

# 14

## Spur and Helical Gears

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This chapter is devoted primarily to analysis and design of spur and helical gears to resist bending failure of the teeth as well as pitting failure of tooth surfaces. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. A surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength. The first two sections present a little of the history of the analyses from which current methodology developed.

The American Gear Manufacturers Association<sup>1</sup> (AGMA) has for many years been the responsible authority for the dissemination of knowledge pertaining to the design and analysis of gearing. The methods this organization presents are in general use in the United States when strength and wear are primary considerations. In view of this fact it is important that the AGMA approach to the subject be presented here.

The general AGMA approach requires a great many charts and graphs—too many for a single chapter in this book. We have omitted many of these here by choosing a single pressure angle and by using only full-depth teeth. This simplification reduces the complexity but does not prevent the development of a basic understanding of the approach. Furthermore, the simplification makes possible a better development of the fundamentals and hence should constitute an ideal introduction to the use of the general AGMA method.<sup>2</sup> Sections 14–1 and 14–2 are elementary and serve as an examination of the foundations of the AGMA method. Table 14–1 is largely AGMA nomenclature.

## 14–1 The Lewis Bending Equation

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in which the tooth form entered into the formulation. The equation, announced in 1892, still remains the basis for most gear design today.

To derive the basic Lewis equation, refer to Fig. 14–1*a*, which shows a rectangular cantilever beam of cross-sectional dimensions  $F$  and  $t$ , having a length  $l$  and a load  $W^t$ , uniformly distributed across the face width  $F$ . The section modulus  $I/c$  is  $Ft^2/6$ , and therefore the bending stress is

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2} \quad (a)$$

Gear designers denote the components of gear-tooth forces as  $W_t$ ,  $W_r$ ,  $W_a$  or  $W^t$ ,  $W^r$ ,  $W^a$  interchangeably. The latter notation leaves room for post-subscripts essential to free-body diagrams. For instance, for gears 2 and 3 in mesh,  $W_{23}^t$  is the transmitted

<sup>1</sup>1001 N. Fairfax Street, Suite 500, Alexandria, VA 22314-1587.

<sup>2</sup>The standards ANSI/AGMA 2001-D04 (revised AGMA 2001-C95) and ANSI/AGMA 2101-D04 (metric edition of ANSI/AGMA 2001-D04), *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, are used in this chapter. The use of American National Standards is completely voluntary; their existence does not in any respect preclude people, whether they have approved the standards or not, from manufacturing, marketing, purchasing, or using products, processes, or procedures not conforming to the standards.

The American National Standards Institute does not develop standards and will in no circumstances give an interpretation of any American National Standard. Requests for interpretation of these standards should be addressed to the American Gear Manufacturers Association. [Tables or other self-supporting sections may be quoted or extracted in their entirety. Credit line should read: “Extracted from ANSI/AGMA Standard 2001-D04 or 2101-D04 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*” with the permission of the publisher, American Gear Manufacturers Association, 1001 N. Fairfax Street, Suite 500, Alexandria, Virginia 22314-1587.] The foregoing is adapted in part from the ANSI foreword to these standards.

**Table 14-1**

 Symbols, Their Names,  
and Locations

Symbol*	Name	Where Found
$C_e$	Mesh alignment correction factor	Eq. (14-35)
$C_f(Z_R)$	Surface condition factor	Eq. (14-16)
$C_H(Z_W)$	Hardness-ratio factor	Eq. (14-18)
$C_{ma}$	Mesh alignment factor	Eq. (14-34)
$C_{mc}$	Load correction factor	Eq. (14-31)
$C_{mf}$	Face load-distribution factor	Eq. (14-30)
$C_p(Z_E)$	Elastic coefficient	Eq. (14-13)
$C_{pf}$	Pinion proportion factor	Eq. (14-32)
$C_{pm}$	Pinion proportion modifier	Eq. (14-33)
$d$	Pitch diameter	Ex. (14-1)
$d_P$	Pitch diameter, pinion	Eq. (14-22)
$d_G$	Pitch diameter, gear	Eq. (14-22)
$F(b)$	Net face width of narrowest member	Eq. (14-15)
$f_P$	Pinion surface finish	Fig. 14-13
$H$	Power	Fig. 14-17
$H_B$	Brinell hardness	Ex. 14-3
$H_{BG}$	Brinell hardness of gear	Sec. 14-12
$H_{BP}$	Brinell hardness of pinion	Sec. 14-12
$hp$	Horsepower	Ex. 14-1
$h_t$	Gear-tooth whole depth	Sec. 14-16
$I(Z_I)$	Geometry factor of pitting resistance	Eq. (14-16)
$J(Y_J)$	Geometry factor for bending strength	Eq. (14-15)
$K_B$	Rim-thickness factor	Eq. (14-40)
$K_f$	Fatigue stress-concentration factor	Eq. (14-9)
$K_m(K_H)$	Load-distribution factor	Eq. (14-30)
$K_o$	Overload factor	Eq. (14-15)
$K_R(Y_Z)$	Reliability factor	Eq. (14-17)
$K_s$	Size factor	Sec. 14-10
$K_T(Y_\theta)$	Temperature factor	Eq. (14-17)
$K_v$	Dynamic factor	Eq. (14-27)
$m$	Module	Eq. (14-15)
$m_B$	Backup ratio	Eq. (14-39)
$m_F$	Face-contact ratio	Eq. (14-19)
$m_G$	Gear ratio (never less than 1)	Eq. (14-22)
$m_N$	Load-sharing ratio	Eq. (14-21)
$m_t$	Transverse module	Eq. (14-15)
$N$	Number of stress cycles	Fig. 14-14
$N_G$	Number of teeth on gear	Eq. (14-22)
$N_P$	Number of teeth on pinion	Eq. (14-22)
$n$	Speed, in rev/min	Eq. (13-34)

(Continued)

**Table 14-1**

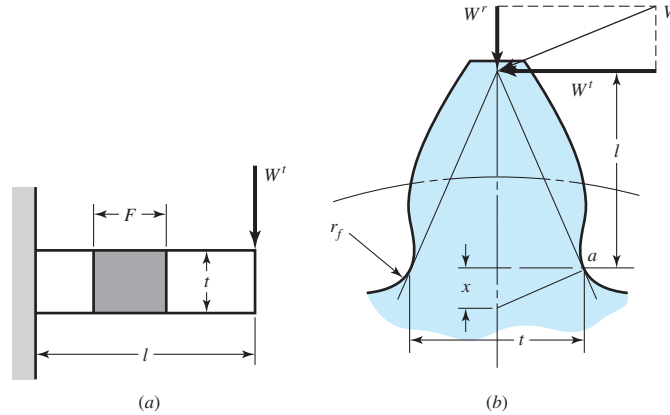
Symbols, Their Names,  
and Locations  
(Continued)

Symbol*	Name	Where Found
$n_p$	Pinion speed, in rev/min	Ex. 14-4
$P$	Diametral pitch	Eq. (14-2)
$P_d$	Transverse diametral pitch	Eq. (14-15)
$p_N$	Normal base pitch	Eq. (14-24)
$p_n$	Normal circular pitch	Eq. (14-24)
$p_x$	Axial pitch	Eq. (14-19)
$Q_v$	Quality number	Eq. (14-29)
$R$	Reliability	Eq. (14-38)
$R_a$	Root-mean-squared roughness	Fig. 14-13
$r_f$	Tooth fillet radius	Fig. 14-1
$r_G$	Pitch-circle radius, gear	In standard
$r_P$	Pitch-circle radius, pinion	In standard
$r_{bP}$	Pinion base-circle radius	Eq. (14-25)
$r_{bG}$	Gear base-circle radius	Eq. (14-25)
$S_C$	Buckingham surface endurance strength	Ex. 14-3
$S_c$	AGMA surface endurance strength	Eq. (14-18)
$S_t$	AGMA bending strength	Eq. (14-17)
$S$	Bearing span	Fig. 14-10
$S_I$	Pinion offset from center span	Fig. 14-10
$S_F$	Safety factor—bending	Eq. (14-41)
$S_H$	Safety factor—pitting	Eq. (14-42)
$W^t$ or $W_t$	Transmitted load	Fig. 14-1
$Y_N$	Stress-cycle factor for bending strength	Fig. 14-14
$Z_N$	Stress-cycle factor for pitting resistance	Fig. 14-15
$\beta$	Exponent	Eq. (14-44)
$\sigma$	Bending stress, AGMA	Eq. (14-15)
$\sigma_C$	Contact stress from Hertzian relationships	Eq. (14-14)
$\sigma_c$	Contact stress from AGMA relationships	Eq. (14-16)
$\sigma_{all}$	Allowable bending stress, AGMA	Eq. (14-17)
$\sigma_{c,all}$	Allowable contact stress, AGMA	Eq. (14-18)
$\phi$	Pressure angle	Eq. (14-12)
$\phi_n$	Normal pressure angle	Eq. (14-24)
$\phi_t$	Transverse pressure angle	Eq. (14-23)
$\psi$	Helix angle	Ex. 14-5

\*Where applicable, the alternate symbol for the metric standard is shown in parenthesis.

force of body 2 on body 3, and  $W_{32}^t$  is the transmitted force of body 3 on body 2. When working with double- or triple-reduction speed reducers, this notation is compact and essential to clear thinking. Since gear-force components rarely take exponents, this causes no complication. Pythagorean combinations, if necessary, can be treated with parentheses or avoided by expressing the relations trigonometrically.

| Figure 14-1



Referring now to Fig. 14-1*b*, we assume that the maximum stress in a gear tooth occurs at point *a*. By similar triangles, you can write

$$\frac{t/2}{x} = \frac{l}{t/2} \quad \text{or} \quad x = \frac{t^2}{4l} \quad \text{or} \quad l = \frac{t^2}{4x} \quad (b)$$

By rearranging Eq. (a),

$$\sigma = \frac{6W'l}{Ft^2} = \frac{W't}{F} \frac{1}{t^2/6l} = \frac{W't}{F} \frac{1}{t^2/4l} \frac{4}{6} \quad (c)$$

If we now substitute the value of *l* from Eq. (b) in Eq. (c) and multiply the numerator and denominator by the circular pitch *p*, we find

$$\sigma = \frac{W'tp}{F(\frac{2}{3})xp} \quad (d)$$

Letting  $y = 2x/(3p)$ , we have

$$\sigma = \frac{W't}{Fpy} \quad (14-1)$$

This completes the development of the original Lewis equation. The factor *y* is called the *Lewis form factor*, and it may be obtained by a graphical layout of the gear tooth or by digital computation.

In using this equation, most engineers prefer to employ the diametral pitch in determining the stresses. This is done by substituting  $p = \pi/P$  and  $y = \pi Y$  in Eq. (14-1). This gives

$$\sigma = \frac{W'P}{FY} \quad (14-2)$$

where

$$Y = \frac{2xP}{3} \quad (14-3)$$

The use of this equation for *Y* means that only the bending of the tooth is considered and that the compression due to the radial component of the force is neglected. Values of *Y* obtained from this equation are tabulated in Table 14-2.

**Table 14-2**

Values of the Lewis Form Factor  $Y$  (These Values Are for a Normal Pressure Angle of  $20^\circ$ , Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	$Y$	Number of Teeth	$Y$
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

The use of Eq. (14-3) also implies that the teeth do not share the load and that the greatest force is exerted at the tip of the tooth. But we have already learned that the contact ratio should be somewhat greater than unity, say about 1.5, to achieve a quality gearset. If, in fact, the gears are cut with sufficient accuracy, the tip-load condition is not the worst, because another pair of teeth will be in contact when this condition occurs. Examination of run-in teeth will show that the heaviest loads occur near the middle of the tooth. Therefore the maximum stress probably occurs while a single pair of teeth is carrying the full load, at a point where another pair of teeth is just on the verge of coming into contact.

### Dynamic Effects

When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present. One of the earliest efforts to account for an increase in the load due to velocity employed a number of gears of the same size, material, and strength. Several of these gears were tested to destruction by meshing and loading them at zero velocity. The remaining gears were tested to destruction at various pitch-line velocities. For example, if a pair of gears failed at 500 lbf tangential load at zero velocity and at 250 lbf at velocity  $V_1$ , then a *velocity factor*, designated  $K_v$ , of 2 was specified for the gears at velocity  $V_1$ . Then another, identical, pair of gears running at a pitch-line velocity  $V_1$  could be assumed to have a load equal to twice the tangential or transmitted load.

Note that the definition of dynamic factor  $K_v$  has been altered. AGMA standards ANSI/AGMA 2001-D04 and 2101-D04 contain this caution:

Dynamic factor  $K_v$  has been redefined as the reciprocal of that used in previous AGMA standards. It is now greater than 1.0. In earlier AGMA standards it was less than 1.0.

Care must be taken in referring to work done prior to this change in the standards.



In the nineteenth century, Carl G. Barth first expressed the velocity factor, and in terms of the current AGMA standards, they are represented as

$$K_v = \frac{600 + V}{600} \quad (\text{cast iron, cast profile}) \quad (14-4a)$$

$$K_v = \frac{1200 + V}{1200} \quad (\text{cut or milled profile}) \quad (14-4b)$$

where  $V$  is the pitch-line velocity in feet per minute. It is also quite probable, because of the date that the tests were made, that the tests were conducted on teeth having a cycloidal profile instead of an involute profile. Cycloidal teeth were in general use in the nineteenth century because they were easier to cast than involute teeth. Equation (14-4a) is called the *Barth equation*. The Barth equation is often modified into Eq. (14-4b), for cut or milled teeth. Later, AGMA added

$$K_v = \frac{50 + \sqrt{V}}{50} \quad (\text{hobbed or shaped profile}) \quad (14-5a)$$

$$K_v = \sqrt{\frac{78 + \sqrt{V}}{78}} \quad (\text{shaved or ground profile}) \quad (14-5b)$$

In SI units, Eqs. (14-4a) through (14-5b) become

$$K_v = \frac{3.05 + V}{3.05} \quad (\text{cast iron, cast profile}) \quad (14-6a)$$

$$K_v = \frac{6.1 + V}{6.1} \quad (\text{cut or milled profile}) \quad (14-6b)$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \quad (\text{hobbed or shaped profile}) \quad (14-6c)$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad (\text{shaved or ground profile}) \quad (14-6d)$$

where  $V$  is in meters per second (m/s).

Introducing the velocity factor into Eq. (14-2) gives

$$\sigma = \frac{K_v W^t P}{FY} \quad (14-7)$$

The metric version of this equation is

$$\sigma = \frac{K_v W^t}{FmY} \quad (14-8)$$

where the face width  $F$  and the module  $m$  are both in millimeters (mm). Expressing the tangential component of load  $W^t$  in newtons (N) then results in stress units of megapascals (MPa).

As a general rule, spur gears should have a face width  $F$  from 3 to 5 times the circular pitch  $p$ .

Equations (14-7) and (14-8) are important because they form the basis for the AGMA approach to the bending strength of gear teeth. They are in general use for

estimating the capacity of gear drives when life and reliability are not important considerations. The equations can be useful in obtaining a preliminary estimate of gear sizes needed for various applications.

**EXAMPLE 14-1**

A stock spur gear is available having a diametral pitch of 4 mm, a 44-mm face, 18 teeth, and a pressure angle of  $20^\circ$  with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of  $n_d = 3.5$  to rate the horsepower output of the gear corresponding to a speed of 25 rev/s and moderate applications.

**Solution**

The term *moderate applications* seems to imply that the gear can be rated by using the yield strength as a criterion of failure. From Table A-20, we find  $S_{ut} = 379$  MPa and  $S_y = 206$  MPa. A design factor of 3.5 means that the allowable bending stress is  $206/3.5 = 58.86$  MPa. The pitch diameter is  $Nm = 18(4) = 72$  mm, so the pitch-line velocity is

$$V = \pi dn = \pi(0.072)25 = 5.56487 \text{ m/s}$$

The velocity factor from Eq. (14-4b) is found to be

$$K_v = \frac{6.1 + V}{6.1} = \frac{6.1 + 5.56487}{6.1} = 1.92703$$

Table 14-2 gives the form factor as  $Y = 0.296$  for 16 teeth. We now arrange and substitute in Eq. (14-7) as follows:

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{0.004(0.004)0.039(58.9)10^6}{1.92703} = 1434.54233 \text{ N}$$

The power that can be transmitted is

**Answer**

$$hp = W^t V = 1434.54(5.65) = 8112.15 \text{ watt}$$

It is important to emphasize that this is a rough estimate, and that this approach must not be used for important applications. The example is intended to help you understand some of the fundamentals that will be involved in the AGMA approach.

**EXAMPLE 14-2**

Estimate the horsepower rating of the gear in the previous example based on obtaining an infinite life in bending.

