34, *I* = 306.8 (103) mm4, and the maximum slope is *ΘC* = 0.0174 rad.

See the solution to Prob. 4-35 for the development of the equation





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**4-41** *IAB* = *π*(0.0254)4/64 = 2.04(-8)m4, *JAB* = 2*IAB* = 4.08(10-8) m4,*IBC* = (0.00635)(0.0381)3/12 = 2.93(10-8) m4, *ICD* = *π* (0.01905)4/64 = 0.646(10-8) m4. For Eq. (3-41), *b/c* = 0.0381/0.00635 = 6 ⇒*β* = 0.299.

Thedeflection can be broken down into several parts

1. The vertical deflection of*B* due to force and moment acting on *B* (*y*1).

2. The vertical deflection due to the slope at *B*, *θB*1, due to the force and moment acting on *B* (*y*2 = *θB*1 = 2*θB*1).

3. The vertical deflection due to the rotation at *B*, *θB*2, due to the torsion acting at *B* (*y*3 = *θB*1 = 5*θB*1).

4. The vertical deflection of *C* due to the force acting on *C* (*y*4).

5. The rotation at *C*, *θC*, due to the torsion acting at *C* (*y*3 = *θC* = 2*θC*).

6. The vertical deflection of *D* due to the force acting on *D* (*y*5).

1. From Table A-9, beams 1 and 4 with *F* = −889.644N and *MB*= 0.0508(889.644) = 45.19N⋅m



2. From Table A-9, beams 1 and 4



*y* 2 = 50.8(0.004074) = 0.207mm

3. The torsion at *B* is *TB* = 0.127(889.644) = 113N⋅m. From Eq. (4-5)



*y* 3 = 127(0.005314) = 0.675mm

4. For bending of *BC*, from Table A-9, beam 1



5. For twist of *BC*, from Eq. (3-41), with *T* = 0.0508(889.644) = 45.19N⋅m



*y* 5 = 50.8(0.02482) = 1.26mm

6. For bending of *CD*, from Table A-9, beam 1



Summing the deflections results in



This problem is solved more easily using Castigliano’s theorem. See Prob. 4-78.

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**4-42** The deflection of *D* in the *x* direction due to *Fz* is from:

1. The deflection due to the slope at *B*, *θB*1, due to the force and moment acting on *B* (*x*1 = *θB*1 = 5*θB*1).

2. The deflection due to the moment acting on *C* (*x*2).

1. For *AB*,*IAB* = *π*(0.02544)/64 = 2.04(10-8)m4. From Table A-9, beams 1 and 4



*x* 1 = 127(−0.002037) = −0.259mm

2. For *BC*, *IBC* = (0.0381)(0.00635)3/12 = 8.13(10-10)m4. From Table A-9, beam 4



The deflection of *D* in the *x* direction due to *Fx* is from:

3. The elongation of *AB* due to the tension. For *AB*, the area is *A* = *π*(0.0254)2/4 = 0.507(10-3)m2



4. The deflection due to the slope at *B*, *θB*2, due to the moment acting on *B* (*x*1 = *θB*2 = 5*θB*2). With *IAB* = 2.04(10-8) m4,



*x*4 = 127(− 0.003056) = − 0.388mm

5. The deflection at *C* due to the bending force acting on *C*. With *IBC* = 8.13(10-10)m4



6. The elongation of *CD* due to the tension. For *CD*, the area is *A* = *π* (0.019052)/4 = 0.285(10-3)m2



Summing the deflections results in



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**4-43** *JOA* =*JBC* = *π* (0.03814)/32 = 2.07(10-7)m4, *JAB* = *π* (0.02544)/32 = 4.09(10-8)m4, *IAB* = *π* (0.02544)/64 = 2.04(10-8) m4, and *ICD* = *π* (0.019054)/64 = 6.38(10-9)m4.

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Simplified

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Simplified is 0.0345/0.0260 = 1.33 times greater*Ans.*

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**4-44** Reverse the deflection equation of beam 7 of Table A-9. I = 12319(106) mm4



The maximum height occurs at *x* = 7.62/2 = 3.81m



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**4-45** From Table A-9, beam 6,









Let and set. Thus,



For the other end view, observe beam 6 of Table A-9 from the back of the page, noting that *a* and *b* interchange as do *x* and –*x*



For a uniform diameter shaft the necessary diameter is the larger of 

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**4-46** The maximum slope will occur at the left bearing. Incorporating a design factor into the solution for of Prob. 4-45,





From Table A-9, beam 6, the maximum deflection will occur in *BC* where *dyBC /dx* = 0







*x* = 136.7 mm is acceptable.

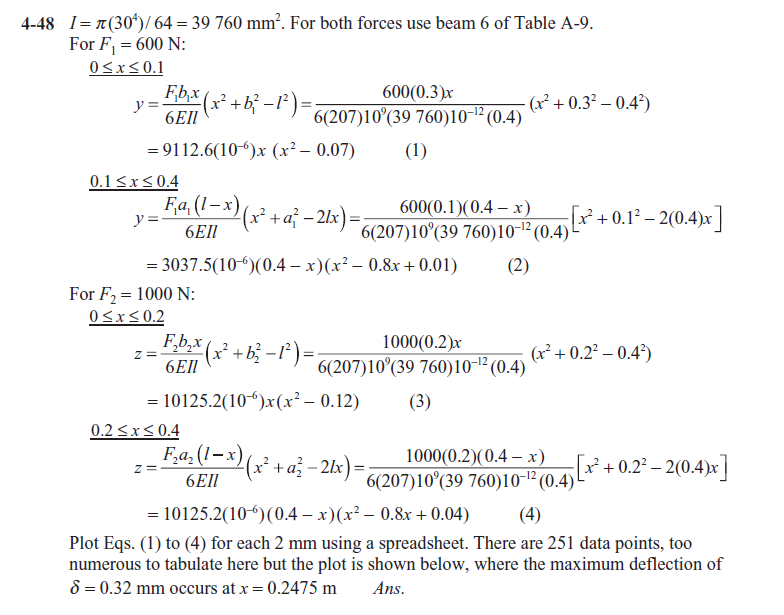


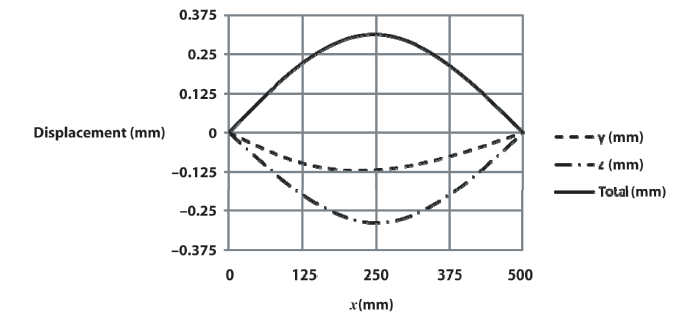
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**4-47** *I* = *π* (0.031754)/64 = 5(10-8)m4. From Table A-9, beam 6



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**4-49** The larger slope will occur at the left end.

From Table A-9, beam 8



With *I = π d* 4/64, the slope at the left bearing is



Solving for *d*



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**4-50** From Table A-5, *E* = 71.7GPa

Σ*MO* = 0 = 0.4572*FBC*−0.1524(444.8) ⇒*FBC* = 148.25N

The cross sectional area of rod*BC* is *A* = *π* (0.01272)/4 = 0.126(10-3)m2.

The deflection at point *B* will be equal to the elongation of the rod *BC*.



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**4-51** Σ*MO* = 0 = 0.1254*FAC*−0.2794(444.8) ⇒*FAC* =815.32N

The deflection at point *A*in the negative *y* direction is equal to the elongation of the rod *AC*. From Table A-5, *Es* = 207GPa.



By similar triangles the deflection at *B* due to the elongation of the rod *AC* is



From Table A-5, *Ea* = 71.7GPa

The bar can then be treated as a simply supported beam with an overhang*AB*. From Table A-9, beam 10,



*yB = yB*1 + *yB*2 = − 0.0284− 0.3653 = − 0.3937mm*Ans.*

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**4-52** From Table A-5, *EA* = 71.7 GPa, *ES* = 207 GPa.

Σ*MO* = 0 = 450 *FCD* – 650(4000) ⇒*FCD* = 5777.8 N



The deflection of *B* due to *yD* is

(*yB*)1 = (650/450) (− 0.2172) = − 0.3137 mm

Treat beam *OADB* as simply-supported at *O* and *D*. Use beam 10 of Table A-9 and use the equation for *yC*,



Superposition:

*yB* = (*yB*)1 + (*yB*)2 = − 0.3137 − 3.868 = − 4.18 mm *Ans.*

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**4-53** From Table A-5, *EA* = 71.7 MPa, *ES* = 207 MPa

Σ*MO* = 0 = 450 *FCD* – 150(4000) ⇒*FCD* = 1333 N



The deflection of *B* due to *yD* is

(*yB*)1 = (650/450) (− 0.0501) = − 0.0724 mm

Treat beam *OADB* as simply-supported at *O* and *D*. Find slope at *B* for beam 6 of Table A-9,





For *OADB*,



Deflection of *B* due to slope at *D* is

(*yB*)2 = 0.004 463(200) = 0.8926 mm

Superposition:

*yB* = (*yB*)1 + (*yB*)2 = − 0.0725 + 0.8926 = 0.820 mm *Ans.*

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**4-54** From Table A-5, *EA* = 71.7GPa, *ES* = 207GPa

Σ*MO* = 0 = 0.4572*FCD* – 0.1524(444.8) ⇒*FCD* = 148.25N



The deflection of *A* due to *yB* is

(*yA*)1 = (152.4/457.2) (−0.0017) = −0.0006mm

Treat beam *OAB* as simply-supported at *O* and *D*. Use beam 6 of Table A-9 

Superposition:

*yA* = (*yA*)1 + (*yA*)2 =−0.0006−0.1407 = −0.1413 mm *Ans.*

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**4-55** From Table A-5, *ES* = 207GPa

Σ*MO* = 0 = 0.4572*FAC* – 0.2794(44.8) ⇒*FAC* = 815.32N



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**4-56** From Table A-5, *EA* = 71.7 MPa, *ES* = 207 MPa

Σ*MO* = 0 = 450 *FCD* – 650(4000) ⇒*FCD* = 5777.8 N



The deflection of *A* due to *yD* is

(*yA*)1 = (150/300) (− 0.2172) = − 0.1086 mm

Treat beam *OADB* as simply-supported at *O* and *D*. Use beam 10 of Table A-9 

Superposition:

*yA* = (*yA*)1 + (*yA*)2 = − 0.1086 + 0.8926 = 0.784 mm *Ans.*

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**4-57** From Table A-5, *EA* = 71.7 MPa, *ES* = 207 MPa

Σ*MO* = 0 = 450 *FCD* – 150(4000) ⇒*FCD* = 1333 N



The deflection of *A* due to *yD* is

(*yA*)1 = (150/300) (− 0.0501) = − 0.02505 mm

Treat beam *OADB* as simply-supported at *O* and *D*. Use beam 6 of Table A-9 

Superposition:

*yA* = (*yA*)1 + (*yA*)2 = − 0.02505 − 0.6695 = − 0.695 mm *Ans.*

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**4-58** From Table A-5, *E* = 207 GPa, and *G* = 79.3 GPa. 

The spring rate is *k = F/* |*yB*|. Thus



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**4-59** For the beam deflection, use beam 5 of Table A-9.

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For *BC*, since Table A-9 does not have an equation (because of symmetry) an equation will need to be developed as the problem is no longer symmetric. This can be done easily using beam 6 of Table A-9 with *a = l* /2



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**4-60**

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**4-61** Let the load be at *x* ≥ *l*/2. The maximum deflection will be in Section*AB*

(Table A-9, beam 6)







For *x*≤*l*/2, 

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**4-62**













, and therefore





*MO* = 9.5 (106) N⋅m. The maximum stress is compressive at the bottom of the beam where*y* = 29.0 −100 = −71 mm



The solutions are the same as Prob. 4-10.

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**4-63** See Prob. 4-11 for reactions: *RO* = 2068N and *RC*= 1147N. Using SI units

*M* =2068*x*−2002〈*x*−1.829〉1−1335〈*x*−3.048〉1



*EIy* = 344.7*x*3−333.6〈*x*−1.828〉3−222.4〈*x*−3.048〉3−*C*1*x*

*y* = 0 at *x* = 0 ⇒*C*2 = 0

*y* = 0 at *x* = 6.096 m

0 = 344.7(6.0963) −333.6(6.096−1.828)3−222.4(6.096−3.048)3 + *C*1*x*⇒*C*1 = −7524.6N⋅m2and,

*EIy* = 344.72*x*3−333.6〈*x*−1.828〉3−222.4〈*x*−3.048〉3−7524.6*x*

Substituting *y* =−0.0127m at *x* = 3.048m gives

207(109) *I* (−0.0127) = 344.72 (3.0483) −333.6(3.048−1.828)3−222.4(3.048−3.048)3−7524.6(3.048)

*I* =5.24(10-6)m4 = 524.45 cm4

Select two 127mm×64 mmchannels; from Table A-7, *I* = 2(482.5) = 965cm4



The maximum moment occurs at *x* = 3.048 m where *M*max = 3863 N⋅m

 O.K.

The solutions are the same as Prob. 4-11.

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**4-64** *I* = *π* (0.03814)/64 = 1.03(10-7 )m4, and *w* = 203.4 N/m.

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Thus,



Evaluating at *x* = 0.381 m,

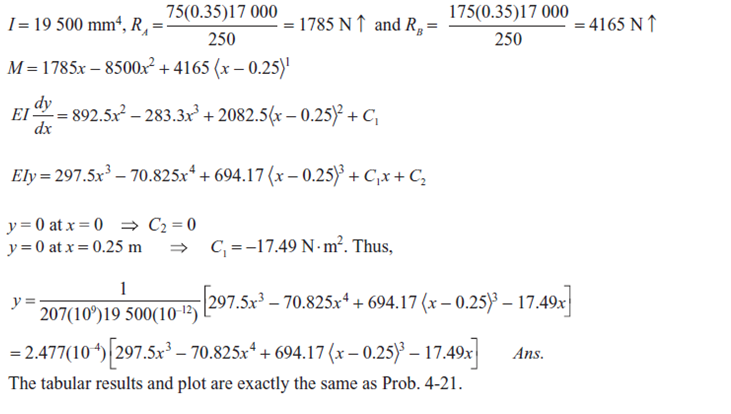




5 % difference *Ans.*

The solutions are the same as Prob. 4-12.

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**4-65** 

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**4-66** *RA* =*RB* = 400 N, and *I* = 6(323) /12 = 16 384 mm4.

First half of beam,

*M* = − 400 *x* + 400 〈*x*− 300 〉1



From symmetry, *dy/dx* = 0 at *x* = 550 mm ⇒ 0 = −200(5502) + 200(550 – 300) 2 + *C*1

⇒*C*1 = 48(106) N·mm2

*EIy* = − 66.67 *x*3 + 66.67 〈*x*− 300 〉3 + 48(106) *x* + *C*2

*y* = 0 at *x* = 300 mm ⇒*C*2 = − 12.60(109)N·mm3.

The term (*EI*)−1 = [207(103)16 384]−1= 2.949 (10−10 )Thus

*y* = 2.949 (10−10) [− 66.67 *x*3 + 66.67 〈*x*− 300 〉3 + 48(106) *x*− 12.60(109)]

*yO* = − 3.72 mm *Ans.*

*y*|*x* = 550 mm =2.949 (10−10) [− 66.67 (5503) + 66.67 (550 − 300)3

+ 48(106) 550 −12.60(109)] = 1.11 mm *Ans.*

The solutions are the same as Prob. 4-13.

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**4-67**

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*y* = 0 at *x* = 0 ⇒*C*2 = 0

*y* = 0 at *x* = *l*⇒. Thus,





In regions,





The solutions reduce to the same as Prob. 4-17.

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**4-68** 







*y* = 0 at *x* = 0 ⇒*C*2 = 0

*y* = 0 at *x* = *l*





The above answer is sufficient. In regions,





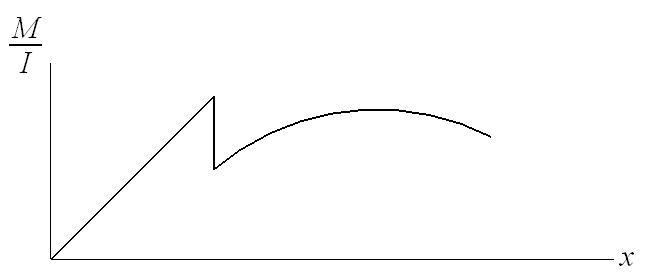


These equations can be shown to be equivalent to the results found in Prob. 4-19.

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**4-69** *I*1 = *π* (0.03494)/64 = 7.3(10-8 )m4, *I*2 = *π* (0.04444)/64 = 1.92(10-7 )m4,

*R*1 = 0.5(31500)(0.254) = 4000 N

 Since the loading and geometry are symmetric, we will only write the equations for the first half of the beam

For 0 ≤*x*≤ 0.2032 m

At *x* = 0.0762, *M* = 305 N⋅m

Writing an equation for *M / I*, as seen in the figure,

the magnitude and slope reduce since *I* 2>*I* 1.