**Solution**

The rotating-beam endurance limit is estimated from Eq. (6-8), p. 290,

$$S'_e = 0.5S_{ut} = 0.5(379 \text{ MPa}) = 189.5 \text{ MPa}$$

To obtain the surface finish Marin factor  $k_a$  we refer to Table 6-3, p. 298, for machined surface, finding  $a = 4.51$  and  $b = -0.265$ . Then Eq. (6-19), p. 295, gives the surface finish Marin factor  $k_a$  as

$$k_a = aS_{ut}^b = 4.51(189.5)^{-0.265} = 0.935$$



The next step is to estimate the size factor  $k_b$ . From Table 13–1, p. 688, the sum of the addendum and dedendum is

$$l = \frac{1}{P} + \frac{1.25}{P} = \frac{1}{0.25} + \frac{1.25}{0.25} = 9 \text{ mm}$$

The tooth thickness  $t$  in Fig. 14–1*b* is given in Sec. 14–1 [Eq. (b)] as  $t = (4lx)^{1/2}$  when  $x = 3Y/(2P)$  from Eq. (14–3). Therefore, since from Ex. 14–1  $Y = 0.296$  and  $P = 0.25$ ,

$$x = \frac{3Y}{2P} = \frac{3(0.296)}{2(0.25)} = 1.776 \text{ mm}$$

then

$$t = (4lx)^{1/2} = [4(9)(1.776)]^{1/2} = 7.996$$

We have recognized the tooth as a cantilever beam of rectangular cross section, so the equivalent rotating-beam diameter must be obtained from Eq. (6–25), p. 297:

$$d_e = 0.808(hb)^{1/2} = 0.808( Ft )^{1/2} = 0.808[44(7.996)]^{1/2} = 15.156 \text{ mm}$$

Then, Eq. (6–20), p. 296, gives  $k_b$  as

$$k_b = \left( \frac{d_e}{0.30} \right)^{-0.107} = \left( \frac{15.156}{7.62} \right)^{-0.107} = 0.929$$

The load factor  $k_c$  from Eq. (6–26), p. 298, is unity. With no information given concerning temperature and reliability we will set  $k_d = k_e = 1$ .

In general, a gear tooth is subjected only to one-way bending. Exceptions include idler gears and gears used in reversing mechanisms. We will account for one-way bending by establishing a miscellaneous-effects Marin factor  $k_f$ .

For one-way bending the steady and alternating stress components are  $\sigma_a = \sigma_m = \sigma/2$  where  $\sigma$  is the largest repeatedly applied bending stress as given in Eq. (14–7). If a material exhibited a Goodman failure locus,

$$\frac{S_a}{S'_e} + \frac{S_m}{S_{ut}} = 1$$

Since  $S_a$  and  $S_m$  are equal for one-way bending, we substitute  $S_a$  for  $S_m$  and solve the preceding equation for  $S_a$ , giving

$$S_a = \frac{S'_e S_{ut}}{S'_e + S_{ut}}$$

Now replace  $S_a$  with  $\sigma/2$ , and in the denominator replace  $S'_e$  with  $0.5S_{ut}$  to obtain

$$\sigma = \frac{2S'_e S_{ut}}{0.5S_{ut} + S_{ut}} = \frac{2S'_e}{0.5 + 1} = 1.33S'_e$$

Now  $k_f = \sigma/S'_e = 1.33S'_e/S'_e = 1.33$ . However, a Gerber fatigue locus gives mean values of

$$\frac{S_a}{S'_e} + \left( \frac{S_m}{S_{ut}} \right)^2 = 1$$

Setting  $S_a = S_m$  and solving the quadratic in  $S_a$  gives

$$S_a = \frac{S_{ut}^2}{2S'_e} \left( -1 + \sqrt{1 + \frac{4S_e'^2}{S_{ut}^2}} \right)$$

Setting  $S_a = \sigma/2$ ,  $S_{ut} = S'_e/0.5$  gives

$$\sigma = \frac{S'_e}{0.5^2} [-1 + \sqrt{1 + 4(0.5)^2}] = 1.66S'_e$$

and  $k_f = \sigma/S'_e = 1.66$ . Since a Gerber locus runs in and among fatigue data and Goodman does not, we will use  $k_f = 1.66$ . The Marin equation for the fully corrected endurance strength is

$$\begin{aligned} S_e &= k_a k_b k_c k_d k_e k_f S'_e \\ &= 0.935(0.929)(1)(1)(1)1.66(189.5) = 273.3 \text{ MPa} \end{aligned}$$

For stress, we will first determine the fatigue stress-concentration factor  $K_f$ . For a  $20^\circ$  full-depth tooth the radius of the root fillet is denoted  $r_f$ , where

$$r_f = \frac{0.300}{P} = \frac{0.300}{0.25} = 1.2 \text{ mm}$$

From Fig. A-15-6

$$\frac{r}{d} = \frac{r_f}{t} = \frac{1.2}{7.996} = 0.15$$

Since  $D/d = \infty$ , we approximate with  $D/d = 3$ , giving  $K_t = 1.68$ . From Fig. 6-20, p. 303,  $q = 0.62$ . From Eq. (6-32), p. 303,

$$K_f = 1 + (0.62)(1.68 - 1) = 1.42$$

For a design factor of  $n_d = 3.5$ , as used in Ex. 14-1, applied to the load or strength, the maximum bending stress is

$$\sigma_{\max} = K_f \sigma_{\text{all}} = \frac{S_e}{n_d}$$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{273.3}{1.42(3.5)} = 54.92 \text{ MPa}$$

The transmitted load  $W'$  is

$$W' = \frac{F Y \sigma_{\text{all}}}{K_v P} = \frac{44(0.296)54.92}{1.927(0.25)} = 1485 \text{ N}$$

and the power is, with  $V = 628 \text{ ft/min}$  from Ex. 14-1,

$$P = \frac{W' V}{33\,000} = \frac{1485(5.6549)}{33\,000} = 8396.4 \text{ Watt}$$

Again, it should be emphasized that these results should be accepted *only* as preliminary estimates to alert you to the nature of bending in gear teeth.

In Ex. 14–2 our resources (Fig. A–15–6) did not directly address stress concentration in gear teeth. A photoelastic investigation by Dolan and Broghamer reported in 1942 constitutes a primary source of information on stress concentration.<sup>3</sup> Mitchiner and Mabie<sup>4</sup> interpret the results in term of fatigue stress-concentration factor  $K_f$  as

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M \quad (14-9)$$

where  $H = 0.34 - 0.458\ 366\ 2\phi$

$L = 0.316 - 0.458\ 366\ 2\phi$

$M = 0.290 + 0.458\ 366\ 2\phi$

$$r = \frac{(b - r_f)^2}{(d/2) + b - r_f}$$

In these equations  $l$  and  $t$  are from the layout in Fig. 14–1,  $\phi$  is the pressure angle,  $r_f$  is the fillet radius,  $b$  is the dedendum, and  $d$  is the pitch diameter. It is left as an exercise for the reader to compare  $K_f$  from Eq. (14–9) with the results of using the approximation of Fig. A–15–6 in Ex. 14–2.

## 14–2 Surface Durability

In this section we are interested in the failure of the surfaces of gear teeth, which is generally called *wear*. *Pitting*, as explained in Sec. 6–16, is a surface fatigue failure due to many repetitions of high contact stresses. Other surface failures are *scoring*, which is a lubrication failure, and *abrasion*, which is wear due to the presence of foreign material.

To obtain an expression for the surface-contact stress, we shall employ the Hertz theory. In Eq. (3–74), p. 138, it was shown that the contact stress between two cylinders may be computed from the equation

$$p_{\max} = \frac{2F}{\pi bl} \quad (a)$$

where  $p_{\max}$  = largest surface pressure

$F$  = force pressing the two cylinders together

$l$  = length of cylinders

and half-width  $b$  is obtained from Eq. (3–73), p. 138, given by

$$b = \left[ \frac{2F}{\pi l} \frac{(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2}{1/d_1 + 1/d_2} \right]^{1/2} \quad (14-10)$$

where  $\nu_1$ ,  $\nu_2$ ,  $E_1$ , and  $E_2$  are the elastic constants and  $d_1$  and  $d_2$  are the diameters, respectively, of the two contacting cylinders.

To adapt these relations to the notation used in gearing, we replace  $F$  by  $W^t/\cos \phi$ ,  $d$  by  $2r$ , and  $l$  by the face width  $F$ . With these changes, we can substitute the value

<sup>3</sup>T. J. Dolan and E. I. Broghamer, *A Photoelastic Study of the Stresses in Gear Tooth Fillets*, Bulletin 335, Univ. Ill. Exp. Sta., March 1942, See also W. D. Pilkey and D. F. Pilkey, *Peterson's Stress-Concentration Factors*, 3rd ed., John Wiley & Sons, Hoboken, NJ, 2008, pp. 407–409, 434–437.

<sup>4</sup>R. G. Mitchiner and H. H. Mabie, "Determination of the Lewis Form Factor and the AGMA Geometry Factor J of External Spur Gear Teeth," *J. Mech. Des.*, Vol. 104, No. 1, Jan. 1982, pp. 148–158.

of  $b$  as given by Eq. (14–10) in Eq. (a). Replacing  $p_{\max}$  by  $\sigma_C$ , the *surface compressive stress (Hertzian stress)* is found from the equation

$$\sigma_C^2 = \frac{W^t}{\pi F \cos \phi} \frac{1/r_1 + 1/r_2}{(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2} \quad (14-11)$$

where  $r_1$  and  $r_2$  are the instantaneous values of the radii of curvature on the pinion- and gear-tooth profiles, respectively, at the point of contact. By accounting for load sharing in the value of  $W^t$  used, Eq. (14–11) can be solved for the Hertzian stress for any or all points from the beginning to the end of tooth contact. Of course, pure rolling exists only at the pitch point. Elsewhere the motion is a mixture of rolling and sliding. Equation (14–11) does not account for any sliding action in the evaluation of stress. We note that AGMA uses  $\mu$  for Poisson's ratio instead of  $\nu$  as is used here.

We have already noted that the first evidence of wear occurs near the pitch line. The radii of curvature of the tooth profiles at the pitch point are

$$r_1 = \frac{d_p \sin \phi}{2} \quad r_2 = \frac{d_G \sin \phi}{2} \quad (14-12)$$

where  $\phi$  is the pressure angle and  $d_p$  and  $d_G$  are the pitch diameters of the pinion and gear, respectively.

Note, in Eq. (14–11), that the denominator of the second group of terms contains four elastic constants, two for the pinion and two for the gear. As a simple means of combining and tabulating the results for various combinations of pinion and gear materials, AGMA defines an *elastic coefficient*  $C_p$  by the equation

$$C_p = \left[ \frac{1}{\pi \left( \frac{1 - \nu_p^2}{E_p} + \frac{1 - \nu_G^2}{E_G} \right)} \right]^{1/2} \quad (14-13)$$

With this simplification, and the addition of a velocity factor  $K_v$ , Eq. (14–11) can be written as

$$\sigma_C = -C_p \left[ \frac{K_v W^t}{F \cos \phi} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \quad (14-14)$$

where the sign is negative because  $\sigma_C$  is a compressive stress.

### EXAMPLE 14-3

The pinion of Examples 14–1 and 14–2 is to be mated with a 50-tooth gear manufactured of ASTM No. 50 cast iron. Using the tangential load of 1700 N, estimate the factor of safety of the drive based on the possibility of a surface fatigue failure.

#### Solution

From Table A–5 we find the elastic constants to be  $E_p = 207$  GPa,  $\nu_p = 0.292$ ,  $E_G = 100$  GPa,  $\nu_G = 0.211$ . We substitute these in Eq. (14–13) to get the elastic coefficient as

$$C_p = \left\{ \frac{1}{\pi \left[ \frac{1 - (0.292)^2}{207(10^9)} + \frac{1 - (0.211)^2}{100(10^9)} \right]} \right\}^{1/2} = 150\,927.3$$

From Example 14–1, the pinion pitch diameter is  $d_p = 48$  mm. The value for the gear is  $d_G = 50(3) = 150$  mm. Then Eq. (14–12) is used to obtain the radii of curvature at the pitch points. Thus

$$r_1 = \frac{48 \sin 20^\circ}{2} = 8.2 \text{ mm} \quad r_2 = \frac{150 \sin 20^\circ}{2} = 25.7 \text{ mm}$$

The face width is given as  $F = 38$  mm. Use  $K_v = 1.5$  from Example 14–1. Substituting all these values in Eq. (14–14) with  $\phi = 20^\circ$  gives the contact stress as

$$\sigma_C = -150\,927.3 \left[ \frac{1.5(1700)}{0.038 \cos 20^\circ} \left( \frac{1}{0.0082} + \frac{1}{0.0257} \right) \right]^{1/2} = -511.5 \text{ MPa}$$

The surface endurance strength of cast iron can be estimated from

$$S_C = 2.206H_B \text{ MPa}$$

for  $10^8$  cycles. Table A–22 gives  $H_B = 262$  for ASTM No. 50 cast iron. Therefore  $S_C = 2.206(262) = 578$  MPa. Contact stress is not linear with respect to the transmitted load [see Eq. (14–14)]. If the factor of safety is defined as the loss-of-function load divided by the imposed load, then the ratio of loads is the ratio of stresses squared. In other words,

$$n = \frac{\text{loss-of-function load}}{\text{imposed load}} = \frac{S_C^2}{\sigma_C^2} = \left( \frac{578}{511.5} \right)^2 = 1.28$$

One is free to define the factor of safety as  $S_C/\sigma_C$ . Awkwardness comes when one compares the factor of safety in bending fatigue with the factor of safety in surface fatigue for a particular gear. Suppose the factor of safety of this gear in bending fatigue is 1.20 and the factor of safety in surface fatigue is 1.28 as above. The threat, since 1.28 is greater than 1.20, is in bending fatigue since both numbers are based on load ratios. If the factor of safety in surface fatigue is based on  $S_C/\sigma_C = \sqrt{1.28} = 1.13$ , then 1.20 is greater than 1.13, but the threat is not from surface fatigue. The surface fatigue factor of safety can be defined either way. One way has the burden of requiring a squared number before numbers that instinctively seem comparable can be compared.

In addition to the dynamic factor  $K_v$  already introduced, there are transmitted load excursions, nonuniform distribution of the transmitted load over the tooth contact, and the influence of rim thickness on bending stress. Tabulated strength values can be means, ASTM minimums, or of unknown heritage. In surface fatigue there are no endurance limits. Endurance strengths have to be qualified as to corresponding cycle count, and the slope of the  $S$ - $N$  curve needs to be known. In bending fatigue there is a definite change in slope of the  $S$ - $N$  curve near  $10^6$  cycles, but some evidence indicates that an endurance limit does not exist. Gearing experience leads to cycle counts of  $10^{11}$  or more. Evidence of diminishing endurance strengths in bending have been included in AGMA methodology.

### 14–3 AGMA Stress Equations

Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). In AGMA terminology, these are called *stress numbers*, as contrasted with actual applied stresses, and are

designated by a lowercase letter  $s$  instead of the Greek lower case  $\sigma$  we have used in this book (and shall continue to use). The fundamental equations are

$$\sigma = \begin{cases} W^t K_o K_v K_s \frac{P_d K_m K_B}{F J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \frac{1}{b m_t} \frac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \quad (14-15)$$

where for U.S. customary units (SI units),

$W^t$  is the tangential transmitted load, lbf (N)

$K_o$  is the overload factor

$K_v$  is the dynamic factor

$K_s$  is the size factor

$P_d$  is the transverse diametral pitch

$F$  ( $b$ ) is the face width of the narrower member, in (mm)

$K_m$  ( $K_H$ ) is the load-distribution factor

$K_B$  is the rim-thickness factor

$J$  ( $Y_J$ ) is the geometry factor for bending strength (which includes root fillet stress-concentration factor  $K_f$ )

$(m_t)$  is the transverse metric module

Before you try to digest the meaning of all these terms in Eq. (14–15), view them as advice concerning items the designer should consider *whether he or she follows the voluntary standard or not*. These items include issues such as

- Transmitted load magnitude
- Overload
- Dynamic augmentation of transmitted load
- Size
- Geometry: pitch and face width
- Distribution of load across the teeth
- Rim support of the tooth
- Lewis form factor and root fillet stress concentration

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \frac{K_m C_f}{d_p F I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \frac{K_H Z_R}{d_{w1} b Z_I}} & \text{(SI units)} \end{cases} \quad (14-16)$$

where  $W^t$ ,  $K_o$ ,  $K_v$ ,  $K_s$ ,  $K_m$ ,  $F$ , and  $b$  are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

$C_p$  ( $Z_E$ ) is an elastic coefficient,  $\sqrt{\text{lbf/in}^2}$  ( $\sqrt{\text{N/mm}^2}$ )

$C_f$  ( $Z_R$ ) is the surface condition factor

$d_p$  ( $d_{w1}$ ) is the pitch diameter of the *pinion*, in (mm)

$I$  ( $Z_I$ ) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.



## 14-4 AGMA Strength Equations

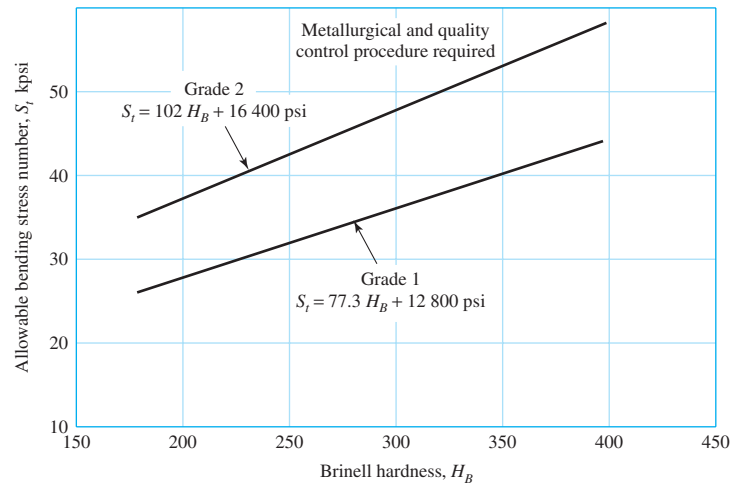
Instead of using the term *strength*, AGMA uses data termed *allowable stress numbers* and designates these by the symbols  $s_{at}$  and  $s_{ac}$ . It will be less confusing here if we continue the practice in this book of using the uppercase letter  $S$  to designate strength and the lowercase Greek letters  $\sigma$  and  $\tau$  for stress. To make it perfectly clear we shall use the term *gear strength* as a replacement for the phrase *allowable stress numbers* as used by AGMA.

Following this convention, values for *gear bending strength*, designated here as  $S_t$ , are to be found in Figs. 14-2, 14-3, and 14-4, and in Tables 14-3 and 14-4. Since gear strengths are not identified with other strengths such as  $S_{ut}$ ,  $S_e$ , or  $S_y$  as used elsewhere in this book, their use should be restricted to gear problems.

In this approach the strengths are modified by various factors that produce limiting values of the bending stress and the contact stress.

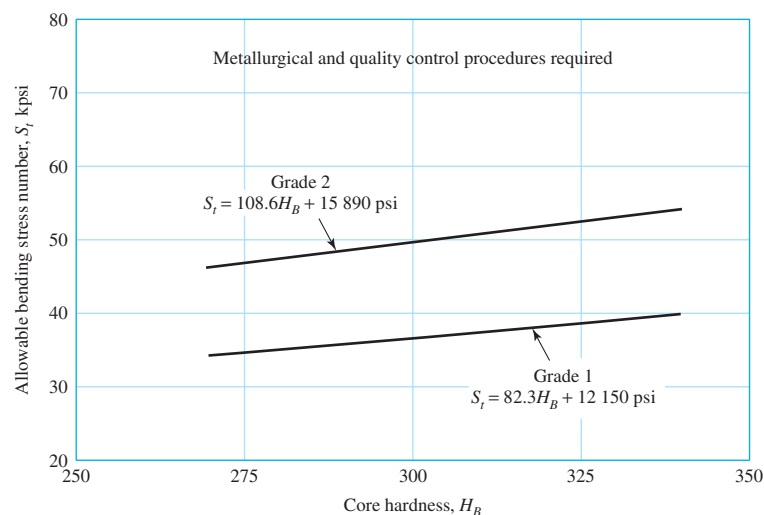
**Figure 14-2**

Allowable bending stress number for through-hardened steels,  $S_t$ . The SI equations are:  $S_t = 0.533H_B + 88.3$  MPa, grade 1, and  $S_t = 0.703H_B + 113$  MPa, grade 2. (Source: ANSI/AGMA 2001-D04 and 2101-D04.)



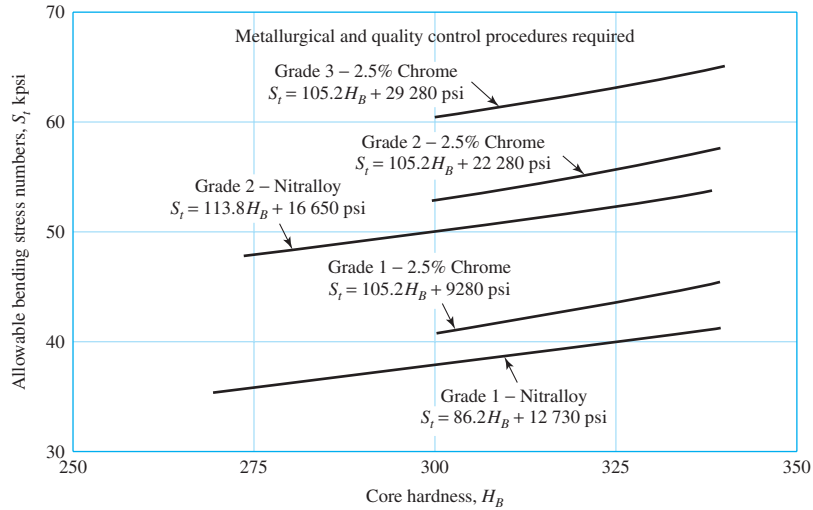
**Figure 14-3**

Allowable bending stress number for nitrided through-hardened steel gears (i.e., AISI 4140, 4340),  $S_t$ . The SI equations are:  $S_t = 0.568H_B + 83.8$  MPa, grade 1, and  $S_t = 0.749H_B + 110$  MPa, grade 2. (Source: ANSI/AGMA 2001-D04 and 2101-D04.)



**Figure 14-4**

Allowable bending stress numbers for nitriding steel gears,  $S_t$ . The SI equations are:  
 $S_t = 0.594H_B + 87.76$  MPa  
 Nitralloy grade 1  
 $S_t = 0.784H_B + 114.81$  MPa  
 Nitralloy grade 2  
 $S_t = 0.7255H_B + 63.89$  MPa  
 2.5% chrome, grade 1  
 $S_t = 0.7255H_B + 153.63$  MPa  
 2.5% chrome, grade 2  
 $S_t = 0.7255H_B + 201.91$  MPa  
 2.5% chrome, grade 3  
 (Source: ANSI/AGMA 2001-D04, 2101-D04.)



**Table 14-3**

Repeatedly Applied Bending Strength  $S_t$  at  $10^7$  Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness <sup>1</sup>	Allowable Bending Stress Number $S_t$ , <sup>2</sup>		
			Grade 1	Grade 2	Grade 3
Steel <sup>3</sup>	Through-hardened	See Fig. 14-2	See Fig. 14-2	See Fig. 14-2	—
	Flame <sup>4</sup> or induction hardened <sup>4</sup> with type A pattern <sup>5</sup>	See Table 8*	45 000 (310)	55 000 (380)	—
	Flame <sup>4</sup> or induction hardened <sup>4</sup> with type B pattern <sup>5</sup>	See Table 8*	22 000 (151)	22 000 (151)	—
	Carburized and hardened	See Table 9*	55 000 (380)	65 000 or (448 or 70 000 <sup>6</sup> 482)	75 000 (517)
	Nitrided <sup>4,7</sup> (through-hardened steels)	83.5 HR15N	See Fig. 14-3	See Fig. 14-3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided <sup>4,7</sup>	87.5 HR15N	See Fig. 14-4	See Fig. 14-4	See Fig. 14-4

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1-7.

<sup>1</sup>Hardness to be equivalent to that at the root diameter in the center of the tooth space and face width.

<sup>2</sup>See tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.

<sup>3</sup>The steel selected must be compatible with the heat treatment process selected and hardness required.

<sup>4</sup>The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.

<sup>5</sup>See figure 12 for type A and type B hardness patterns.

<sup>6</sup>If bainite and microcracks are limited to grade 3 levels, 70 000 psi may be used.

<sup>7</sup>The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design. [7]

\*Tables 8 and 9 of ANSI/AGMA 2001-D04 are comprehensive tabulations of the major metallurgical factors affecting  $S_t$  and  $S_c$  of flame-hardened and induction-hardened (Table 8) and carburized and hardened (Table 9) steel gears.

**Table 14-4**

 Repeatedly Applied Bending Strength  $S_t$  for Iron and Bronze Gears at  $10^7$  Cycles and 0.99 Reliability

Source: ANSI/AGMA 2001-D04.

Material	Material Designation <sup>1</sup>	Heat Treatment	Typical Minimum Surface Hardness <sup>2</sup>	Allowable Bending Stress Number, $S_t$ , <sup>3</sup> psi (MPa)
ASTM A48 gray cast iron	Class 20	As cast	—	5000 (35)
	Class 30	As cast	174 HB	8500 (58)
	Class 40	As cast	201 HB	13 000 (90)
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000 (151–227)
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000 (151–227)
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000 (186–275)
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700 (39)
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600 (163)

Notes:

<sup>1</sup>See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual*.

<sup>2</sup>Measured hardness to be equivalent to that which would be measured at the root diameter in the center of the tooth space and face width.

<sup>3</sup>The lower values should be used for general design purposes. The upper values may be used when:

High quality material is used.

Section size and design allow maximum response to heat treatment.

Proper quality control is effected by adequate inspection.

Operating experience justifies their use.

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \frac{S_t}{S_F} \frac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \frac{S_t}{S_F} \frac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \quad (14-17)$$

where for U.S. customary units (SI units),

 $S_t$  is the allowable bending stress, lbf/in<sup>2</sup> (N/mm<sup>2</sup>)

 $Y_N$  is the stress-cycle factor for bending stress

 $K_T$  ( $Y_\theta$ ) are the temperature factors

 $K_R$  ( $Y_Z$ ) are the reliability factors

 $S_F$  is the AGMA factor of safety, a stress ratio

The equation for the allowable contact stress  $\sigma_{c,all}$  is

$$\sigma_{c,all} = \begin{cases} \frac{S_c Z_N C_H}{S_H K_T K_R} & \text{(U.S. customary units)} \\ \frac{S_c Z_N Z_W}{S_H Y_\theta Y_Z} & \text{(SI units)} \end{cases} \quad (14-18)$$

where the upper equation is in U.S. customary units and the lower equation is in SI units, Also,

- $S_c$  is the allowable contact stress, lbf/in<sup>2</sup> (N/mm<sup>2</sup>)
- $Z_N$  is the stress-cycle factor
- $C_H$  ( $Z_W$ ) are the hardness ratio factors for pitting resistance
- $K_T$  ( $Y_\theta$ ) are the temperature factors
- $K_R$  ( $Y_Z$ ) are the reliability factors
- $S_H$  is the AGMA factor of safety, a stress ratio

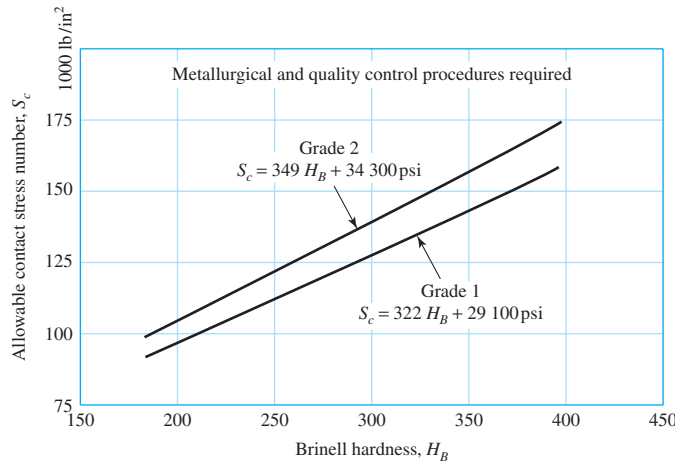
The values for the allowable contact stress, designated here as  $S_c$ , are to be found in Fig. 14-5 and Tables 14-5, 14-6, and 14-7.

AGMA allowable stress numbers (strengths) for bending and contact stress are for

- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

**Figure 14-5**

Contact-fatigue strength  $S_c$  at  $10^7$  cycles and 0.99 reliability for through-hardened steel gears. The SI equations are:  $S_c = 2.22H_B + 200$  MPa, grade 1, and  $S_c = 2.41H_B + 237$  MPa, grade 2. (Source: ANSI/AGMA 2001-D04 and 2101-D04.)



**Table 14-5**

Nominal Temperature Used in Nitriding and Hardnesses Obtained

Source: Darle W. Dudley, *Handbook of Practical Gear Design*, rev. ed., McGraw-Hill, New York, 1984.

Steel	Temperature Before Nitriding, °F	Nitriding, °F	Hardness, Rockwell C Scale	
			Case	Core
Nitralloy 135*	1150	975	62-65	30-35
Nitralloy 135M	1150	975	62-65	32-36
Nitralloy N	1000	975	62-65	40-44
AISI 4340	1100	975	48-53	27-35
AISI 4140	1100	975	49-54	27-35
31 Cr Mo V 9	1100	975	58-62	27-33

\*Nitralloy is a trademark of the Nitralloy Corp., New York.

**Table 14-6**Repeatedly Applied Contact Strength  $S_c$  at  $10^7$  Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness <sup>1</sup>	Allowable Contact Stress Number, <sup>2</sup> $S_c$ , psi ( $\sigma_{HP}$ , MPa)		
			Grade 1	Grade 2	Grade 3
Steel <sup>3</sup>	Through hardened <sup>4</sup>	See Fig. 14-5	See Fig. 14-5	See Fig. 14-5	—
	Flame <sup>5</sup> or induction hardened <sup>5</sup>	50 HRC	170 000 (1172)	190 000 (1310)	—
		54 HRC	175 000 (1206)	195 000 (1344)	—
	Carburized and hardened <sup>5</sup>	See Table 9*	180 000 (1240)	225 000 (1551)	275 000 (1896)
	Nitrided <sup>5</sup> (through hardened steels)	83.5 HR15N	150 000 (1035)	163 000 (1123)	175 000 (1206)
		84.5 HR15N	155 000 (1068)	168 000 (1158)	180 000 (1240)
2.5% chrome (no aluminum)	Nitrided <sup>5</sup>	87.5 HR15N	155 000 (1068)	172 000 (1186)	189 000 (1303)
Nitralloy 135M	Nitrided <sup>5</sup>	90.0 HR15N	170 000 (1172)	183 000 (1261)	195 000 (1344)
Nitralloy N	Nitrided <sup>5</sup>	90.0 HR15N	172 000 (1186)	188 000 (1296)	205 000 (1413)
2.5% chrome (no aluminum)	Nitrided <sup>5</sup>	90.0 HR15N	176 000 (1213)	196 000 (1351)	216 000 (1490)

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–5.

<sup>1</sup>Hardness to be equivalent to that at the start of active profile in the center of the face width.<sup>2</sup>See Tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.<sup>3</sup>The steel selected must be compatible with the heat treatment process selected and hardness required.<sup>4</sup>These materials must be annealed or normalized as a minimum.<sup>5</sup>The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.\*Table 9 of ANSI/AGMA 2001-D04 is a comprehensive tabulation of the major metallurgical factors affecting  $S_f$  and  $S_c$  of carburized and hardened steel gears.

The factors in this section, too, will be evaluated in subsequent sections.

When two-way (reversed) loading occurs, as with idler gears, AGMA recommends using 70 percent of  $S_f$  values. This is equivalent to  $1/0.70 = 1.43$  as a value of  $k_e$  in Ex. 14-2. The recommendation falls between the value of  $k_e = 1.33$  for a Goodman failure locus and  $k_e = 1.66$  for a Gerber failure locus.

## 14-5 Geometry Factors $I$ and $J$ ( $Z_I$ and $Y_J$ )

We have seen how the factor  $Y$  is used in the Lewis equation to introduce the effect of tooth form into the stress equation. The AGMA factors<sup>5</sup>  $I$  and  $J$  are intended to accomplish the same purpose in a more involved manner.

The determination of  $I$  and  $J$  depends upon the *face-contact ratio*  $m_F$ . This is defined as

$$m_F = \frac{F}{p_x} \quad (14-19)$$

where  $p_x$  is the axial pitch and  $F$  is the face width. For spur gears,  $m_F = 0$ .

<sup>5</sup>A useful reference is AGMA 908-B89, *Geometry Factors for Determining Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*.

**Table 14-7**Repeatedly Applied Contact Strength  $S_c$   $10^7$  Cycles and 0.99 Reliability for Iron and Bronze Gears

Source: ANSI/AGMA 2001-D04.

Material	Material Designation <sup>1</sup>	Heat Treatment	Typical Minimum Surface Hardness <sup>2</sup>	Allowable Contact Stress Number, <sup>3</sup> $S_c$ psi ( $\sigma_{HP}$ , MPa)
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000 (344–415)
	Class 30	As cast	174 HB	65 000–75 000 (448–517)
	Class 40	As cast	201 HB	75 000–85 000 (517–586)
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000 (530–634)
		Quenched and tempered	179 HB	77 000–92 000 (530–634)
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000 (634–772)
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000 (710–868)
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000 (206)
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000 (448)

Notes:

<sup>1</sup>See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual*.<sup>2</sup>Hardness to be equivalent to that at the start of active profile in the center of the face width.<sup>3</sup>The lower values should be used for general design purposes. The upper values may be used when:

High-quality material is used.

Section size and design allow maximum response to heat treatment.

Proper quality control is effected by adequate inspection.

Operating experience justifies their use.

Low-contact-ratio (LCR) helical gears having a small helix angle or a thin face width, or both, have face-contact ratios less than unity ( $m_F \leq 1$ ), and will not be considered here. Such gears have a noise level not too different from that for spur gears. Consequently we shall consider here only spur gears with  $m_F = 0$  and conventional helical gears with  $m_F > 1$ .