To reduce the magnitude at *x* = 0.0762 m, we add the

term, − 305(1/*I* 1− 1/ *I* 2)〈*x*− 0.0762〉0. The slope of 4000 at *x* = 0.0762 m is also reduced. We account for this with a ramp function, 〈*x*− 0.0762〉1 . Thus,





Boundary Condition: 

⇒

*C*1 = − 197.10 N\* m2

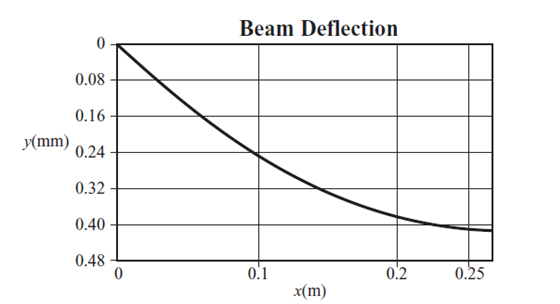


*y* = 0 at *x* = 0 ⇒*C*2 = 0

Thus, for 0 ≤*x*≤ 0.203 m

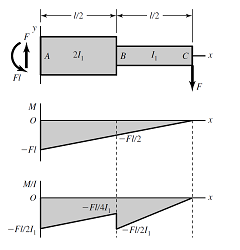


Using a spreadsheet, the following graph represents the deflection equation found above



The maximum is 

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**4-70** The force and moment reactions at the left support

are *F* and *Fl* respectively. The bending moment

equation is

*M = Fx*−*Fl*

Plots for *M* and *M /I* are shown.

*M /I* can be expressed using singularity functions



where the step down and increase in slope at*x = l* /2 are given by the last two terms. Integrate



*dy/dx* = 0 at *x* = 0 ⇒*C*1 = 0



*y* = 0 at *x* = 0 ⇒*C*2 = 0







The answers are identical to Ex. 4-10.

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**4-71** Place a fictitious force, *Q*, at the center. The reaction, *R*1 = *wl* /2 + *Q* / 2



Integrating for half the beam and doubling the results



Note, after differentiating with respect to *Q*, it can be set to zero



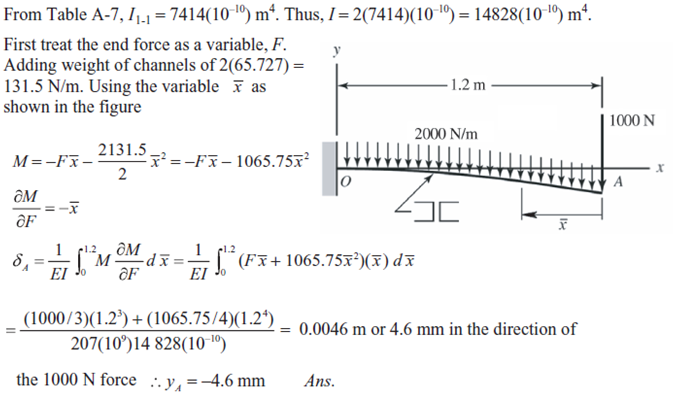
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**4-72** Place a fictitious force *Q* pointing downwards at the end. Use the variable originating at the free end and positive to the left

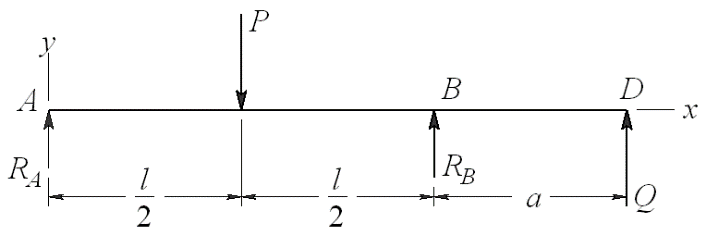




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**4-73** 

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**4-74**

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We observe that for section *BD,* the only force is *Q,* which is zero, so there is no contribution to the deflection at *D* from the strain energy in section *BD*.

****

The first two integrals can be combined from 0 to *l*,



**(b)** Table A-9, beam 5 with *F* = *P*,

Slope: 

At *x* = 0,  . By symmetry, 

 This agrees with part (a)

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**4-75** The energy includes torsion in *AC*, torsion in *CO*, and bending in *AB*.

Neglecting transverse shear in *AB*



In *AC* and *CO*,



The total energy is



The deflection at the tip is







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**4-76** *I*1 = *π* (0.034924)/64 = 7.30(10-8 )m4, *I*2 = *π* (0.04444)/64 = 1.92(10-7 )m4

Place a fictitious force *Q* pointing downwards at the midspan of the beam, *x* = 0.203 m



For 0≤*x*≤0.0762 m 

For0.0762≤*x*≤ 0.33 m 

By symmetry it is equivalent to use twice the integral from 0 to 8



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**4-77** *I* = *π* (0.01274)/64 = 1.28 (10−9) m4, *J*= 2*I* =2.56 (10−9) m4

*A =π* (0.01272)/4 = 0.13(10-3 )m2.

Consider *x* to be in the direction of *OA*, *y* vertically upward, and *z* in the direction of *AB*.

Resolve the force *F* into components in the *x* and *y* directions obtaining 0.6 *F* in the horizontal direction and 0.8 *F* in the negative verticaldirection. The 0.6 *F* force creates strain energy in the form of bending in *AB* and *OA*, and tension in *OA*. The 0.8 *F* forcecreates strain energy in the form of bending in *AB* and *OA*, and torsion in *OA*. Use the dummy variable to originate at the end where the loads are applied on each segment,

0.6 *F*: *AB* 

*OA* 



0.8*F*: *AB* 

*OA* 



Once the derivatives are taken the value of *F* = 66.72 N can be substituted in. The deflection of *B* in the direction of *F* is\*



\*Note.Be careful, this is not the actual deflection of point *B*. For this, fictitious forces must be placed on *B* in the *x*, *y*, and *z* directions. Determine the energy due to each, take derivatives, and then substitute the values of *Fx* = 40 N, *Fy* = −53.4 N, and *Fz* = 0. This can be done separately and then added by vector addition. The actual deflections of *B* are found to be

***δB*** = 2.11 **i**− 7.26 **j**− 0.2 **k** mm

From this, the deflection of *B*in the direction of *F* is



which agrees with our result.

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**4-78** Strain energy. *AB*: Bending and torsion, *BC*: Bending and torsion, *CD*: Bending.

*IAB* = *π* (0.02544)/64 = 2.04(10-8 )m4, *JAB* = 2 *IAB* = 4.08(10-8 )m4, *IBC*= 0.00635(0.03813)/12 = 2.92(10-8 ) m4, *ICD* = *π* (0.019054)/64 = 6.46(10-9 )m4.

For the torsion of bar *BC*, Eq. (3-41) is in the form of *θ =TL/*(*JG*), where the equivalent of *J* is *J*eq = *βbc* 3. With *b/c* = 0.0381/0.00635 = 6,*JBC* = *βbc* 3 = 0.299(0.0381)0.006353 = 2.92 (10−9) m4.

Use the dummy variable to originate at the end where the loads are applied on each segment,

*AB*: Bending 

Torsion 

*BC*: Bending 

Torsion 

*CD*: Bending 



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**4-79** *AAB* = *π* (0.02542)/4 = 0.00051 m2, *IAB* = *π* (0.02544)/64 = 2.04(10-8 )m4, *IBC* = 0.0381 (0.006353)/12 = 8.13 (10−10) m4, *ACD* = *π* (0.0192)/4 = .00029 m2, *IAB* = *π* (0.0194)/64 = 6.46(10-9 )m4. For (*δD* )*x* let

*F* = *Fx* = − 667.2 N and *Fz* = − 444.8 N. Use the dummy variable to originate at the end where the loads are applied on each segment,

*CD*: 



*BC*: 



*AB*: 





Substituting *F* = *Fx* = − 667.2 N and *Fz* = − 444.8 N gives



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**4-80** *IOA* = *IBC* = *π* (0.03814)/64 = 1.03(10-7 )m4, *JOA* = *JBC* = 2 *IOA* = 2.07(10-7 )m4, *IAB* = *π* (0.02544)/64 = 2.04(10-8 )m4,*JAB* = 2 *IAB* = 4.09(10-8 )m4, *ICD* = *π* (0.019054)/64 = 6.46(10-9 )m4

Let*Fy = F,* and use the dummy variable to originate at the end where the loads are applied on each segment,

*OC*: ****

*DC*: 

****

The terms involving the torsion and bending moments in *OC* must be split up because of the changing second-area moments.