### Bending-Strength Geometry Factor $J$ ( $Y_J$ )

The AGMA factor  $J$  employs a modified value of the Lewis form factor, also denoted by  $Y$ ; a *fatigue stress-concentration factor*  $K_f$ ; and a *tooth load-sharing ratio*  $m_N$ . The resulting equation for  $J$  for spur and helical gears is

$$J = \frac{Y}{K_f m_N} \quad (14-20)$$

It is important to note that the form factor  $Y$  in Eq. (14-20) is *not* the Lewis factor at all. The value of  $Y$  here is obtained from calculations within AGMA 908-B89, and is often based on the highest point of single-tooth contact.



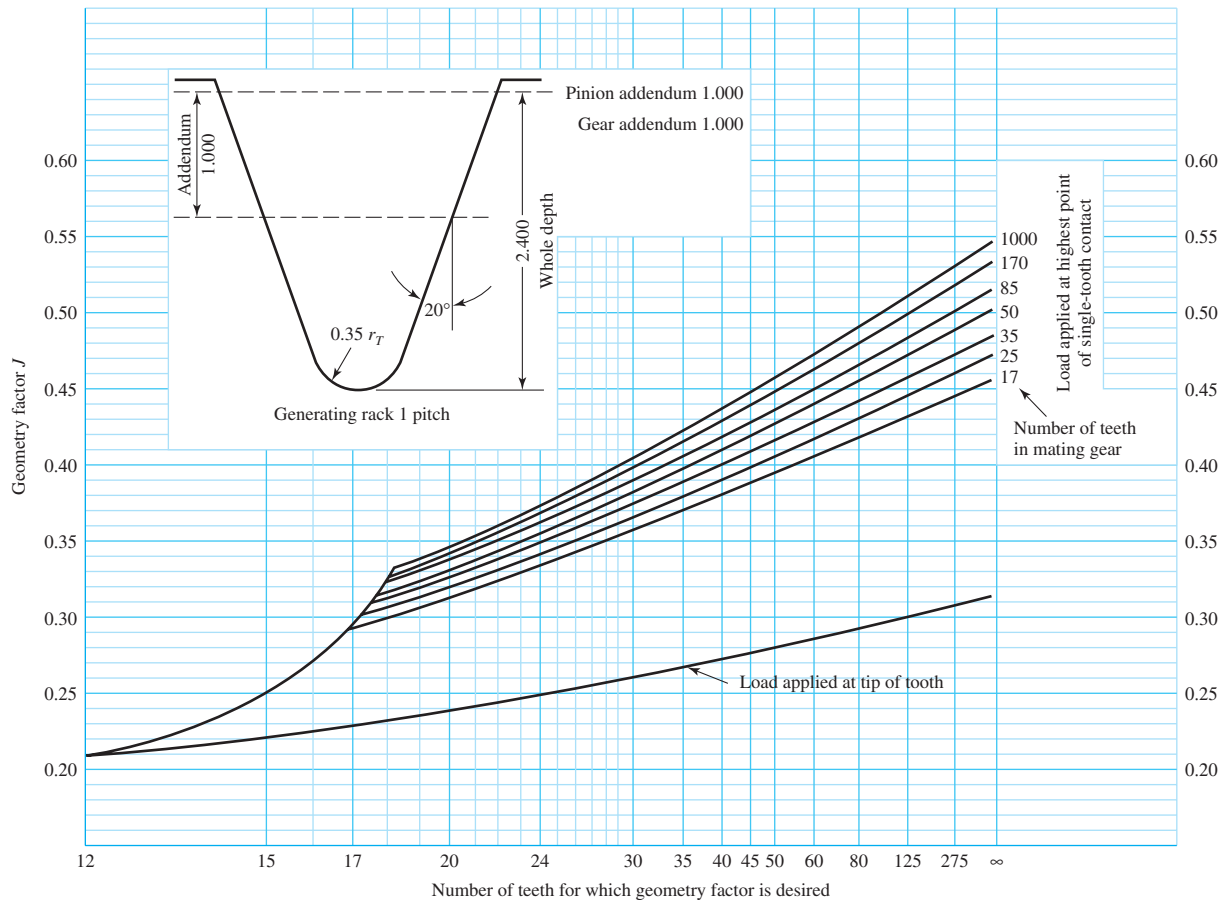
The factor  $K_f$  in Eq. (14-20) is called a *stress-correction factor* by AGMA. It is based on a formula deduced from a photoelastic investigation of stress concentration in gear teeth over 50 years ago.

The load-sharing ratio  $m_N$  is equal to the face width divided by the minimum total length of the lines of contact. This factor depends on the transverse contact ratio  $m_p$ , the face-contact ratio  $m_F$ , the effects of any profile modifications, and the tooth deflection. For spur gears,  $m_N = 1.0$ . For helical gears having a face-contact ratio  $m_F > 2.0$ , a conservative approximation is given by the equation

$$m_N = \frac{p_N}{0.95Z} \quad (14-21)$$

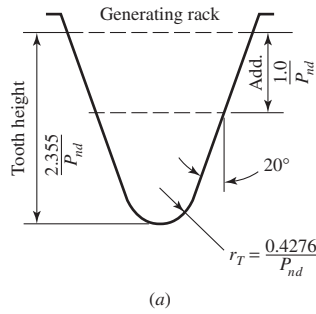
where  $p_N$  is the normal base pitch and  $Z$  is the length of the line of action in the transverse plane (distance  $L_{ab}$  in Fig. 13-15, p. 676).

Use Fig. 14-6 to obtain the geometry factor  $J$  for spur gears having a  $20^\circ$  pressure angle and full-depth teeth. Use Figs. 14-7 and 14-8 for helical gears having a  $20^\circ$  normal pressure angle and face-contact ratios of  $m_F = 2$  or greater. For other gears, consult the AGMA standard.



**Figure 14-6**

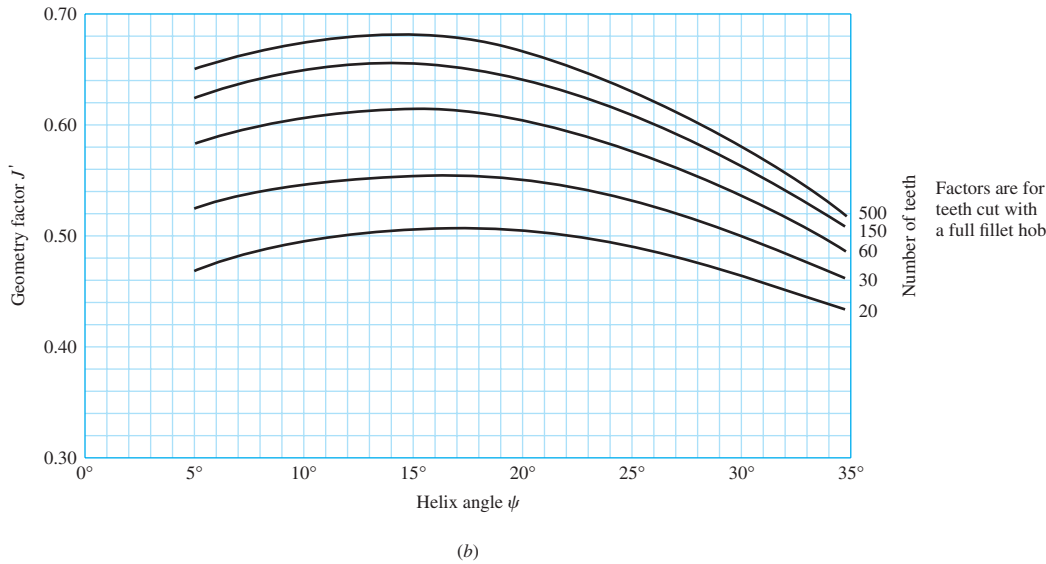
Spur-gear geometry factors  $J$ . Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



$$m_N = \frac{P_N}{0.95Z}$$

Value for Z is for an element of indicated numbers of teeth and a 75-tooth mate

Normal tooth thickness of pinion and gear tooth each reduced 0.024 in to provide 0.048 in total backlash for one normal diametral pitch



**Figure 14-7**

Helical-gear geometry factors  $J'$ . Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

### Surface-Strength Geometry Factor $I (Z_I)$

The factor  $I$  is also called the *pitting-resistance geometry factor* by AGMA. We will develop an expression for  $I$  by noting that the sum of the reciprocals of Eq. (14-14), from Eq. (14-12), can be expressed as

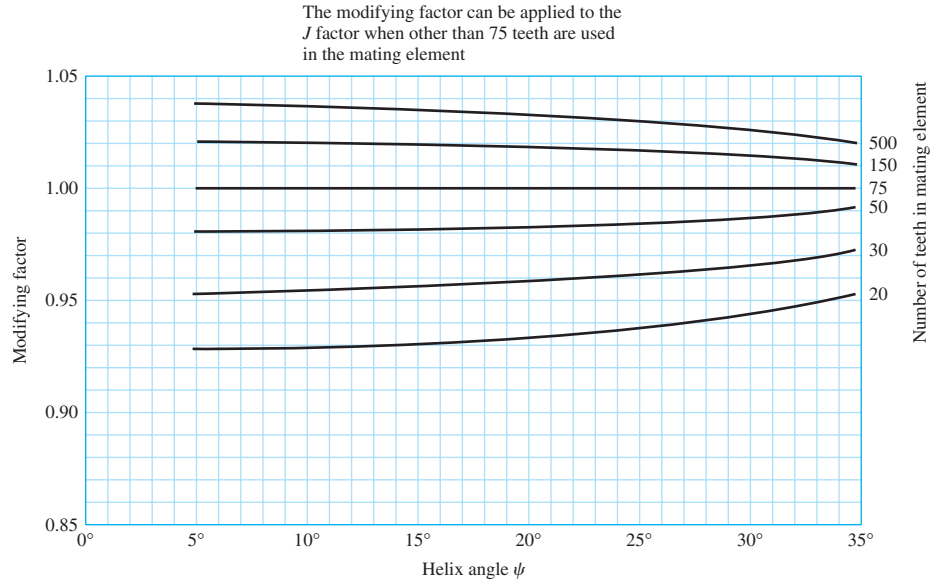
$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin \phi_t} \left( \frac{1}{d_p} + \frac{1}{d_G} \right) \tag{a}$$

where we have replaced  $\phi$  by  $\phi_t$ , the transverse pressure angle, so that the relation will apply to helical gears too. Now define *speed ratio*  $m_G$  as

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \tag{14-22}$$

**Figure 14-8**

$J'$ -factor multipliers for use with Fig. 14-7 to find  $J$ .  
 Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



Equation (a) can now be written

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_p \sin \phi_t} \frac{m_G + 1}{m_G} \quad (b)$$

Now substitute Eq. (b) for the sum of the reciprocals in Eq. (14-14). The result is found to be

$$\sigma_c = |\sigma_c| = C_p \left[ \frac{K_v W^t}{d_p F} \frac{1}{\frac{\cos \phi_t \sin \phi_t}{2} \frac{m_G}{m_G + 1}} \right]^{1/2} \quad (c)$$

The geometry factor  $I$  for external spur and helical gears is the denominator of the second term in the brackets in Eq. (c). By adding the load-sharing ratio  $m_N$ , we obtain a factor valid for both spur and helical gears. The equation is then written as

$$I = \begin{cases} \frac{\cos \phi_t \sin \phi_t}{2m_N} \frac{m_G}{m_G + 1} & \text{external gears} \\ \frac{\cos \phi_t \sin \phi_t}{2m_N} \frac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \quad (14-23)$$

where  $m_N = 1$  for spur gears. In solving Eq. (14-21) for  $m_N$ , note that

$$p_N = p_n \cos \phi_n \quad (14-24)$$

where  $p_n$  is the normal circular pitch. The quantity  $Z$ , for use in Eq. (14-21), can be obtained from the equation

$$Z = [(r_P + a)^2 - r_{bP}^2]^{1/2} + [(r_G + a)^2 - r_{bG}^2]^{1/2} - (r_P + r_G) \sin \phi_t \quad (14-25)$$

where  $r_P$  and  $r_G$  are the pitch radii and  $r_{bP}$  and  $r_{bG}$  the base-circle radii of the pinion and gear, respectively.<sup>6</sup> Recall from Eq. (13-6), the radius of the base circle is

$$r_b = r \cos \phi_t \quad (14-26)$$

<sup>6</sup>For a development, see Joseph E. Shigley and John J. Uicker Jr., *Theory of Machines and Mechanisms*, McGraw-Hill, New York, 1980, p. 262.

Certain precautions must be taken in using Eq. (14–25). The tooth profiles are not conjugate below the base circle, and consequently, if either one or the other of the first two terms in brackets is larger than the third term, then it should be replaced by the third term. In addition, the effective outside radius is sometimes less than  $r + a$ , owing to removal of burrs or rounding of the tips of the teeth. When this is the case, always use the effective outside radius instead of  $r + a$ .

## 14-6 The Elastic Coefficient $C_p$ ( $Z_E$ )

Values of  $C_p$  may be computed directly from Eq. (14–13) or obtained from Table 14–8.

## 14-7 Dynamic Factor $K_v$

As noted earlier, dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action. *Transmission error* is defined as the departure from uniform angular velocity of the gear pair. Some of the effects that produce transmission error are:

- Inaccuracies produced in the generation of the tooth profile; these include errors in tooth spacing, profile lead, and runout
- Vibration of the tooth during meshing due to the tooth stiffness
- Magnitude of the pitch-line velocity
- Dynamic unbalance of the rotating members
- Wear and permanent deformation of contacting portions of the teeth
- Gearshaft misalignment and the linear and angular deflection of the shaft
- Tooth friction

In an attempt to account for these effects, AGMA has defined a set of *quality numbers*,  $Q_v$ .<sup>7</sup> These numbers define the tolerances for gears of various sizes manufactured to a specified accuracy. Quality numbers 3 to 7 will include most commercial-quality gears. Quality numbers 8 to 12 are of precision quality. The following equations for the dynamic factor are based on these  $Q_v$  numbers:

$$K_v = \begin{cases} \left( \frac{A + \sqrt{V}}{A} \right)^B & V \text{ in ft/min} \\ \left( \frac{A + \sqrt{200V}}{A} \right)^B & V \text{ in m/s} \end{cases} \quad (14-27)$$

where

$$\begin{aligned} A &= 50 + 56(1 - B) \\ B &= 0.25(12 - Q_v)^{2/3} \end{aligned} \quad (14-28)$$

<sup>7</sup>AGMA 2000-A88. ANSI/AGMA 2001-D04, adopted in 2004, replaced the quality number  $Q_v$  with the transmission accuracy level number  $A_v$  and incorporated ANSI/AGMA 2015-1-A01.  $A_v$  ranges from 6 to 12, with lower numbers representing greater accuracy. The  $Q_v$  approach was maintained as an alternate approach, and resulting  $K_v$  values are comparable.

**Table 14-8**

Elastic Coefficient  $C_p$  ( $Z_E$ ),  $\sqrt{\text{psi}}$  ( $\sqrt{\text{MPa}}$ ) Source: AGMA 218.01

Pinion Material	Pinion Modulus of Elasticity $E_p$ psi (MPa)*	Gear Material and Modulus of Elasticity $E_G$ , lbf/in <sup>2</sup> (MPa)*					
		Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	$30 \times 10^6$ ( $2 \times 10^5$ )	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	$25 \times 10^6$ ( $1.7 \times 10^5$ )	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	$24 \times 10^6$ ( $1.7 \times 10^5$ )	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	$22 \times 10^6$ ( $1.5 \times 10^5$ )	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	$17.5 \times 10^6$ ( $1.2 \times 10^5$ )	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	$16 \times 10^6$ ( $1.1 \times 10^5$ )	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

\*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

**Figure 14-9**

Dynamic factor  $K_v$ . The equations to these curves are given by Eq. (14-27) and the end points by Eq. (14-29). (ANSI/AGMA 2001-D04, Annex A)

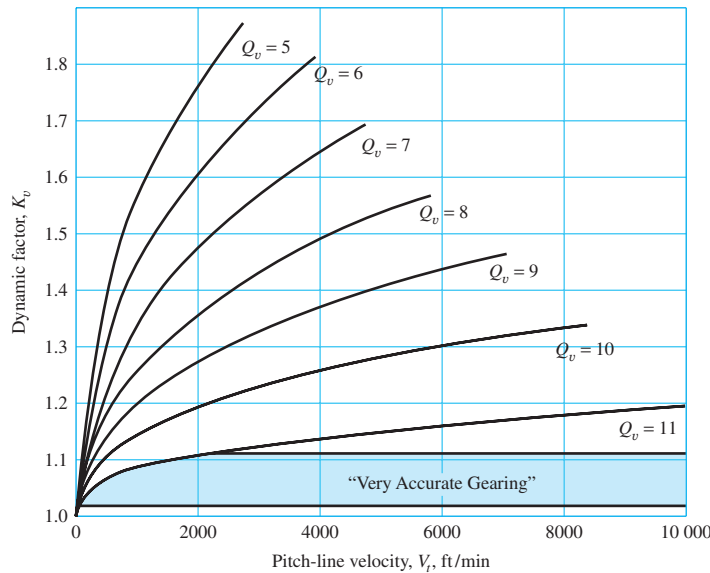


Figure 14-9 graphically represents Eq. (14-27). The maximum recommended pitch-line velocity for a given quality number is represented by the end point of each  $Q_v$  curve, and is given by

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \frac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \quad (14-29)$$

**14-8 Overload Factor  $K_o$** 

The overload factor  $K_o$  is intended to make allowance for all externally applied loads in excess of the nominal tangential load  $W^t$  in a particular application (see Figs. 14-17 and 14-18 for tables). Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive. There are other similar factors such as application factor or service factor. These factors are established after considerable field experience in a particular application.<sup>8</sup>

**14-9 Surface Condition Factor  $C_f$  ( $Z_R$ )**

The surface condition factor  $C_f$  or  $Z_R$  is used only in the pitting resistance equation, Eq. (14-16). It depends on

- Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shotpeening
- Residual stress
- Plastic effects (work hardening)

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value of  $C_f$  greater than unity.

<sup>8</sup>An extensive list of service factors appears in Howard B. Schwerdlin, "Couplings," Chap. 16 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design*, 3rd ed., McGraw-Hill, New York, 2004.



## 14-10 Size Factor $K_s$

The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. In such cases AGMA recommends a size factor greater than unity. If there is no detrimental size effect, use unity.

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests  $K_s = 1$ , which makes  $K_s$  a placeholder in Eqs. (14-15) and (14-16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13-1, p. 688,  $l = a + b = 2.25/P$ . The tooth thickness  $t$  in Fig. 14-6 is given in Sec. 14-1, Eq. (b), as  $t = \sqrt{4lx}$  where  $x = 3Y/(2P)$  from Eq. (14-3). From Eq. (6-25), p. 297, the equivalent diameter  $d_e$  of a rectangular section in bending is  $d_e = 0.808\sqrt{Ft}$ . From Eq. (6-20), p. 296,  $k_b = (d_e/0.3)^{-0.107}$ . Noting that  $K_s$  is the reciprocal of  $k_b$ , we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192 \left( \frac{F\sqrt{Y}}{P} \right)^{0.0535} \quad (a)$$

$K_s$  can be viewed as Lewis's geometry incorporated into the Marin size factor in fatigue. You may set  $K_s = 1$ , or you may elect to use the preceding Eq. (a). This is a point to discuss with your instructor. We will use Eq. (a) to remind you that you have a choice. If  $K_s$  in Eq. (a) is less than 1, use  $K_s = 1$ .

## 14-11 Load-Distribution Factor $K_m$ ( $K_H$ )

The load-distribution factor modified the stress equations to reflect nonuniform distribution of load across the line of contact. The ideal is to locate the gear "midspan" between two bearings at the zero slope place when the load is applied. However, this is not always possible. The following procedure is applicable to

- Net face width to pinion pitch diameter ratio  $F/d_p \leq 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

The load-distribution factor under these conditions is currently given by the *face load distribution factor*,  $C_{mf}$ , where

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \quad (14-30)$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \quad (14-31)$$

$$C_{pf} = \begin{cases} \frac{F}{10d} - 0.025 & F \leq 1 \text{ in} \\ \frac{F}{10d} - 0.0375 + 0.0125F & 1 < F \leq 17 \text{ in} \\ \frac{F}{10d} - 0.1109 + 0.0207F - 0.000228F^2 & 17 < F \leq 40 \text{ in} \\ \frac{b}{10d} - 0.025 & b \leq 25 \text{ mm} \\ \frac{b}{10d} - 0.0375 + 4.92(10^{-4})b & 25 < b \leq 425 \text{ mm} \\ \frac{b}{10d} - 0.1109 + 8.15(10^{-4})b - 3.53(10^{-7})b^2 & 425 < b \leq 1000 \text{ mm} \end{cases} \quad (14-32)$$

Note that for values of  $F/(10d_p) < 0.05$ ,  $F/(10d_p) = 0.05$  is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases} \quad (14-33)$$

$$C_{ma} = A + BF + CF^2 \quad (\text{see Table 14-9 for values of } A, B, \text{ and } C) \quad (14-34)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \quad (14-35)$$

See Fig. 14-10 for definitions of  $S$  and  $S_1$  for use with Eq. (14-33), and see Fig. 14-11 for graph of  $C_{ma}$ .

**Table 14-9**

Empirical Constants  $A$ ,  $B$ , and  $C$  for Eq. (14-34), Face Width  $F$  in Inches\*

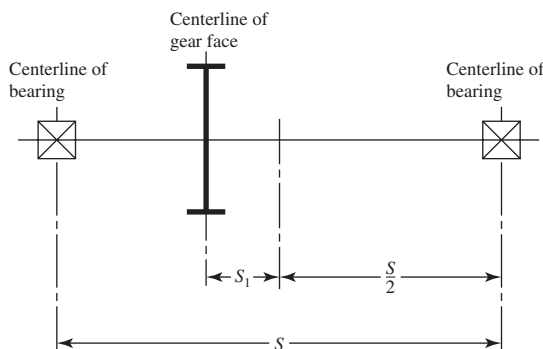
Source: ANSI/AGMA 2001-D04.

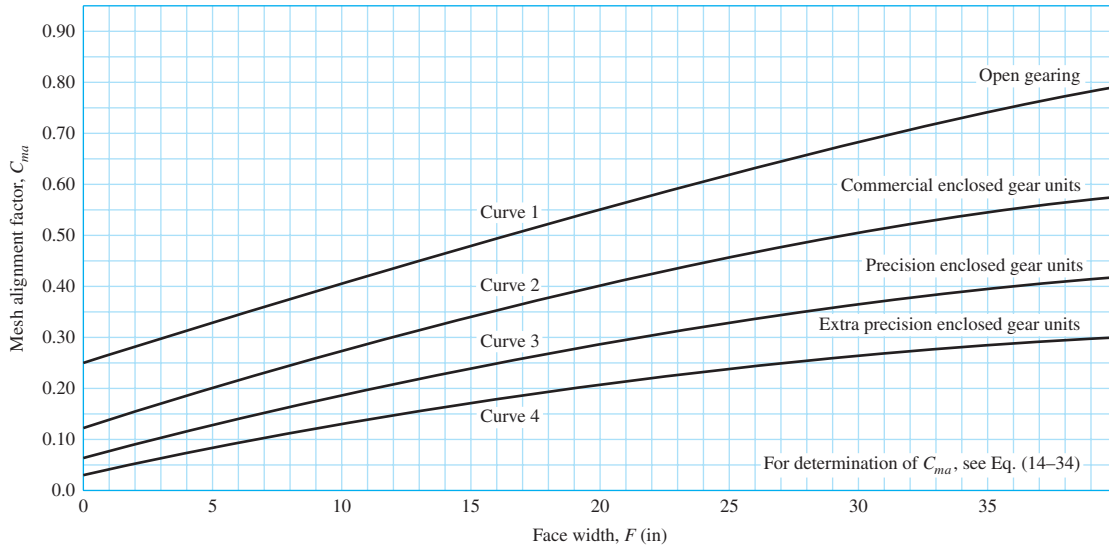
Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

\*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

**Figure 14-10**

Definition of distances  $S$  and  $S_1$  used in evaluating  $C_{pm}$ , Eq. (14-33). (ANSI/AGMA 2001-D04.)





**Figure 14-11**

Mesh alignment factor  $C_{ma}$ . Curve-fit equations in Table 14-9. (ANSI/AGMA 2001-D04.)

## 14-12 Hardness-Ratio Factor $C_H (Z_W)$

The pinion generally has a smaller number of teeth than the gear and consequently is subjected to more cycles of contact stress. If both the pinion and the gear are through-hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear. A similar effect can be obtained when a surface-hardened pinion is mated with a through-hardened gear. The hardness-ratio factor  $C_H$  is used *only for the gear*. Its purpose is to adjust the surface strengths for this effect. For the pinion,  $C_H = 1$ . For the gear,  $C_H$  is obtained from the equation

$$C_H = 1.0 + A'(m_G - 1.0) \quad (14-36)$$

where

$$A' = 8.98(10^{-3}) \left( \frac{H_{BP}}{H_{BG}} \right) - 8.29(10^{-3}) \quad 1.2 \leq \frac{H_{BP}}{H_{BG}} \leq 1.7$$

The terms  $H_{BP}$  and  $H_{BG}$  are the Brinell hardness (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term  $m_G$  is the speed ratio and is given by Eq. (14-22). See Fig. 14-12 for a graph of Eq. (14-36). For

$$\frac{H_{BP}}{H_{BG}} < 1.2, \quad A' = 0$$

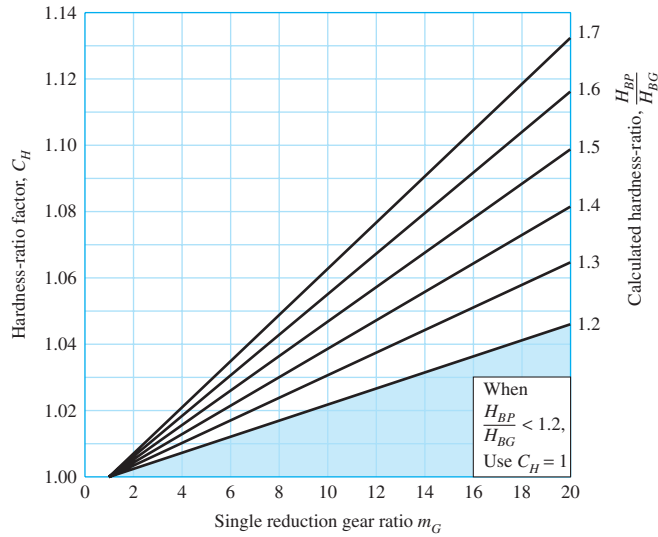
$$\frac{H_{BP}}{H_{BG}} > 1.7, \quad A' = 0.00698$$

When surface-hardened pinions with hardnesses of 48 Rockwell C scale (Rockwell C48) or harder are run with through-hardened gears (180–400 Brinell), a work hardening occurs. The  $C_H$  factor is a function of pinion surface finish  $f_p$  and the mating gear hardness. Figure 14-13 displays the relationships:

$$C_H = 1 + B'(450 - H_{BG}) \quad (14-37)$$

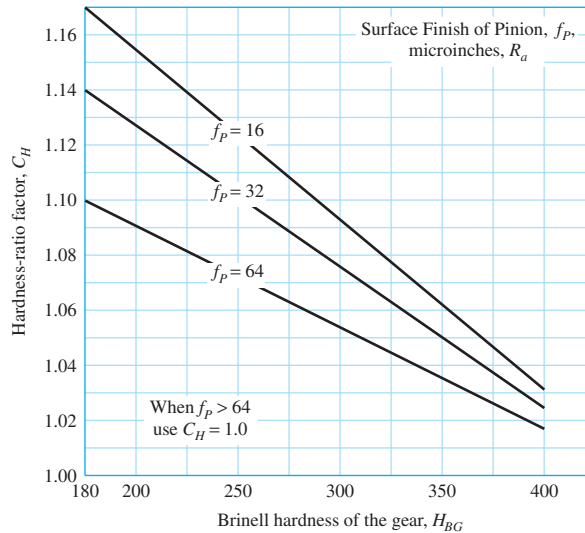
**Figure 14-12**

Hardness-ratio factor  $C_H$   
(through-hardened steel).  
(ANSI/AGMA 2001-D04.)



**Figure 14-13**

Hardness-ratio factor  $C_H$   
(surface-hardened steel pinion).  
(ANSI/AGMA 2001-D04.)



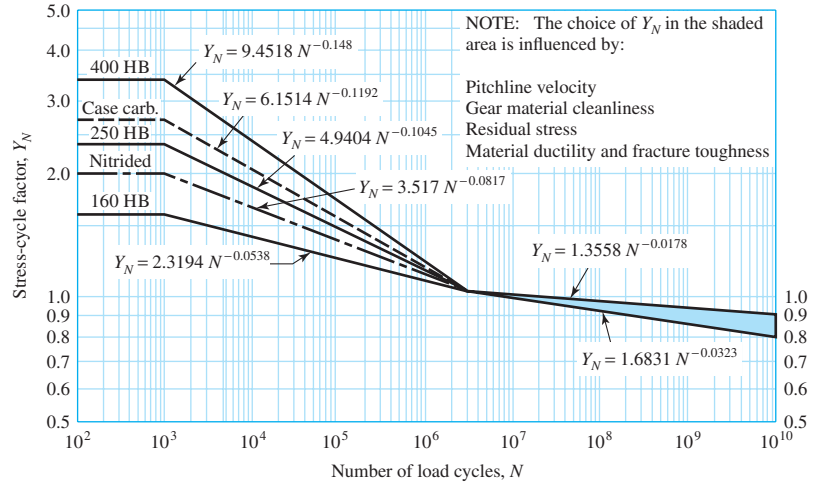
where  $B' = 0.00075 \exp[-0.0112f_p]$  and  $f_p$  is the surface finish of the pinion expressed as root-mean-square roughness  $R_a$  in  $\mu$  in.

### 14-13 Stress-Cycle Factors $Y_N$ and $Z_N$

The AGMA strengths as given in Figs. 14-2 through 14-4, in Tables 14-3 and 14-4 for bending fatigue, and in Fig. 14-5 and Tables 14-5 and 14-6 for contact-stress fatigue are based on  $10^7$  load cycles applied. The purpose of the stress-cycle factors  $Y_N$  and  $Z_N$  is to modify the gear strength for lives other than  $10^7$  cycles. Values for these factors are given in Figs. 14-14 and 14-15. Note that for  $10^7$  cycles  $Y_N = Z_N = 1$  on each graph. Note also that the equations for  $Y_N$  and  $Z_N$  change on either side of  $10^7$  cycles. For life goals slightly higher than  $10^7$  cycles, the mating gear may be experiencing fewer than  $10^7$  cycles and the equations for  $(Y_N)_P$  and  $(Y_N)_G$  can be different. The same comment applies to  $(Z_N)_P$  and  $(Z_N)_G$ .

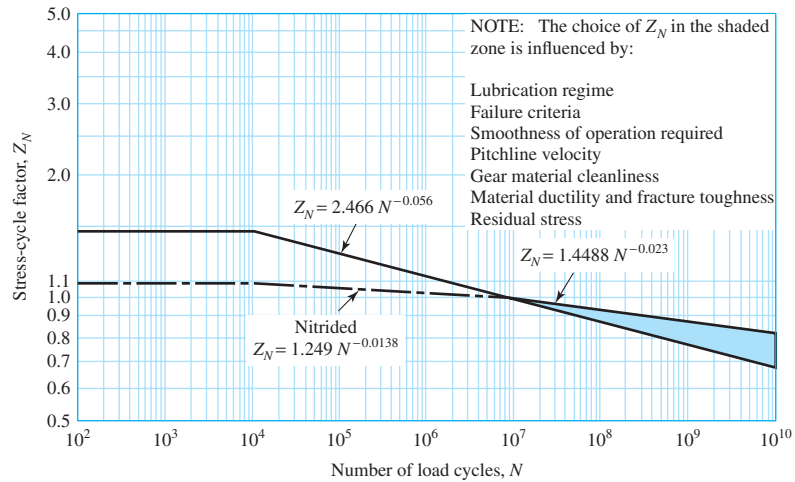
**Figure 14-14**

Repeatedly applied bending strength stress-cycle factor  $Y_N$ . (ANSI/AGMA 2001-D04.)



**Figure 14-15**

Pitting resistance stress-cycle factor  $Z_N$ . (ANSI/AGMA 2001-D04.)



### 14-14 Reliability Factor $K_R$ ( $Y_Z$ )

The reliability factor accounts for the effect of the statistical distributions of material fatigue failures. Load variation is not addressed here. The gear strengths  $S_t$  and  $S_c$  are based on a reliability of 99 percent. Table 14-10 is based on data developed by the U.S. Navy for bending and contact-stress fatigue failures.

The functional relationship between  $K_R$  and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \leq R \leq 0.9999 \end{cases} \quad (14-38)$$

For cardinal values of  $R$ , take  $K_R$  from the table. Otherwise use the logarithmic interpolation afforded by Eqs. (14-38).

**Table 14-10**Reliability Factors  $K_R (Y_Z)$ Source: ANSI/AGMA  
2001-D04.

Reliability	$K_R (Y_Z)$
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

**14-15 Temperature Factor  $K_T (Y_\theta)$** 

For oil or gear-blank temperatures up to 250°F (120°C), use  $K_T = Y_\theta = 1.0$ . For higher temperatures, the factor should be greater than unity. Heat exchangers may be used to ensure that operating temperatures are considerably below this value, as is desirable for the lubricant.