For the simplified shaft *OC*,



Simplified is 19.37/16.12 = 1.20 times greater*Ans.*

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**4-81** Place a fictitious force *Q* pointing downwards at point *B.* The reaction at *C* is

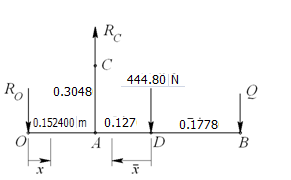
*RC* = *Q* +(0.1524/0.4572)444.8 = *Q* + 148.25

This is the axial force in member *BC*. Isolating the beam, we find that the moment is not a function of *Q*, and thus does not contribute to the strain energy. Thus, only energy in the member *BC* needs to be considered. Let the axial force in *BC* be *F*, where





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**4-82** *IOB*= 0.00635(0.05083)/12 = 6.94(10-8 )m4

*AAC*=*π* (0.01272)/4 = 0.127(10-3 )m2

Σ*MO*= 0 =0.1524 *RC*− 0.2794(444.8) − 0.4572 *Q*

*RC* = 3*Q* + 815.32

Σ*MA*= 0 =0.1524 *RO*− 0.127(444.8) − 0.3048 *Q* ⇒ *RO* = 2*Q* + 370.52

Bending in *OB*.

*BD*: Bending in *BD* is only due to *Q* which when set to zero after differentiation gives no contribution.

*AD*: Using the variable as shown in the figure above



*OA*: Using the variable *x*as shown in the figure above

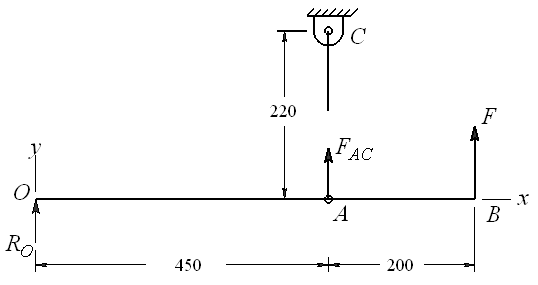


Axial in *AC*:





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**4-83** Table A-5, *EA* = 71.7 GPa, *ES* = 207 GPa, *EI* = 71.7(109)0.012(0.053)/12 = 8962.5 N۰m2

*F* = − 4000 lbf

Σ*MO* = 0 = 450 *FAC* + 650*F*⇒*FAC* = − 1.444 *F*



Σ*MA* = 0 = – 450 *RO* + 200*F*⇒*RO* = 0.4444 *F*

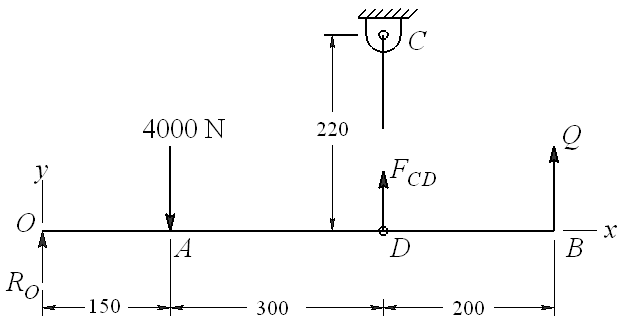






*yB* = (*yB*)1 + (*yB*)2 = − 0.3135 − 3.867 = − 4.18 mm Ans.

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**4-84** Table A-5, *EA* = 71.7 GPa, *ES* = 207 GPa, *EI* = 71.7(109)0.012(0.053)/12 = 8962.5 N۰m2

Σ*MO* = 0 = 450 *FCD* + 650*Q* – 150(4000) ⇒*FCD* = 1333 − 1.444 *Q*



Σ*Fy* = 0 = *RO* – 4000 + 1333 – 1.444 *Q* + *Q* ⇒*RO* = 2667 + 0.444 *Q*

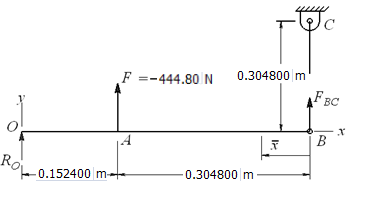






*yB* = (*yB*)1 + (*yB*)2 = − 0.0724 + 0.8922 = 0.820 mm *Ans.*

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**4-85** Table A-5, *EA* = 71.7 GPa, *ES* = 207 GPa, *EI* = 71.7(109)0.00635(0.050803)/12 = 4.97 (103) N۰m3

Σ*MO* = 0 = 0.4572 *FBC* + 0.1524*F*⇒*FBC* = −*0.33F*



Σ*Fy* = 0 = *RO* +*F* + *FBC* ⇒ *RO* = − 0.66 *F*

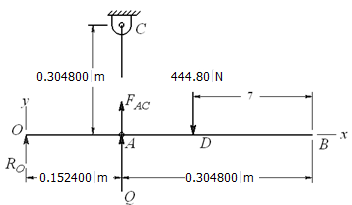






*yA* = (*yA*)1 + (*yA*)2 = − 0.1 (10−5) − 0.141 (10−3) = − 1.41 (10−4) m *Ans.*

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**4-86** Table A-5, *EA* = 71.7 GPa, *ES* = 207 GPa

Σ*MO* = 0 =0.1524(*FAC* +*Q*) − 0.2794(444.8)

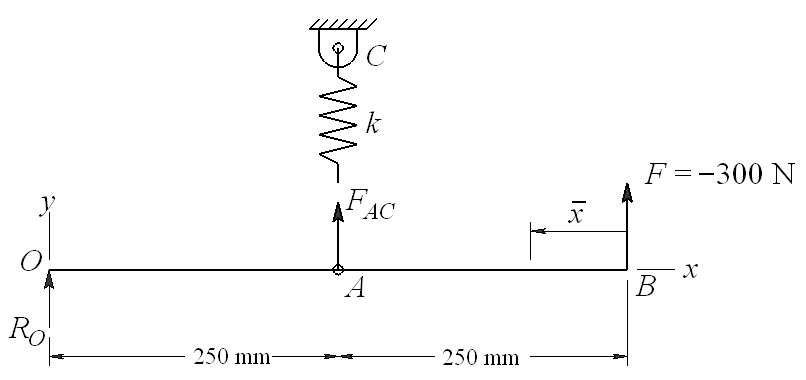
*FAC* = 815.32 −*Q* 



Treating beam *OADB* as a simply-supported beam pinned at *O* and *A* we see that the force *Q* does not induce any bending. Thus,

*yA* = (*yA*)1 = − 0.95 (10−5) m *Ans.*

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**4-87** Table A-5, *EA* = 71.7 GPa,

*k* = *P* / *δ* = 10 (103)/(10−3)

= 10(106) N/m,

*EI* = 71.7 (109)0.005(0.033)/12

= 806.6 N۰m2

**(a)***OA*: *RO* = *F*

*M* = *ROx* = *Fx*



**(b)***AB*: 

Since *AB* is the same length as *OA*, the integration is identical to part (a). Thus

(*yB*)2 = − 1.937 mm *Ans.*

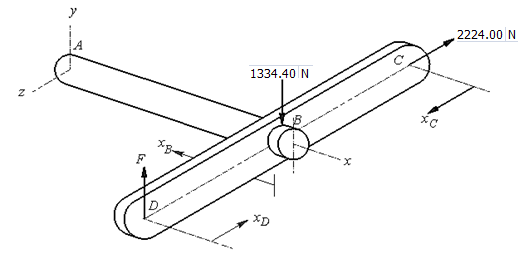
**(c)***AC*: Eq. (4-15): Σ*MO* = 0 = 250 *FAC* + 500 *F*⇒*FAC* = − 2*F*



**(d)** *yB* = (*yB*)1 + (*yB*)2 + (*yB*)3 = 2(− 1.937) − 0.12 = − 3.994 mm *Ans.*

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**4-88** Table A-5, *E* = 207 GPa, *G* = 79.2 GPa.(*EI*)*AB* = 207(109)(π/64)0.02544 = 4225 N۰m2, (*JG*)*AB* = (π/32)0.02544 (79.3)109 = 3240 N۰m2, (*EI*)*BD* = 207(109) 0.00635 (0.031753)/12 = 3500 N۰m2. *F* = 889.6 N.



**(a)***AB* at *xB*: 





*Tx*= 0.1016*F*  

(*yD*)1 = (*yD*)*i* + (*yD*)*ii* = − 0.124 (10−3) +0.432(10−3) = 0.307(10−3) m

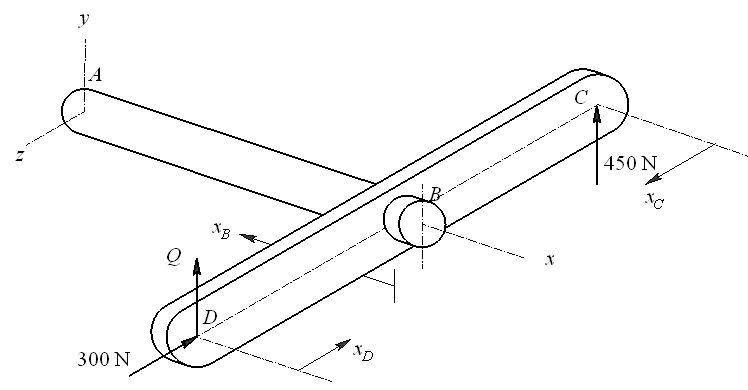
**(b)***BC* at *xC* has no *F*. Therefore, (*yD*)2 = 0 *Ans.*

**(c)***BD* at *xD*: 

**(d)***yD*= (*yD*)1 + (*yD*)2 + (*yD*)3 = 0.307(10−3) + 0 + 8.87(10−5) = 39.6(10−5) m *Ans.*

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**4-89** Table A-5, *E* = 207 GPa, *G* = 79.3 GPa, (*EI*)*AB* = 207 (109)(π/64) 0.0254 = 3.969 (103) m4,

 (*JG*)*AB* = (π/32) 0.0254 (79.3)109 = 3.041 (103) m4

**(a)***AB*: Bending:



No contribution from *Q*.