**14-16 Rim-Thickness Factor  $K_B$** 

When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet. In such cases, the use of a stress-modifying factor  $K_B$  is recommended. This factor, the *rim-thickness factor*  $K_B$ , adjusts the estimated bending stress for the thin-rimmed gear. It is a function of the backup ratio  $m_B$ ,

$$m_B = \frac{t_R}{h_t} \quad (14-39)$$

where  $t_R$  = rim thickness below the tooth, and  $h_t$  = the tooth height. The geometry is depicted in Fig. 14-16. The rim-thickness factor  $K_B$  is given by

$$K_B = \begin{cases} 1.6 \ln \frac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \quad (14-40)$$

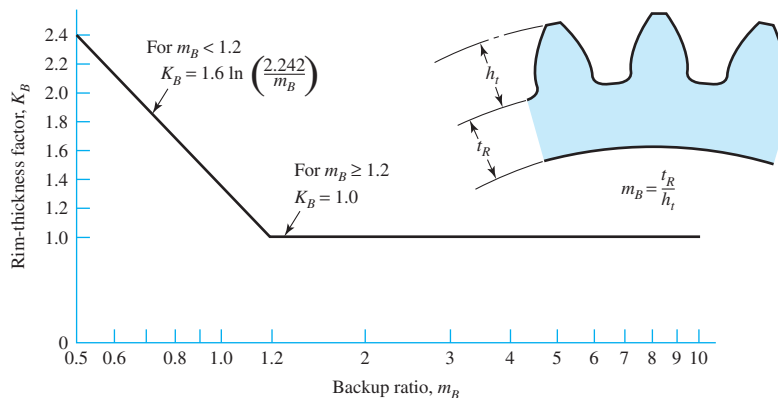
**Figure 14-16**Rim-thickness factor  $K_B$ .  
(ANSI/AGMA 2001-D04.)



Figure 14–16 also gives the value of  $K_B$  graphically. The rim-thickness factor  $K_B$  is applied in addition to the 0.70 reverse-loading factor when applicable.

## 14–17 Safety Factors $S_F$ and $S_H$

The ANSI/AGMA standards 2001-D04 and 2101-D04 contain a safety factor  $S_F$  guarding against bending fatigue failure and safety factor  $S_H$  guarding against pitting failure.

The definition of  $S_F$ , from Eq. (14–17), for U.S. customary units, is

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \quad (14-41)$$

where  $\sigma$  is estimated from Eq. (14–15), for U.S. customary units. It is a strength-over-stress definition in a case where the stress is linear with the transmitted load.

The definition of  $S_H$ , from Eq. (14–18), is

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \quad (14-42)$$

when  $\sigma_c$  is estimated from Eq. (14–16). This, too, is a strength-over-stress definition but in a case where the stress is *not* linear with the transmitted load  $W^t$ .

While the definition of  $S_H$  does not interfere with its intended function, a caution is required when comparing  $S_F$  with  $S_H$  in an analysis in order to ascertain the nature and severity of the threat to loss of function. To render  $S_H$  linear with the transmitted load,  $W^t$  it could have been defined as

$$S_H = \left( \frac{\text{fully corrected contact strength}}{\text{contact stress imposed}} \right)^2 \quad (14-43)$$

with the exponent 2 for linear or helical contact, or an exponent of 3 for crowned teeth (spherical contact). With the definition, Eq. (14–42), compare  $S_F$  with  $S_H^2$  (or  $S_H^3$  for crowned teeth) when trying to identify the threat to loss of function with confidence.

The role of the overload factor  $K_o$  is to include predictable excursions of load beyond  $W^t$  based on experience. A safety factor is intended to account for unquantifiable elements in addition to  $K_o$ . When designing a gear mesh, the quantity  $S_F$  becomes a design factor  $(S_F)_d$  within the meanings used in this book. The quantity  $S_F$  evaluated as part of a design assessment is a factor of safety. This applies equally well to the quantity  $S_H$ .

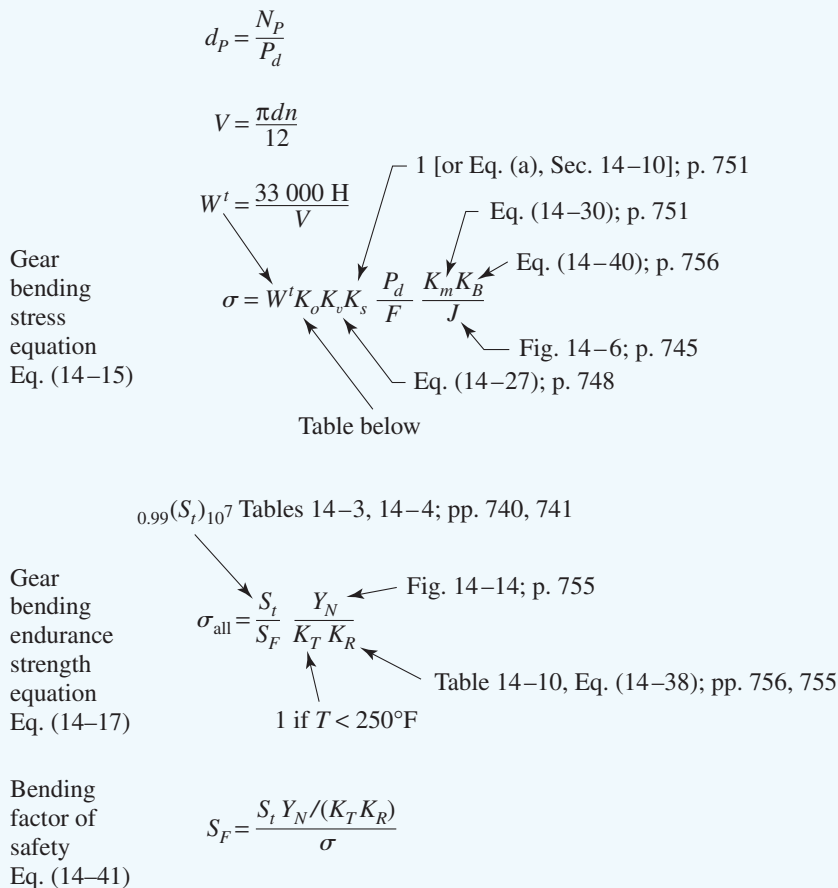
## 14–18 Analysis

Description of the procedure based on the AGMA standard is highly detailed. The best review is a “road map” for bending fatigue and contact-stress fatigue. Figure 14–17 identifies the bending stress equation, the endurance strength in bending equation, and the factor of safety  $S_F$ . Figure 14–18 displays the contact-stress equation, the contact fatigue endurance strength equation, and the factor of safety  $S_H$ . The equations in these figures are in terms of U.S. customary units. Similar roadmaps can readily be generated in terms of SI units.

The following example of a gear mesh analysis is intended to make all the details presented concerning the AGMA method more familiar.

SPUR GEAR BENDING

Based on ANSI/AGMA 2001-D04 (U.S. customary units)



Remember to compare  $S_F$  with  $S_H^2$  when deciding whether bending or wear is the threat to function. For crowned gears compare  $S_F$  with  $S_H^3$ .

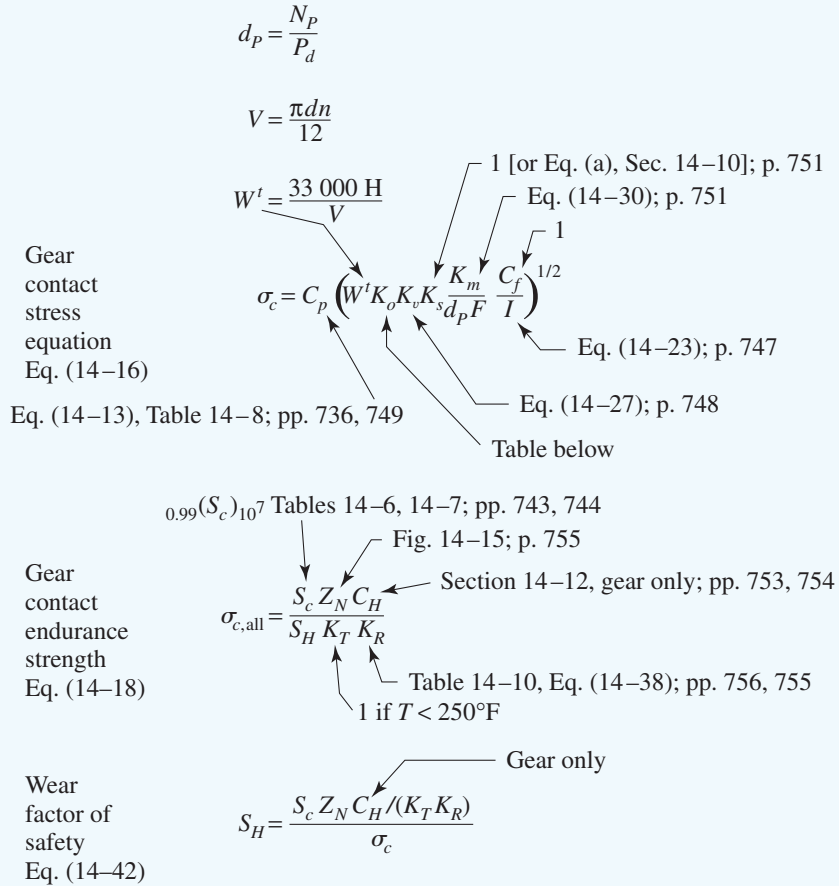
Table of Overload Factors,  $K_o$

Power source	Driven Machine		
	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14-17

Roadmap of gear bending equations based on AGMA standards. (ANSI/AGMA 2001-D04.)

SPUR GEAR WEAR  
Based on ANSI/AGMA 2001-D04 (U.S. customary units)



Remember to compare  $S_F$  with  $S_H^2$  when deciding whether bending or wear is the threat to function. For crowned gears compare  $S_F$  with  $S_H^3$ .

Figure 14-18

Roadmap of gear wear equations based on AGMA standards. (ANSI/AGMA 2001-D04.)

**EXAMPLE 14-4**

A 17-tooth  $20^\circ$  pressure angle spur pinion rotates at 1800 rev/min and transmits 2982.8 Watt to a 52-tooth disk gear. The 0.4 teeth/mm, the 38.1 mm, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30,  $J_P = 0.30$ ,  $J_G = 0.40$ , and Young's modulus is 206.8 GPa. The loading is smooth because of motor and load. Assume a pinion life of  $10^8$  cycles and a reliability of 0.90, and use  $Y_N = 1.3558N^{-0.0178}$ ,  $Z_N = 1.4488N^{-0.023}$ . The tooth profile is uncrowned. This is a commercial enclosed gear unit.

- Find the factor of safety of the gears in bending.
- Find the factor of safety of the gears in wear.
- By examining the factors of safety, identify the threat to each gear and to the mesh.

**Solution**

There will be many terms to obtain so use Figs. 14-17 and 14-18 as guides to what is needed.

$$d_p = N_p/P_d = 17/0.4 = 42.5 \text{ m} \quad d_G = 52/0.4 = 130 \text{ mm}$$

$$V = \frac{\pi d_p n_p}{60} = \frac{\pi(42.5)1800}{60} = 4005.53 \text{ mm/s}$$

$$W^t = \frac{33\,000 H}{V} = \frac{33\,000(2982.8)}{0.40055} = 744.67 \text{ N}$$

Assuming uniform loading,  $K_o = 1$ . To evaluate  $K_v$ , from Eq. (14-28) with a quality number  $Q_v = 6$ ,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14-27) the dynamic factor is

$$K_v = \left( \frac{59.77 + \sqrt{200(0.4)}}{59.77} \right)^{0.8255} = 1.37$$

To determine the size factor,  $K_s$ , the Lewis form factor is needed. From Table 14-2, with  $N_p = 17$  teeth,  $Y_P = 0.303$ . Interpolation for the gear with  $N_G = 52$  teeth yields  $Y_G = 0.412$ . Thus from Eq. (a) of Sec. 14-10, with  $F = 38.1$  mm,

$$(K_s)_P = 1.192 \left( \frac{38.1\sqrt{0.303}}{0.4} \right)^{0.0535} = 1.4708$$

$$(K_s)_G = 1.192 \left( \frac{38.1\sqrt{0.412}}{0.4} \right)^{0.0535} = 1.4829$$

The load distribution factor  $K_m$  is determined from Eq. (14–30), where five terms are needed. They are, where 38.1 mm when needed:

Uncrowned, Eq. (14–30):  $C_{mc} = 1$ ,

Eq. (14–32):  $C_{pf} = 38.1/[10(42.5)] - 0.0375 + 4.92(10e-4)(38.1) = 0.071$

Bearings immediately adjacent, Eq. (14–33):  $C_{pm} = 1$

Commercial enclosed gear units (Fig. 14–11):  $C_{ma} = 0.15$

Eq. (14–35):  $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.071(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor  $K_B = 1$ . The speed ratio is  $m_G = N_G/N_P = 52/17 = 3.059$ . The load cycle factors given in the problem statement, with  $N(\text{pinion}) = 10^8$  cycles and  $N(\text{gear}) = 10^8/m_G = 10^8/3.059$  cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9,  $K_R = 0.85$ . From Fig. 14–18, the temperature and surface condition factors are  $K_T = 1$  and  $C_f = 1$ . From Eq. (14–23), with  $m_N = 1$  for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8,  $C_p = 191\sqrt{\text{MPa}}$ .

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with  $H_{BP} = 240$  and  $H_{BG} = 200$ , we use Fig. 14–2, which gives

$$(S_t)_P = 0.533(240) + 88.3 = 216.22 \text{ MPa}$$

$$(S_t)_G = 0.533(200) + 88.3 = 194.9 \text{ MPa}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 2.22(240) + 200 = 732.8 \text{ MPa}$$

$$(S_c)_G = 2.22(200) + 200 = 644 \text{ MPa}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio  $C_H$ , the hardness ratio is  $H_{BP}/H_{BG} = 240/200 = 1.2$ . Then, from Sec. 14–12,

$$\begin{aligned} A' &= 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3}) \\ &= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.00249 \end{aligned}$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.00249(3.059 - 1) = 1.005$$

(a) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$\begin{aligned}(\sigma)_P &= \left( W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \right)_P = 744.67(1) 1.377(1.4708) \frac{0.4}{38.1} \frac{1.22(1)}{0.30} \\ &= 64.44 \text{ MPa}\end{aligned}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer 
$$(S_F)_P = \left( \frac{S_t Y_N / (K_T K_R)}{\sigma} \right)_P = \frac{216.22(0.977) / [1(0.85)]}{64.44} = 3.856$$

**Gear tooth bending.** Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 744.67(1) 1.377(1.4829) \frac{0.4}{38.1} \frac{1.22(1)}{0.40} = 48.73 \text{ MPa}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer 
$$(S_F)_G = \frac{194.9(0.996) / [1(0.85)]}{48.73} = 4.688$$

(b) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$\begin{aligned}(\sigma_c)_P &= C_P \left( W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I} \right)_P^{1/2} \\ &= 191 \left[ 744.67(1) 1.377(1.4708) \frac{1.22}{42.5(38.1)} \frac{1}{0.121} \right]^{1/2} = 585.3 \text{ MPa}\end{aligned}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer 
$$(S_H)_P = \left[ \frac{S_c Z_N / (K_T K_R)}{\sigma_c} \right]_P = \frac{732.8(0.948) / [1(0.85)]}{585.3} = 1.397$$

**Gear tooth wear.** The only term in Eq. (14–16) that changes for the gear is  $K_s$ . Thus,

$$(\sigma_c)_G = \left[ \frac{(K_s)_G}{(K_s)_P} \right]^{1/2} (\sigma_c)_P = \left( \frac{1.4829}{1.4708} \right)^{1/2} 585.3 = 587.71 \text{ MPa}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with  $C_H = 1.005$  gives

Answer 
$$(S_H)_G = \frac{644(0.973) 1.005 / [1(0.85)]}{587.71} = 1.261$$

(c) For the pinion, we compare  $(S_F)_P$  with  $(S_H)_P^2$ , or 3.856 with  $1.397^2 = 1.953$ , so, the threat in the pinion is from wear. For the gear, we compare  $(S_F)_G$  with  $(S_H)_G^2$ , or 4.688 with  $1.261^2 = 1.59$ , so the threat in the gear is also from wear.

There are perspectives to be gained from Ex. 14–4. First, the pinion is overly strong in bending compared to wear. The performance in wear can be improved by surface-hardening techniques, such as flame or induction hardening, nitriding, or carburizing and case hardening, as well as shot peening. This in turn permits the gearset

to be made smaller. Second, in bending, the gear is stronger than the pinion, indicating that both the gear core hardness and tooth size could be reduced; that is, we may increase  $P$  and reduce the diameters of the gears, or perhaps allow a cheaper material. Third, in wear, surface strength equations have the ratio  $(Z_N)/K_R$ . The values of  $(Z_N)_P$  and  $(Z_N)_G$  are affected by gear ratio  $m_G$ . The designer can control strength by specifying surface hardness. This point will be elaborated later.

Having followed a spur-gear analysis in detail in Ex. 14–4, it is timely to analyze a helical gearset under similar circumstances to observe similarities and differences.

### EXAMPLE 14-5

A 17-tooth  $20^\circ$  normal pitch-angle helical pinion with a right-hand helix angle of  $30^\circ$  rotates at 1800 rev/min when transmitting 2982.8 Watt to a 52-tooth helical gear. The normal diametral pitch 0.4 teeth/mm, the face width is 38.1 mm, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of  $10^8$  cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(a) Find the factors of safety of the gears in bending.