Torsion: 



(*yD*)1 = (*yD*)*i* + (*yD*)*ii* = 0.1275 − 0.2775 = − 0.150 mm *Ans*.

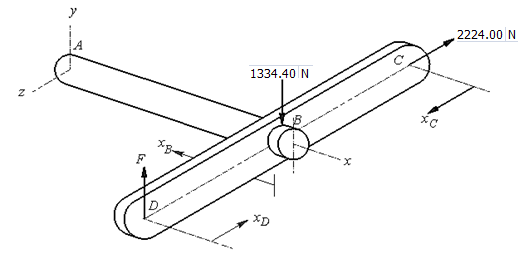
**(b)***BC*: Break at *xC* has no *Q* in it. Thus, (*yD*)2 = 0 *Ans*.

**(c)***BD*: Axial has no *Q*. Thus no contribution. Bending only has *Q* and since *Q* = 0, no contribution. Therefore, (*yD*)3 = 0 *Ans*.

**(d)** *yD* = (*yD*)1 +(*yD*)2 +(*yD*)3 =− 0.150 + 0 + 0 = − 0.150 mm *Ans*.

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**4-90** Table A-5, *E* = 207 GPa, *G* = 79.3 GPa. (*EI*)*AB* = 207 (109) (π/64) 0.02544 = 4225 N۰m2.

**(a)***AB*: Break at *xB*: 



No contribution of *Q* to *Tx*, Mz. Thus,

(*zD*)1 = (*zD*)*i* = − 0.62(10-3 ) m *Ans*.

**(b)***BC*: Break *xC* shows no contributions from *Q*. Thus, (*zD*)2 = 0 *Ans*.

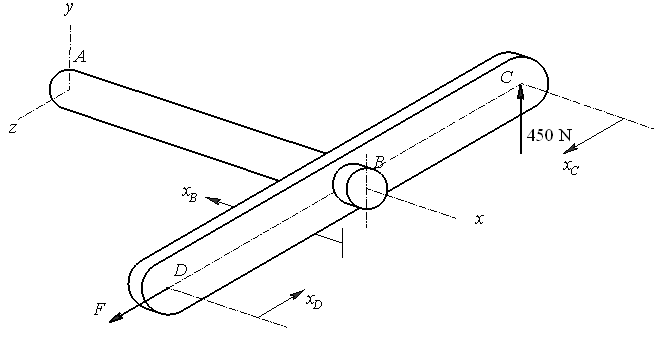
**(c)***BD*: Break *xD* shows no contributions from *Q* for *Mx*, *My*, or *Tz*. For axial,

 but setting *Q* = 0 gives nothing. Thus, (*zD*)3 = 0 *Ans*.

**(d)***zD* = (*zD*)1 = − 0.62(10-3 ) m *Ans*.

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**4-91** Table A-5, *E* = 207 GPa, (*EI*)*AB* = 207 (109)(π/64) 0.0254 = 3.969 (103) m4,



(*EA*)*BD* = 207(109) (π/4) 0.0252 = 101.6 (106) m2. *F* = − 300 N.

**(a)***AB*: Break at *xB*shows only contribution to *My* from *F*





**(b)***BC*: Break at *xB* shows no contribution from *F*. Thus, (*zD*)2 = 0 *Ans*.

**(c)***BD*: Break at *xD* shows only contribution to axial force from *F*

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**(d)***zD* = − 0.0850 + 0 − 2.95 (10−4) = − 0.0853 mm *Ans*.

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**4-92** There is no bending in *AB*. Using the variable*θ*, rotating counterclockwise from *B*

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From Table 3-4, for a rectangular cross section

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From Eq. (4-33), the eccentricity is *e = R − rn* =40 − 39.92489 = 0.07511 mm

From Table A-5, *E* = 207(103) MPa, *G* = 79.3(103) MPa

From Table 4-1, *C* = 1.2

From Eq. (4-38)









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**4-93** Place a fictitious force *Q* pointing downwards at point *A*. Bending in *AB* is only due to *Q* which when set to zero after differentiation gives no contribution. For section *BC* use the variable*θ*, rotating counterclockwise from *B*







But after differentiation, we can set *Q* = 0. Thus,





From Table 3-4, for a rectangular cross section

****

From Eq. (4-33), the eccentricity is *e = R − rn* =40 − 39.92489 = 0.07511 mm

From Table A-5, *E* = 207(103) MPa, *G* = 79.3(103) MPa

From Table 4-1, *C* = 1.2

From Eq. (4-38),



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**4-94** *A* = 0.0762(0.05715) −0.05715(0.0381) = 2.18(10-3 )m2



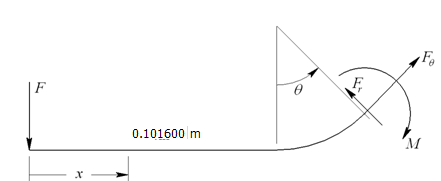
Section is equivalent to the “T” section of Table 3-4,





For the straightsection





For 0 ≤*x*≤ 0.1016 m



For*θ*≤*π* /2





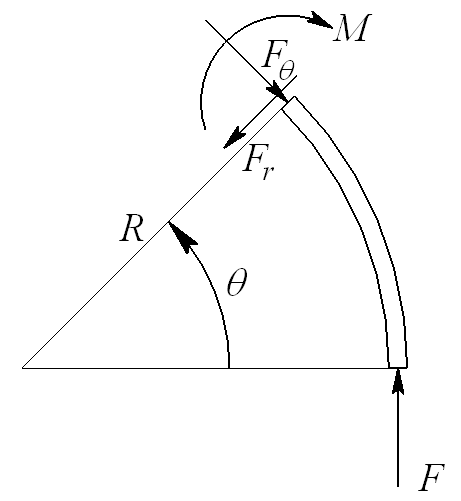
Use Eqs. (4-31) and (4-24) (with *C* = 1)for the straight part, and Eq. (4-38) for the curved part, integrating from 0 to π/2, and double the results



Substitute *I* = 1.12(10-6 )m4, *F* = 29802 N, *E* = 207 GPa, *G* = 79.3 GPa



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**4-95** Since *R/h* = 35/4.5 = 7.78 use Eq. (4-38), integrate from 0 to *π* , and double the results









From Eq. (4-38),



*A* = 4.5(3) = 13.5 mm2, *E* = 207 (103) N/mm2, *G* = 79.3 (103) N/mm2, and from Table 3-4,



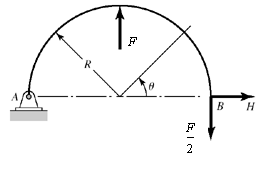
and *e* = *R − rn* = 35 −34.95173 = 0.04827 mm. Thus,



where *F* is in N. For *δ* = 1 mm,

Note: The first term in the equation for *δ* dominates and this is from the bending moment. Try Eq. (4-41), and compare the results.

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**4-96** *R*/*h* = 20 > 10 so Eq. (4-41) can be used to determine deflections. Consider the horizontal reaction to be applied at *B* and subject to the constraint 



By symmetry, we may consider only half of the wire form and use twice the strain energy

Eq. (4-41) then becomes,







The reaction at *A* is the same where *H* goes to the left. Substituting *H* into the moment equation we get,





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**4-97** The radius is sufficiently large compared to the wire diameter to use Eq. (4-41) for the curved beam portion. The shear and axial components will be negligible compared to bending.

Place a fictitious force *Q* pointing to the left at point *A*.



Note that the strain energy in the straight portion is zero since there is no real force in that section.

From Eq. (4-41), 

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**4-98** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

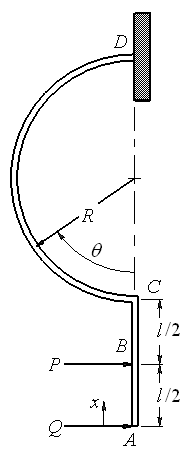
Straight portion: 

Curved portion: 

From Eq. (4-41) with the addition of the bending strain energy in the straight portion of the wire,



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-99** *R* = 0.0635 m, *d* = 0.003175 m, *l* = 0.0508 m, *P* = 4.45 N, *E* = 207 GPa.