(b) Find the factors of safety of the gears in wear.

(c) By examining the factors of safety identify the threat to each gear and to the mesh.

### Solution

All of the parameters in this example are the same as in Ex. 14–4 with the exception that we are using helical gears. Thus, several terms will be the same as Ex. 14–4. The reader should verify that the following terms remain unchanged:  $K_o = 1$ ,  $Y_P = 0.303$ ,  $Y_G = 0.412$ ,  $m_G = 3.059$ ,  $(K_s)_P = 1.4708$ ,  $(K_s)_G = 1.4829$ ,  $(Y_N)_P = 0.977$ ,  $(Y_N)_G = 0.996$ ,  $K_R = 0.85$ ,  $K_T = 1$ ,  $C_f = 1$ ,  $C_p = 191 \sqrt{\text{MPa}}$ ,  $(S_t)_P = 216.22 \text{ MPa}$ ,  $(S_t)_G = 194.9 \text{ MPa}$ ,  $(S_c)_P = 732.8 \text{ MPa}$ ,  $(S_c)_G = 644 \text{ MPa}$ ,  $(Z_N)_P = 0.948$ ,  $(Z_N)_G = 0.973$ , and  $C_H = 1.005$ .

For helical gears, the transverse diametral pitch, given by Eq. (13–18), p. 684, is

$$P_t = P_n \cos \psi = 0.4 \cos 30^\circ = 0.3464 \text{ teeth/mm}$$

Thus, the pitch diameters are  $d_p = N_p/P_t = 17/0.3464 = 49.075 \text{ mm}$  and  $d_G = 52/0.3464 = 150.11 \text{ mm}$ . The pitch-line velocity and transmitted force are

$$V = \frac{\pi d_p n_p}{60} = \frac{\pi(49.075)1800}{60} = 4625.19 \text{ mm/s}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(2982.8)}{4625.19} = 644.9 \text{ N}$$

As in Ex. 14–4, for the dynamic factor,  $B = 0.8255$  and  $A = 59.77$ . Thus, Eq. (14–27) gives

$$K_v = \left( \frac{59.77 + \sqrt{200(4.625)}}{59.77} \right)^{0.8255} = 1.40$$

The geometry factor  $I$  for helical gears requires a little work. First, the transverse pressure angle is given by Eq. (13–19) p. 684,

$$\phi_t = \tan^{-1} \left( \frac{\tan \phi_n}{\cos \psi} \right) = \tan^{-1} \left( \frac{\tan 20^\circ}{\cos 30^\circ} \right) = 22.80^\circ$$

The radii of the pinion and gear are  $r_p = 49.075/2 = 24.537$  in and  $r_G = 150.111/2 = 22.62$  in, respectively. The addendum is  $a = 1/P_n = 1/0.4 = 2.5$ , and the base-circle radii of the pinion and gear are given by Eq. (13–6), p. 672, with  $\phi = \phi_i$ :

$$(r_b)_P = r_p \cos \phi_i = 24.537 \cos 22.80^\circ = 22.62 \text{ mm}$$

$$(r_b)_G = 3.002 \cos 22.80^\circ = 69.191$$

From Eq. (14–25), the surface strength geometry factor

$$\begin{aligned} Z &= \sqrt{(24.537 + 2.5)^2 - 22.62^2} + \sqrt{(75.056 + 2.5)^2 - 69.191^2} \\ &\quad - (24.537 + 75.056) \sin 22.80^\circ \\ &= 14.8105 + 35.0354 - 38.5938 = 11.2520 \text{ in} \end{aligned}$$

Since the first two terms are less than 38.5938, the equation for  $Z$  stands. From Eq. (14–24) the normal circular pitch  $p_N$  is

$$p_N = p_n \cos \phi_n = \frac{\pi}{P_n} \cos 20^\circ = \frac{\pi}{0.4} \cos 20^\circ = 7.3803 \text{ mm}$$

From Eq. (14–21), the load sharing ratio

$$m_N = \frac{p_N}{0.95Z} = \frac{7.3803}{0.95(11.2520)} = 0.6904$$

Substituting in Eq. (14–23), the geometry factor  $I$  is

$$I = \frac{\sin 22.80^\circ \cos 22.80^\circ}{2(0.6904)} \frac{3.06}{3.06 + 1} = 0.195$$

From Fig. 14–7, geometry factors  $J'_p = 0.45$  and  $J'_G = 0.54$ . Also from Fig. 14–8 the  $J$ -factor multipliers are 0.94 and 0.98, correcting  $J'_p$  and  $J'_G$  to

$$J_p = 0.45(0.94) = 0.423$$

$$J_G = 0.54(0.98) = 0.529$$

The load-distribution factor  $K_m$  is estimated from Eq. (14–32):

$$C_{pf} = \frac{38.1}{10(49.075)} - 0.0375 + 4.92e-4(38.1) = 0.0569$$

with  $C_{mc} = 1$ ,  $C_{pm} = 1$ ,  $C_{ma} = 0.15$  from Fig. 14–11, and  $C_e = 1$ . Therefore, from Eq. (14–30),

$$K_m = 1 + (1)[0.0569(1) + 0.15(1)] = 1.207$$

(a) **Pinion tooth bending.** Substituting the appropriate terms into Eq. (14–15) using  $P_t$  gives

$$\begin{aligned} (\sigma)_P &= \left( W' K_o K_v K_s \frac{P_t}{F} \frac{K_m K_B}{J} \right)_P = 644.904(1)1.404(1.043) \frac{0.3464}{38.1} \frac{1.208(1)}{0.423} \\ &= 34.5545 \text{ MPa} \end{aligned}$$



Substituting the appropriate terms for the pinion into Eq. (14–41) gives

$$\text{Answer} \quad (S_F)_P = \left( \frac{S_t Y_N / (K_T K_R)}{\sigma} \right)_P = \frac{216.22(0.977) / [1(0.85)]}{3445} = 7.19$$

**Gear tooth bending.** Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 644.9035(1)1.404(1.052) \frac{0.3464}{38.1} \frac{1.208(1)}{0.529} = 27.85 \text{ MPa}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

$$\text{Answer} \quad (S_F)_G = \frac{194.9(0.996) / [1(0.85)]}{27.85} = 8.2$$

(b) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$\begin{aligned} (\sigma_c)_P &= C_p \left( W' K_o K_v K_s \frac{K_m C_f}{d_p F I} \right)_P^{1/2} \\ &= 191 \left[ 644.9035(1)1.404(1.043) \frac{1.208}{49.075(38.1)} \frac{1}{0.195} \right]^{1/2} = 403 \text{ MPa} \end{aligned}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

$$\text{Answer} \quad (S_H)_P = \left( \frac{S_c Z_N / (K_T K_R)}{\sigma_c} \right)_P = \frac{732.8(0.948) / [1(0.85)]}{403} = 2.03$$

**Gear tooth wear.** The only term in Eq. (14–16) that changes for the gear is  $K_s$ . Thus,

$$(\sigma_c)_G = \left[ \frac{(K_s)_G}{(K_s)_P} \right]^{1/2} (\sigma_c)_P = \left( \frac{1.052}{1.043} \right)^{1/2} 403 = 404 \text{ MPa}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with  $C_H = 1.005$  gives

$$\text{Answer} \quad (S_H)_G = \frac{644(0.973)1.005 / [1(0.85)]}{404} = 1.8325$$

(c) For the pinion we compare  $S_F$  with  $S_H^2$ , or 7.19 with  $2.03^2 = 4.12$ , so the threat in the pinion is from wear. For the gear we compare  $S_F$  with  $S_H^2$ , or 8.20 with  $1.83^2 = 3.358$ , so the threat is also from wear in the gear. For the meshing gearset wear controls.

It is worthwhile to compare Ex. 14–4 with Ex. 14–5. The spur and helical gearsets were placed in nearly identical circumstances. The helical gear teeth are of greater length because of the helix and identical face widths. The pitch diameters of the helical gears are larger. The  $J$  factors and the  $I$  factor are larger, thereby reducing stresses. The result is larger factors of safety. In the design phase the gearsets in Ex. 14–4 and Ex. 14–5 can be made smaller with control of materials and relative hardnesses.

Now that examples have given the AGMA parameters substance, it is time to examine some desirable (and necessary) relationships between material properties of spur gears in mesh. In bending, the AGMA equations are displayed side by side:

$$\sigma_P = \left( W^t K_o K_v K_s \frac{P_d K_m K_B}{F J} \right)_P \quad \sigma_G = \left( W^t K_o K_v K_s \frac{P_d K_m K_B}{F J} \right)_G$$

$$(S_F)_P = \left( \frac{S_t Y_N / (K_T K_R)}{\sigma} \right)_P \quad (S_F)_G = \left( \frac{S_t Y_N / (K_T K_R)}{\sigma} \right)_G$$

Equating the factors of safety, substituting for stress and strength, canceling identical terms ( $K_s$  virtually equal or exactly equal), and solving for  $(S_t)_G$  gives

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P J_P}{(Y_N)_G J_G} \quad (a)$$

The stress-cycle factor  $Y_N$  comes from Fig. 14-14, where for a particular hardness,  $Y_N = \alpha N^\beta$ . For the pinion,  $(Y_N)_P = \alpha N_P^\beta$ , and for the gear,  $(Y_N)_G = \alpha (N_P/m_G)^\beta$ . Substituting these into Eq. (a) and simplifying gives

$$(S_t)_G = (S_t)_P m_G^\beta \frac{J_P}{J_G} \quad (14-44)$$

Normally,  $m_G > 1$  and  $J_G > J_P$ , so Eq. (14-44) shows that the gear can be less strong (lower Brinell hardness) than the pinion for the same safety factor.

### EXAMPLE 14-6

In a set of spur gears, a 250-Brinell 14-tooth 16-pitch 20° full-depth pinion meshes with a 60-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using  $\beta = -0.023$ , what hardness can the gear have for the same factor of safety?

#### Solution

For through-hardened grade 1 steel the pinion strength  $(S_t)_P$  is given in Fig. 14-2:

$$(S_t)_P = 0.533(250) + 88.3 = 221.55 \text{ MPa}$$

From Fig. 14-6 the form factors are  $J_P = 0.32$  and  $J_G = 0.41$ . Equation (14-44) gives

$$(S_t)_G = 221.55 \left( \frac{60}{14} \right)^{-0.023} \frac{0.32}{0.41} = 167.23 \text{ MPa}$$

Use the equation in Fig. 14-2 again.

#### Answer

$$(H_B)_G = \frac{167.23 - 88.3}{77.3} = 148.26 \text{ Brinell}$$

The AGMA contact-stress equations also are displayed side by side:

$$(\sigma_c)_P = C_p \left( W^t K_o K_v K_s \frac{K_m C_f}{d_p F I} \right)_P^{1/2} \quad (\sigma_c)_G = C_p \left( W^t K_o K_v K_s \frac{K_m C_f}{d_p F I} \right)_G^{1/2}$$

$$(S_H)_P = \left( \frac{S_c Z_N / (K_T K_R)}{\sigma_c} \right)_P \quad (S_H)_G = \left( \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} \right)_G$$

Equating the factors of safety, substituting the stress relations, and canceling identical terms including  $K_s$  gives, after solving for  $(S_c)_G$ ,

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G} \left( \frac{1}{C_H} \right)_G = (S_c)_P m_G^\beta \left( \frac{1}{C_H} \right)_G$$

where, as in the development of Eq. (14-44),  $(Z_N)_P/(Z_N)_G = m_G^\beta$  and the value of  $\beta$  for wear comes from Fig. 14-15. Since  $C_H$  is so close to unity, it is usually neglected; therefore

$$(S_c)_G = (S_c)_P m_G^\beta \quad (14-45)$$

**EXAMPLE 14-7** For  $\beta = -0.056$  for a through-hardened steel, grade 1, continue Ex. 14-6 for wear.

**Solution** From Fig. 14-5,

$$(S_c)_P = 2.22(300) + 200 = 866 \text{ MPa}$$

From Eq. (14-45),

$$(S_c)_G = (S_c)_P \left( \frac{64}{18} \right)^{-0.056} = 866 \left( \frac{64}{18} \right)^{-0.056} = 807 \text{ MPa}$$

**Answer**

$$(H_B)_G = \frac{807 - 200}{2.22} = 273 \text{ Brinell}$$

which is slightly less than the pinion hardness of 300 Brinell.

Equations (14-44) and (14-45) apply as well to helical gears.

## 14-19 Design of a Gear Mesh

A useful decision set for spur and helical gears includes

- Function: load, speed, reliability, life,  $K_o$
  - Unquantifiable risk: design factor  $n_d$
  - Tooth system:  $\phi$ ,  $\psi$ , addendum, dedendum, root fillet radius
  - Gear ratio  $m_G$ ,  $N_p$ ,  $N_G$
  - Quality number  $Q_v$
  - Diametral pitch  $P_d$
  - Face width  $F$
  - Pinion material, core hardness, case hardness
  - Gear material, core hardness, case hardness
- $\left. \begin{array}{l} \bullet \text{ Function: load, speed, reliability, life, } K_o \\ \bullet \text{ Unquantifiable risk: design factor } n_d \\ \bullet \text{ Tooth system: } \phi, \psi, \text{ addendum, dedendum, root fillet radius} \\ \bullet \text{ Gear ratio } m_G, N_p, N_G \\ \bullet \text{ Quality number } Q_v \end{array} \right\} \text{ a priori decisions}$ 
  
 $\left. \begin{array}{l} \bullet \text{ Diametral pitch } P_d \\ \bullet \text{ Face width } F \\ \bullet \text{ Pinion material, core hardness, case hardness} \\ \bullet \text{ Gear material, core hardness, case hardness} \end{array} \right\} \text{ design decisions}$

The first item to notice is the dimensionality of the decision set. There are four design decision categories, eight different decisions if you count them separately. This is a larger number than we have encountered before. It is important to use a design strategy that is convenient in either longhand execution or computer implementation. The design

decisions have been placed in order of importance (impact on the amount of work to be redone in iterations). The steps, after the a priori decisions have been made are

- Choose a diametral pitch.
- Examine implications on face width, pitch diameters, and material properties. If not satisfactory, return to pitch decision for change.
- Choose a pinion material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.
- Choose a gear material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.

With these plan steps in mind, we can consider them in more detail.

First select a trial diametral pitch.

*Pinion bending:*

- Select a median face width for this pitch,  $4\pi/P$
- Find the range of necessary ultimate strengths
- Choose a material and a core hardness
- Find face width to meet factor of safety in bending
- Choose face width
- Check factor of safety in bending

*Gear bending:*

- Find necessary companion core hardness
- Choose a material and core hardness
- Check factor of safety in bending

*Pinion wear:*

- Find necessary  $S_c$  and attendant case hardness
- Choose a case hardness
- Check factor of safety in wear

*Gear wear:*

- Find companion case hardness
- Choose a case hardness
- Check factor of safety in wear

Completing this set of steps will yield a satisfactory design. Additional designs with diametral pitches adjacent to the first satisfactory design will produce several among which to choose. A figure of merit is necessary in order to choose the best. Unfortunately, a figure of merit in gear design is complex in an academic environment because material and processing costs vary. The possibility of using a process depends on the manufacturing facility if gears are made in house.

After examining Ex. 14–4 and Ex. 14–5 and seeing the wide range of factors of safety, one might entertain the notion of setting all factors of safety equal.<sup>9</sup> In

<sup>9</sup>In designing gears it makes sense to define the factor of safety in wear as  $(S)_H^2$  for uncrowned teeth, so that there is no mix-up. ANSI, in the preface to ANSI/AGMA 2001-D04 and 2101-D04, states “the use is completely voluntary . . . does not preclude anyone from using . . . procedures . . . not conforming to the standards.”

steel gears, wear is usually controlling and  $(S_H)_P$  and  $(S_H)_G$  can be brought close to equality. The use of softer cores can bring down  $(S_F)_P$  and  $(S_F)_G$ , but there is value in keeping them higher. A tooth broken by bending fatigue not only can destroy the gear set, but can bend shafts, damage bearings, and produce inertial stresses up and downstream in the power train, causing damage elsewhere if the gear box locks.