*EI* = 207 (109) (π/64) 0.0031754 = 1.032 N۰m2.

Member *ABC*: 

Since *Q* = 0, there is no contribution.

****

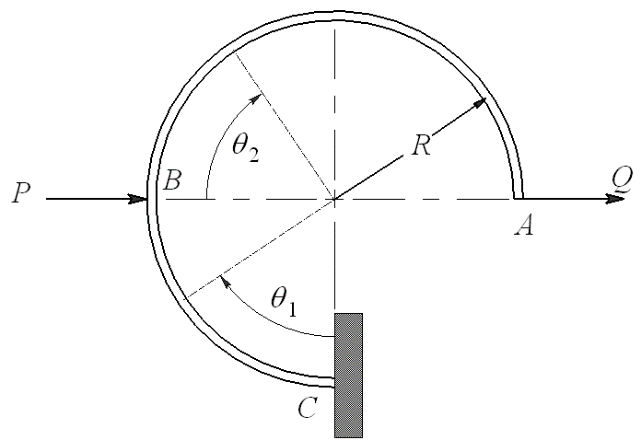


Member *CD*:



**** *δA* = (*δA*)1 +(*δA*)2 =  +  = 10.53(10-3 ) m *Ans*.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-100** *E* = 207 GPa, *EI* = 207 (109) (π/64) (0.0.003175)4 = 1.032 N۰m2.



No contribution in *θ*2 range since *Q* = 0. Thus



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-101** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a fictitious force, *Q*, at *A* vertically downward. The only load in the straight section is the axial force, *Q*. Since this will be zero, there is no contribution.

In the curved section



From Eq. (4-41)



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-102** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a fictitious force, *Q*, at *A* vertically downward. The load in the straight section is the axial force, *Q*, whereas the bending moment is only a function of *P* and is not a function of *Q*. When setting *Q* = 0, there is no axial or bending contribution.

In the curved section



From Eq. (4-41)

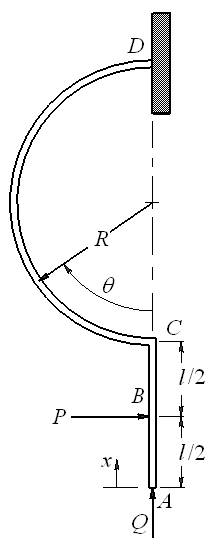


Since the deflection is negative, *δ* is in the opposite direction of *Q*. Thus the deflection is



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

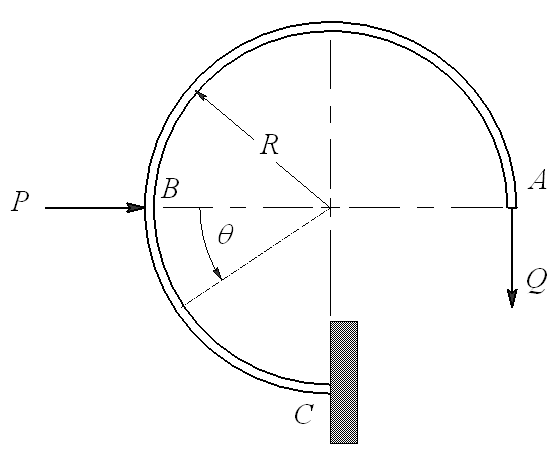
**4-103** *EI* =207(109) (π/64)(0.003175)4 = 1.032 N۰m2.

 *AB*: Only *Q* and since *Q* = 0, no contribution to (*δA*)*V*.

*CD*:





\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-104** *EI* =207(109) (π/64)(0.003175)4 = 1.032 N۰m2.

*AB*: Only *Q*. Since *Q* = 0, no contribution.

*BC*:





\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-105** Consider the force of the mass to be *F*, where *F* = 9.81(1) = 9.81 N. The load in *AB* is tension

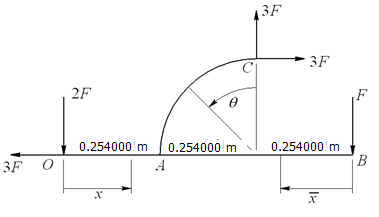


For the curved section, the radius is sufficiently large to use Eq. (4-41). There is no bending in section *DE*. For section *BCD*, let *θ* be counterclockwise originating at *D*  
 

Using Eqs. (4-29) and (4-41)



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



**4-106** *AOA* = 2(0.00635) = 0.00032 m2,

*IOAB* = 0.00635(0.05083)/12 = 6.94(10-8 )m4,

*IAC* = *π* (0.01274)/64 = 1.28 (10-9) m4

Applying a force *F* at point *B*, using

statics (*AC* is a two-force member),

the reaction forces at *O* and *C*are as

shown.

*OA*: Axial 

Bending 

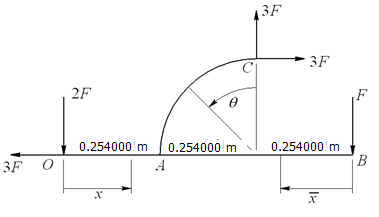
*AB*: Bending 

*AC*: Isolating the upper curved section





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**4-107** *AOA* = 2(0.00635) = 0.00032 m2,

*IOAB* = 0.00635(0.05083)/12

= 6.94(10-8 )m4,

*IAC* = *π* (0.01274)/64 = 1.28 (10-9) m4

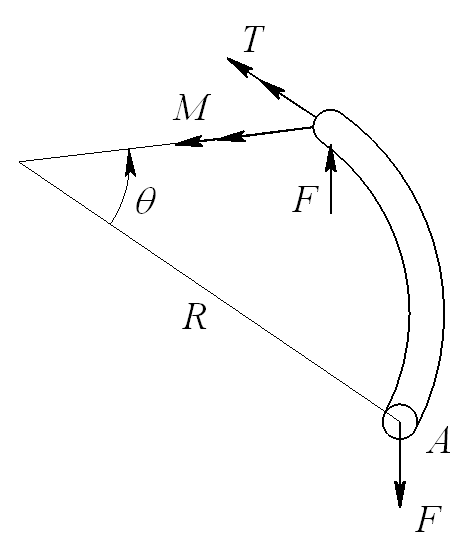
Applying a vertical fictitious force, *Q*, at *A*,from statics the reactions are as shown. Thefictitious force is transmitted through section*OA*and member *AC*.

*OA*: 

*AC*:



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**4-108** *I* = *π* (64)/64 = 63.62 mm4

0 ≤*θ*≤*π* / 2



According to Castigliano’s theorem, a positive∂*U/*∂*F* will yield a deflection of *A* in the negative *y* direction. Thus the deflection in the positive *y* direction is



Integrating and substituting 



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-109** The force applied to the copper and steel wire assembly is

 (1)

Since the deflections are equal, 





Yields,*Fc* = 1.60 *Fs*. Substituting this into Eq. (1) gives









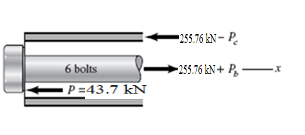
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**4-110** **(a)**Bolt stress 

Total bolt force 

Cylinder stress 

**(b)**Force from pressure

Σ*Fx* = 0

*Pb + Pc* = 43.7 kN (1)

Since 



*Pc* = 3.5 *Pb* (2)

Substituting this into Eq. (1)

*Pb +* 3.5 *Pb* = 4.5 *Pb* = 43.7 ⇒*Pb*= 9.705 kN. From Eq. (2), *Pc*= 33.97 kN

Using the results of (**a**) above, the total bolt and cylinder stresses are





\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-111** *Tc + Ts = T* (1)

*θc* = *θs*⇒

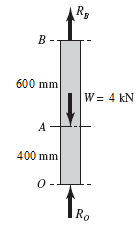
Substitute this into Eq. (1)



The percentage of the total torque carried by the shell is



\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-112** *RO* + *RB* = *W* (1)

*δOA* = *δAB*





Substitute this unto Eq. (1)



From Eq. (2) 



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**4-113** See figure in Prob. 4-112 solution.

Procedure 1:

1. Let *RB* be the redundant reaction.

2. Statics. *RO* + *RB* = 4 000 N ⇒*RO* = 4 000 −*RB* (1)

3. Deflection of point *B*. 

4. From Eq. (2), *AE* cancels and *RB* = 1 600 N *Ans.*

and from Eq. (1), *RO* = 4 000 − 1 600 = 2 400 N *Ans.*



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**4-114** (**a**) Without the right-hand wall, the deflection of point *C* would be



(**b**) Let *RC* be the reaction of the wall at *C* acting to the left (←). Thus, the deflection of point *C* is now





Σ*Fx* = (13.344+8.896) +*RA*− 10.74 = 0 ⇒*RA* = 11.5 kN ←*Ans*.

Deflection. *AB* is 11.5 kN in tension. Thus



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**4-115** Since*θOA* = *θAB*,



Statics. *TOA* + *TAB* = 22.6 (2)

Substitute Eq. (1) into Eq. (2), 

From Eq. (1) 







\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4-116** Since *θOA* = *θAB*,



Statics. *TOA* + *TAB* = 22.6 Nm (2)

Substitute Eq. (1) into Eq. (2),



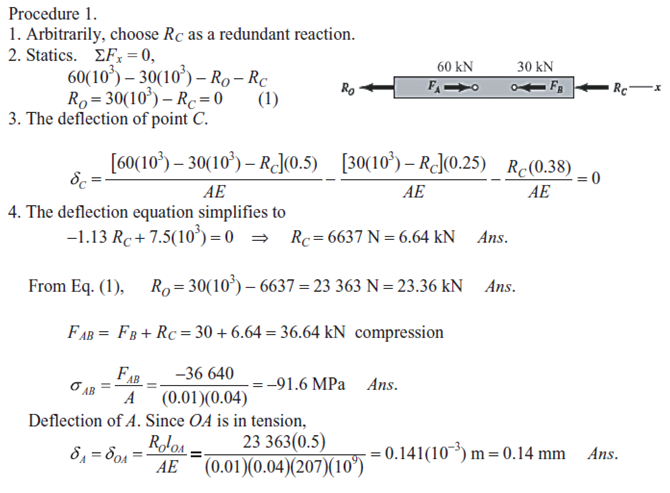
From Eq. (1) 



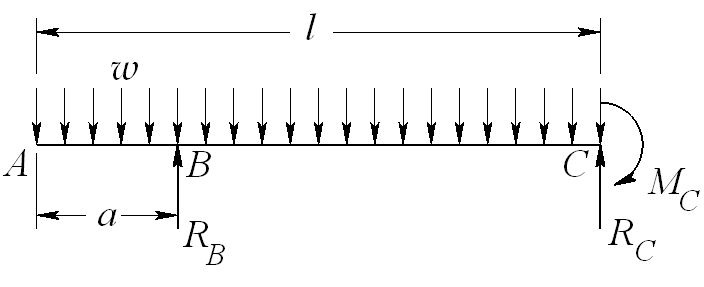




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**4-117** 

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**4-118** Procedure 1

1. Choose *RB* as redundant reaction.

2. Statics. *RC* = *wl*−*RB* (1)



3. Deflection equation for point *B*. Superposition of beams 2 and 3 of Table A-9,



4. Solving for *RB*.



Substituting this into Eqs. (1) and (2) gives





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**4-119** See figure in Prob. 4-118 solution.

Procedure 1

1. Choose *RB* as redundant reaction.

2. Statics. *RC* = *wl*−*RB* (1)



3. Deflection equation for point *B*. Let the variable *x* start at point *A* and to the right. Using singularity functions, the bending moment as a function of *x* is





or,



Solving for *RB* gives

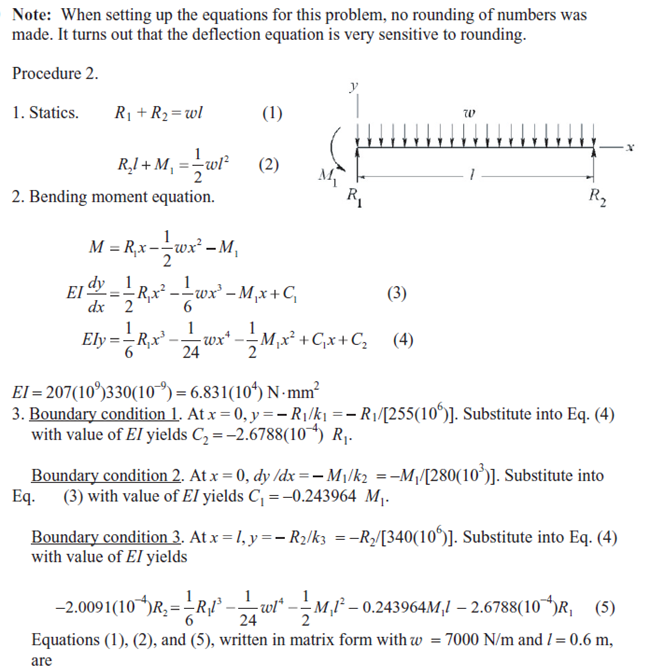
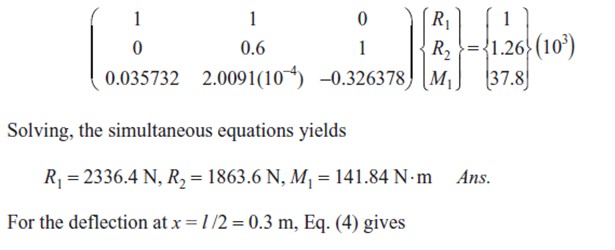


From Eqs. (1) and (2)

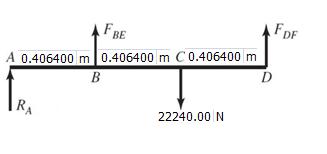




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**4-120** 

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**4-121** Cable area, ****

Procedure 2

1. Statics. *RA + FBE + FDF* = 22240 (1)

3 *FDF + FBE* = 44480 (2)

2. Bending moment equation.





3. B.C. 1: At  *x* = 0, *y* = 0 ⇒*C*2 = 0

B.C. 2: At *x* = 0.4064 m,



Substituting into Eq. (4) and evaluating at *x* = 0.4064 m, I =5(10-7) m4 

Simplifying gives 0.01118 *RA* + 3.81(10-3 )*FBE* + 0.4064 *C*1 = 0 (5)

B.C. 2: At *x* = 1.2192 m,



Substituting into Eq. (4) and evaluating at *x* = 1.2192 m,



Simplifying gives 0.302 *RA* + 0.089 *FBE* + 3.81(10-3 )*FDF* + 1.2192 *C*1 = 248.8 (6)

Equations (1), (2), (5) and (6) in matrix form are



Solve simultaneously or use software. The results are

*RA* = − 4317 N, *FBE* = 17596 N, *FDF* = 8963 N, and *C*1 = − 45.97 N⋅m2.



*EI* = 207(109)(5(10-7)) = 1.03(105 ) N⋅m2



*B*: *x* = 0.4064 m, 

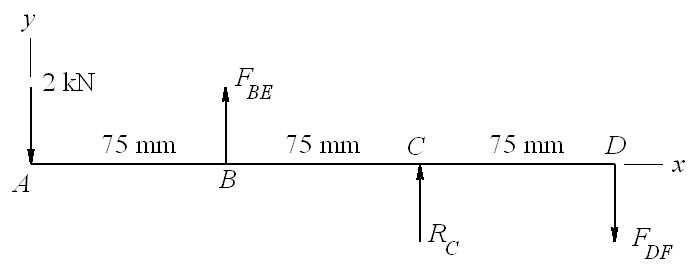
*C*: *x* = 0.8128 m,



*D*: *x* = 1.2192 m,



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**4-122** Beam:*EI* = 207(103)21(103)

= 4.347(109) N⋅mm2.

Rods: *A* = (*π* /4)82 = 50.27 mm2.

Procedure 2

1. Statics.

*RC* + *FBE*−*FDF* = 2 000 (1)

*RC* + 2*FBE* = 6 000 (2)

2. Bending moment equation.

*M* = − 2 000 *x* + *FBE*〈*x*− 75 〉1 + *RC*〈*x*− 150 〉1



3. B.C 1. At *x* = 75 mm,



Substituting into Eq. (4) at *x* = 75 mm,



Simplifying gives



B.C 2. At *x* = 150 mm, *y* = 0. From Eq. (4),



or,



B.C 3. At *x* = 225 mm,



Substituting into Eq. (4) at *x* = 225 mm,



Simplifying gives



Equations (1), (2), (5), (6), and (7) in matrix form are



Solve simultaneously or use software. The results are

*RC* = − 2378 N, *FBE* = 4189 N, *FDF* = − 189.2 N *Ans*.

and *C*1 = 1.036 (107) N⋅mm2, *C*2 = − 7.243 (108) N⋅mm3.

The bolt stresses are *σBE* = 4189/50.27 = 83.3 MPa, *σDF* = − 189/50.27= − 3.8 MPa *Ans*.

The deflections are

From Eq. (4) 

For points *B* and *D* use the axial deflection equations\*.





\*Note. The terms in Eq. (4) are quite large, and due to rounding are not very accurate for calculating the very small deflections, especially for point *D*.

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**4-123** Everything in Ex. 4-15 is the same except *kB* = 40 MN/m.

*yB* = −*FB* / *kB* = −*FB* / 40 (106) = − 2.5 (10−8) *FB*

Equation (7) is replaced by



With *EI* = 1.25 (104) N۰m2, the equation reduces to

− 1.3333 (10−3) *FA* + 3.125 (10−4) *FB* + 0.2 *C*1 + *C*2

The remaining equations come from Ex. 4-15



Solving for the unknowns gives



Equation (3): 

Which reduces to *y* = − 0.03133 *x*3 + 0.07312 〈*x*− 0.2〉3 +7.619 (10−3) *x*− 1.410 (10−3)

At *x* = 0, *y* = *yA* = − 1.410 (10−3) m = − 1.41 mm *Ans*.

At *x* = 0.2 m, *y*=*yB* = − 0.03133(0.2)3 +7.619(10−3)0.2 − 1.410(10−3)

= − 1.37 (10−4) m = − 0.137 mm *Ans*.

At *x* = 0.35 m,

*y*=*yC* = − 0.03133(0.35)3 + 0.07312 (0.35 − 0.2)3 +7.619(10−3)0.35 − 1.410(10−3)

= 1.60 (10−4) m = − 0.160 mm *Ans*.

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**4-124** (**a**) The cross section at *A* does not rotate. Thus, for a single quadrant we have

****

The bending moment at an angle *θ* to the *x* axis is



The rotation at *A* is



Thus, 

or,



Substituting this into the equation for *M* gives

 (1)

The maximum occurs at *B* where *θ = π* /2



(**b**) Assume *B* is supported on a knife edge. The deflection of point *D* is *∂ U/∂ F*. We will deal with the quarter-ring segment and multiply the results by 4. From Eq. (1)



Thus,



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**4-125**







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**4-126**  where *K = d / D.*

The radius of gyration, *k*, is given by



From Eq. (4-46)









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**4-127** **(a)** 

Using *nd* = 4, design for *F*cr = *ndFBO* = 4(1373) = 5492 N



In-plane:







Since use Johnson formula.

Try 25 mm x 12 mm,



This is significantly greater than the design load of 5492 N found earlier. Check out-of-plane.

Out-of-plane:



Since use Euler equation.



This is greater than the design load of 5492 N found earlier. It is also significantly less than the in-plane *P*cr found earlier, so the out-of-plane condition will dominate. Iterate the process to find the minimum *h* that gives *P*cr greater than the design load.

With*h* = 0.010, *P*cr= 4815 N (too small)

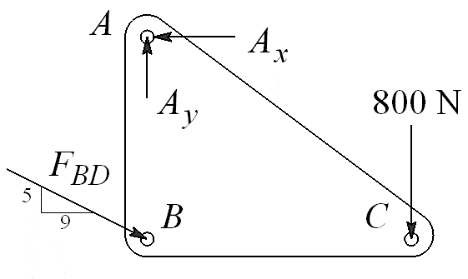
*h* = 0.011, *P*cr = 6409 N (acceptable)

Use 25 mm x 11 mm. If standard size is preferred, use 25 mm x 12 mm. *Ans.*

(**b**) 

*No,* bearing stress is not significant. *Ans.*

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**4-128** **(a)**



**(b)**

**(c)**

In plane, Table 4-2 ⇒ *C* = 1



*l* / *k* = 1.030/0.007 217 = 142.7.

Eq. (4-45): 

Since (*l* / *k*)1>*l* / *k*, use Johnson formula, Eq. (4-48)





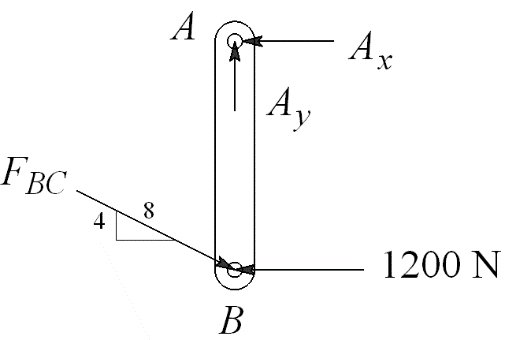
**(d)** Outof plane, Table 4-2, *C* = 1.2, *k* = 0.011/ = 3.175 (10−3) m,

*l* / *k* = 1.030/3.175 (10−3) = 324.4. Since *l* / *k* > (*l* / *k*)1, use the Euler formula, Eq. (4-44)





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**4-129** Out of plane bending, *C* =1.2.





*l* / *k* = 0.8/[2.887(10−3)] = 277.1

Eq. (4-45): 

Since *l* / *k* > (*l* / *k*)1, use the Euler formula, Eq. (4-44)





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**4-130** This is an open-ended design problem with no one distinct solution.

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**4-131** *F* = 10.34(106)(*π* /4)0.05082 = 20.96 k N. From Table A-20, *Sy* = 258.55 MPa

*P*cr = *nd F* = 2.5(20.96) = 52.4 kN

(**a**) Assume Euler with *C* = 1



Use *d* = 0.03175 m. The radius of gyration, *k* = ( *I / A*)1/2 = *d* /4 = 7.94(10-3 )m



Since 63.13 kN > 52.4 kN, *d* = 0.03175 m is satisfactory. *Ans*.

(**b**) so use *d* = 0.01905 m

*k* = 0.01905/4 = 4.76(10-3 )m



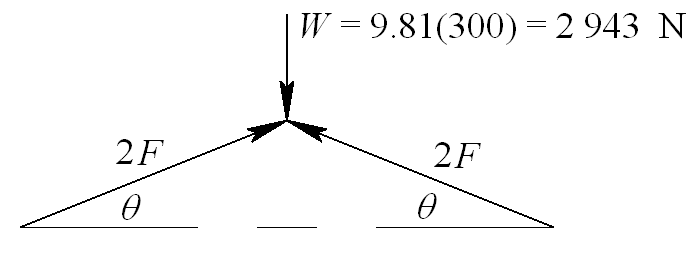


Use *d* = 0.75 in.

(**c**)



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**4-132** From Table A-20, *Sy* = 180 MPa

4*F* sin*θ* = 2 943

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In range of operation, *F* is maximum when *θ* = 15°



*P*cr = *ndF*max = 3.50 (2 843) = 9 951 N

*l* = 350 mm, *h* = 30 mm

Try *b* = 5mm. Out of plane, *k* = *b* / = 5/ = 1.443 mm



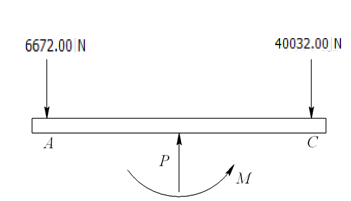
Too low. Try *b* = 6 mm. *k* = 6/ = 1.732 mm



O.K. Use 25 × 6 mm bars *Ans.* The factor of safety is



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**4-133** *P* = 6672 + 40032 = 46 704 N *Ans*.

Σ*MA* = 46 704 (0.1143/2) − 40032 (0.1143) +*M* = 0

*M* = 1906 N⋅m

*e* = *M / P* = 1906/46704 = 40.8(10-3 )m *Ans*.

From Table A-8, *A* = 1.39(10-3 )m2, and *I* = 8.57(10-7 )m4. The stresses are determined using Eq. (4-55)



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**4-134** This is a design problem which has no single distinct solution.

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**4-135** Loss of potential energy of weight = *W* (*h +δ* )

Increase inpotential energy of spring = 

*W* (*h +δ* )= 

or, . *W* = 133.44 N, *k* = 17513 N/m, *h* = 0.0508 m yields

*δ* 2− 0.0152 *δ*− 0.000774 = 0

Taking the positive root [see discussion after Eq. (*b*), Sec. 4-17]



*F*max = *k δ* max = 17512.7 (0.0364) = 638.73 N *Ans*.

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**4-136** The drop of weight *W*1 converts potential energy, *W*1*h*, to kinetic energy. Equating these provides the velocity of *W*1at impact with *W*2.

 (1)

Since the collision is inelastic, momentum is conserved. That is, (*m*1 + *m*2) *v*2 = *m*1 *v*1,where *v*2 is the velocity of *W*1 + *W*2 after impact. Thus

 (2)

The kinetic and potential energies of *W*1 + *W*2 are then converted to potential energy of the spring. Thus,



Substituting in Eq. (1) and rearranging results in

 (3)

Solving for the positive root [see discussion after Eq. (*b*), Sec. 4-17]

 (4)

*W*1 = 40 N, *W*2 = 400 N, *h* = 200 mm, *k* = 32 kN/m = 32 N/mm.



*F*max = *kδ* = 32(29.06) = 930 N *Ans*.

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**4-137** The initial potential energy of the *k*1 spring is *Vi* = . The movement of the weight *W* the distance *y* gives a final potential of *Vf* =. Equating the two energies give



Simplifying gives



This has two roots, *y* = 0,. Without damping the weight will vibrate between these two limits. The maximum displacement is thus *y* max = *Ans*.

With *W* = 22.24N, *k*1 = 1751N/m, *k*2 = 3502N/m, and *a* = 0.00635m



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