### EXAMPLE 14-8

Design a 4:1 spur-gear reduction for a 74.57 kW, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at  $10^9$  revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

#### Solution

Make the a priori decisions:

- Function: 74.57 kW, 1120 rev/min,  $R = 0.95$ ,  $N = 10^9$  cycles,  $K_o = 1$
- Design factor for unquantifiable exigencies:  $n_d = 2$
- Tooth system:  $\phi_n = 20^\circ$
- Tooth count:  $N_P = 18$  teeth,  $N_G = 72$  teeth (no interference, Sec. 13-7, p. 677)
- Quality number:  $Q_v = 6$ , use grade 1 material
- Assume  $m_B \geq 1.2$  in Eq. (14-40),  $K_B = 1$

*Pitch:* Select a trial diametral pitch of  $P_d = 0.1575$  teeth/mm. Thus,  $d_p = 18/0.1575 = 114.3$  mm and  $d_G = 72/0.1575 = 457.2$  mm. From Table 14-2,  $Y_P = 0.309$ ,  $Y_G = 0.4324$  (interpolated). From Fig. 14-6,  $J_P = 0.32$ ,  $J_G = 0.415$ .

$$V = \frac{\pi d_p n_p}{60} = \frac{\pi(114.3)1120}{60} = 6702.90 \text{ mm/s}$$

$$W^t = \frac{33\,000 H}{V} = \frac{33\,000(74570)}{6702.9 \times 1e-3} = 11125.03 \text{ N}$$

From Eqs. (14-28) and (14-27),

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left( \frac{59.77 + \sqrt{1319}}{59.77} \right)^{0.8255} = 1.480$$

From Eq. (14-38),  $K_R = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$ . From Fig. 14-14,

$$(Y_N)_P = 1.3558(10^9)^{-0.0178} = 0.938$$

$$(Y_N)_G = 1.3558(10^9/4)^{-0.0178} = 0.961$$

From Fig. 14-15,

$$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.900$$

$$(Z_N)_G = 1.4488(10^9/4)^{-0.023} = 0.929$$

From the recommendation after Eq. (14-8),  $3p \leq F \leq 5p$ . Try  $F = 4p = 4\pi/P = 4\pi/0.1575 = 79.8$  mm. From Eq. (a), Sec. 14-10,

$$K_s = 1.192 \left( \frac{F\sqrt{Y}}{P} \right)^{0.0535} = 1.192 \left( \frac{79.8\sqrt{0.309}}{0.1576} \right)^{0.0535} = 1.612$$

From Eqs. (14-31), (14-33) and (14-35),  $C_{mc} = C_{pm} = C_e = 1$ . From Fig. 14-11,  $C_{ma} = 0.175$  for commercial enclosed gear units. From Eq. (14-32),  $F/(10d_p) = 79.8/[10(114.30)] = 0.0698$ . Thus,

$$C_{pf} = 0.0698 - 0.0375 + 4.92 \times 1e-4 (79.8) = 0.0717$$

From Eq. (14-30),

$$K_m = 1 + (1)[0.0717(1) + 0.175(1)] = 1.247$$

From Table 14-8, for steel gears,  $C_p = 191\sqrt{\text{MPa}}$ . From Eq. (14-23), with  $m_G = 4$  and  $m_N = 1$ ,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{4}{4+1} = 0.1286$$

**Pinion tooth bending.** With the above estimates of  $K_s$  and  $K_m$  from the trial diametral pitch, we check to see if the mesh width  $F$  is controlled by bending or wear considerations. Equating Eqs. (14-15) and (14-17), substituting  $n_d W^t$  for  $W^t$ , and solving for the face width  $(F)_{\text{bend}}$  necessary to resist bending fatigue, we obtain

$$(F)_{\text{bend}} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B K_T K_R}{J_p S_t Y_N} \quad (1)$$

Equating Eqs. (14-16) and (14-18), substituting  $n_d W^t$  for  $W^t$ , and solving for the face width  $(F)_{\text{wear}}$  necessary to resist wear fatigue, we obtain

$$(F)_{\text{wear}} = \left( \frac{C_p K_T K_R}{S_c Z_N} \right)^2 n_d W^t K_o K_v K_s \frac{K_m C_f}{d_p I} \quad (2)$$

From Table 14-5 the hardness range of Nitralloy 135M is Rockwell C32-36 (302-335 Brinell). Choosing a midrange hardness as attainable, using 320 Brinell. From Fig. 14-4,

$$S_t = 0.594(320) + 87.76 = 277.84 \text{ MPa}$$

Inserting the numerical value of  $S_t$  in Eq. (1) to estimate the face width gives

$$(F)_{\text{bend}} = 2(11125.03)(1)1.48(1.14)0.1575 \frac{1.247(1)(1)0.885}{0.32(277.84)0.938} = 110.95 \text{ mm}$$

From Table 14-6 for Nitralloy 135M,  $S_c = 1172$  MPa. Inserting this in Eq. (2), we find

$$(F)_{\text{wear}} = \left( \frac{191(1)(0.885)}{1172(0.900)} \right)^2 2(11125.03)1(1.48)1.14 \frac{1.247(1)}{114.3(0.1286)} = 116.15 \text{ mm}$$

**Decision** Make face width 120 mm. Correct  $K_s$  and  $K_m$ :

$$K_s = 1.192 \left( \frac{120\sqrt{0.309}}{0.1575} \right)^{0.0535} = 1.6475$$

$$\frac{F}{10d_p} = \frac{120}{10(114.3)} = 0.1050$$

$$C_{pf} = 0.1050 - .0375 + 4.92 \times 10^{-4} (120) = 0.1268$$

$$K_m = 1 + (1)[0.1268(1) + .175(1)] = 1.3018$$

The bending stress induced by  $W^t$  in bending, from Eq. (14–15), is

$$(\sigma)_P = 11125.03(1)1.48(1.147) \frac{0.1575}{120} \frac{1.259(1)}{0.32} = 145.16 \text{ MPa}$$

The AGMA factor of safety in bending of the pinion, from Eq. (14–41), is

$$(S_F)_P = \frac{277.84(0.938)/1(0.885)]}{145.16} = 2.03$$

**Decision** **Gear tooth bending.** Use cast gear blank because of the 18-in pitch diameter. Use the same material, heat treatment, and nitriding. The load-induced bending stress is in the ratio of  $J_P/J_G$ . Then

$$(\sigma)_G = 145.16 \frac{0.32}{0.415} = 112 \text{ MPa}$$

The factor of safety of the gear in bending is

$$(S_F)_G = \frac{277.84(0.961)/1(0.885)]}{112} = 2.69$$

**Pinion tooth wear.** The contact stress, given by Eq. (14–16), is

$$(\sigma_c)_P = 191 \left[ 11125.03(1)1.48(1.147) \frac{1.259}{114.3(120)} \frac{1}{0.129} \right]^{1/2} = 828 \text{ MPa}$$

The factor of safety from Eq. (14–42), is

$$(S_H)_P = \frac{1172(0.900)/[1(0.885)]}{828} = 1.437$$

By our definition of factor of safety, pinion bending is  $(S_F)_P = 2.03$ , and wear is  $(S_H)_P^2 = (1.437)^2 = 2.07$ .

**Gear tooth wear.** The hardness of the gear and pinion are the same. Thus, from Fig. 14–12,  $C_H = 1$ , the contact stress on the gear is the same as the pinion,  $(\sigma_c)_G = 828 \text{ MPa}$ . The wear strength is also the same,  $S_c = 1172 \text{ MPa}$ . The factor of safety of the gear in wear is

$$(S_H)_G = \frac{1172 (0.929)/[1(0.885)]}{828} = 1.48$$

So, for the gear in bending,  $(S_F)_G = 2.69$ , and wear  $(S_H)_G^2 = (1.48)^2 = 2.20$ .

**Rim.** Keep  $m_B \geq 1.2$ . The whole depth is  $h_t = \text{addendum} + \text{dedendum} = 1/P_d + 1.25/P_d = 2.25/P_d = 2.25/0.1575 = 14.2875$ . The rim thickness  $t_R$  is

$$t_R \geq m_B h_t = 1.2(14.2875) = 17.1450 \text{ mm}$$

In the design of the gear blank, be sure the rim thickness exceeds 17.1450 mm; if it does not, review and modify this mesh design.

This design example showed a satisfactory design for a four-pitch spur-gear mesh. Material could be changed, as could pitch. There are a number of other satisfactory designs, thus a figure of merit is needed to identify the best.

One can appreciate that gear design was one of the early applications of the digital computer to mechanical engineering. A design program should be interactive, presenting results of calculations, pausing for a decision by the designer, and showing the consequences of the decision, with a loop back to change a decision for the better. The program can be structured in totem-pole fashion, with the most influential decision at the top, then tumbling down, decision after decision, ending with the ability to change the current decision or to begin again. Such a program would make a fine class project. Troubleshooting the coding will reinforce your knowledge, adding flexibility as well as bells and whistles in subsequent terms.

Standard gears may not be the most economical design that meets the functional requirements, because no application is standard in all respects.<sup>10</sup> Methods of designing custom gears are well understood and frequently used in mobile equipment to provide good weight-to-performance index. The required calculations including optimizations are within the capability of a personal computer.

## PROBLEMS

Problems marked with an asterisk (\*) are linked to problems in other chapters, as summarized in Table 1–2 of Sec. 1–17, p. 34.

Because the results will vary depending on the method used, the problems are presented by section.

- 14-1** A steel spur pinion has a pitch of 3 mm, 22 full-depth teeth, and a 20° pressure angle. The pinion runs at a speed of 1200 rev/min and transmits 11 kW to a 60-tooth gear. If the face width is 50 mm, estimate the bending stress.
- 14-2** A steel spur pinion has a diametral pitch of 10 teeth/in, 18 teeth cut full-depth with a 20° pressure angle, and a face width of 1 in. This pinion is expected to transmit 2 hp at a speed of 600 rev/min. Determine the bending stress.
- 14-3** A steel spur pinion has a module of 1.25 mm, 18 teeth cut on the 20° full-depth system, and a face width of 12 mm. At a speed of 1800 rev/min, this pinion is expected to carry a steady load of 0.5 kW. Determine the bending stress.

<sup>10</sup>See H. W. Van Gerpen, C. K. Reece, and J. K. Jensen, *Computer Aided Design of Custom Gears*, Van Gerpen-Reece Engineering, Cedar Falls, Iowa, 1996.



- 14-4** A steel spur pinion has 16 teeth cut on the  $20^\circ$  full-depth system with a module of 8 mm and a face width of 90 mm. The pinion rotates at 150 rev/min and transmits 6 kW to the mating steel gear. What is the bending stress?
- 14-5** A steel spur pinion has a module of 1 mm and 16 teeth cut on the  $20^\circ$  full-depth system and is to carry 0.15 kW at 400 rev/min. Determine a suitable face width based on an allowable bending stress of 150 MPa.
- 14-6** A  $20^\circ$  full-depth steel spur pinion has 20 teeth and a module of 2 mm and is to transmit 0.5 kW at a speed of 200 rev/min. Find an appropriate face width if the bending stress is not to exceed 75 MPa.
- 14-7** A  $20^\circ$  full-depth steel spur pinion has a module of 5 mm and 24 teeth and transmits 4.5 kW at a speed of 50 rev/min. Find an appropriate face width if the allowable bending stress is 140 MPa.
- 14-8** A steel spur pinion is to transmit 20 hp at a speed of 400 rev/min. The pinion is cut on the  $20^\circ$  full-depth system and has a diametral pitch of 4 teeth/in and 16 teeth. Find a suitable face width based on an allowable stress of 12 kpsi.
- 14-9** A  $20^\circ$  full-depth steel spur pinion with 18 teeth is to transmit 2.5 hp at a speed of 600 rev/min. Determine appropriate values for the face width and diametral pitch based on an allowable bending stress of 10 kpsi.
- 14-10** A  $20^\circ$  full-depth steel spur pinion is to transmit 1.5 kW hp at a speed of 900 rev/min. If the pinion has 18 teeth, determine suitable values for the module and face width. The bending stress should not exceed 75 MPa.
- 14-11** A speed reducer has  $20^\circ$  full-depth teeth and consists of a 20-tooth steel spur pinion driving a 60-tooth cast-iron gear. The horsepower transmitted is 11 kW at a pinion speed of 1200 rev/min. For a module of 4 mm and a face width of 50 mm, find the contact stress.
- 14-12** A gear drive consists of a 16-tooth  $20^\circ$  steel spur pinion and a 48-tooth cast-iron gear having a pitch of 12 teeth/in. For a power input of 1.5 hp at a pinion speed of 700 rev/min, select a face width based on an allowable contact stress of 100 kpsi.
- 14-13** A gearset has a module of 5 mm, a  $20^\circ$  pressure angle, and a 24-tooth cast-iron spur pinion driving a 48-tooth cast-iron gear. The pinion is to rotate at 50 rev/min. What horsepower input can be used with this gearset if the contact stress is limited to 690 MPa and  $F = 60$  mm?
- 14-14** A  $20^\circ$  20-tooth cast-iron spur pinion having a module of 4 mm drives a 32-tooth cast-iron gear. Find the contact stress if the pinion speed is 1000 rev/min, the face width is 50 mm, and 10 kW of power is transmitted.
- 14-15** A steel spur pinion and gear have a diametral pitch of 12 teeth/in, milled teeth, 17 and 30 teeth, respectively, a  $20^\circ$  pressure angle, a face width of  $\frac{7}{8}$  in, and a pinion speed of 525 rev/min. The tooth properties are  $S_{ut} = 76$  kpsi,  $S_y = 42$  kpsi and the Brinell hardness is 149. Use the Gerber criteria to compensate for one-way bending. For a design factor of 2.25, what is the power rating of the gearset?
- 14-16** A milled-teeth steel pinion and gear pair have  $S_{ut} = 113$  kpsi,  $S_y = 86$  kpsi and a hardness at the involute surface of 262 Brinell. The diametral pitch is 3 teeth/in, the face width is 2.5 in, and the pinion speed is 870 rev/min. The tooth counts are 20 and 100. Use the Gerber criteria to compensate for one-way bending. For a design factor of 1.5, rate the gearset for power considering both bending and wear.

- 14-17** A  $20^\circ$  full-depth steel spur pinion rotates at 1145 rev/min. It has a module of 6 mm, a face width of 75 mm, and 16 milled teeth. The ultimate tensile strength at the involute is 900 MPa exhibiting a Brinell hardness of 260. The gear is steel with 30 teeth and has identical material strengths. Use the Gerber criteria to compensate for one-way bending. For a design factor of 1.3 find the power rating of the gearset based on the pinion and the gear resisting bending and wear fatigue.
- 14-18** A steel spur pinion has a pitch of 6 teeth/in, 17 full-depth milled teeth, and a pressure angle of  $20^\circ$ . The pinion has an ultimate tensile strength at the involute surface of 116 kpsi, a Brinell hardness of 232, and a yield strength of 90 kpsi. Its shaft speed is 1120 rev/min, its face width is 2 in, and its mating gear has 51 teeth. Use a design factor of 2.
- Pinion bending fatigue imposes what power limitation? Use the Gerber criteria to compensate for one-way bending.
  - Pinion surface fatigue imposes what power limitation? The gear has identical strengths to the pinion with regard to material properties.
  - Determine power limitations due to gear bending and wear.
  - Specify the power rating for the gearset.
- 14-19** A commercial enclosed gear drive consists of a  $20^\circ$  spur pinion having 16 teeth driving a 48-tooth gear. The pinion speed is 300 rev/min, the face width 50 mm, and the diametral pitch 4 mm. The gears are grade 1 steel, through-hardened at 200 Brinell, made to No. 6 quality standards, uncrowned, and are to be accurately and rigidly mounted. Assume a pinion life of  $10^8$  cycles and a reliability of 0.90. Determine the AGMA bending and contact stresses and the corresponding factors of safety if 4 kW is to be transmitted.
- 14-20** A  $20^\circ$  spur pinion with 20 teeth and a module of 2.5 mm transmits 120 W to a 36-tooth gear. The pinion speed is 100 rev/min, and the gears are grade 1, 18-mm face width, through-hardened steel at 200 Brinell, uncrowned, manufactured to a No. 6 quality standard, and considered to be of open gearing quality installation. Find the AGMA bending and contact stresses and the corresponding factors of safety for a pinion life of  $10^8$  cycles and a reliability of 0.95.
- 14-21** Repeat Prob. 14-19 using helical gears each with a  $20^\circ$  normal pitch angle and a helix angle of  $30^\circ$  and a normal diametral pitch of 6 teeth/in.
- 14-22** A spur gearset has 17 teeth on the pinion and 51 teeth on the gear. The pressure angle is  $20^\circ$  and the overload factor  $K_o = 1$ . The diametral pitch is 6 teeth/in and the face width is 2 in. The pinion speed is 1120 rev/min and its cycle life is to be  $10^8$  revolutions at a reliability  $R = 0.99$ . The quality number is 5. The material is a through-hardened steel, grade 1, with Brinell hardnesses of 232 core and case of both gears. For a design factor of 2, rate the gearset for these conditions using the AGMA method.
- 14-23** In Sec. 14-10, Eq. (a) is given for  $K_s$  based on the procedure in Ex. 14-2. Derive this equation.
- 14-24** A speed-reducer has  $20^\circ$  full-depth teeth, and the single-reduction spur-gear gearset has 22 and 60 teeth. The diametral pitch is 4 teeth/in and the face width is  $3\frac{1}{4}$  in. The pinion shaft speed is 1145 rev/min. The life goal of 5-year 24-hour-per-day service is about  $3(10^9)$  pinion revolutions. The absolute value of the pitch variation is such that the quality number is 6. The materials are 4340 through-hardened grade 1 steels, heat-treated to 250 Brinell, core and case, both gears. The load is moderate shock and the power is smooth. For a reliability of 0.99, rate the speed reducer for power.

- 14-25** The speed reducer of Prob. 14-24 is to be used for an application requiring 40 hp at 1145 rev/min. For the gear and the pinion, estimate the AGMA factors of safety for bending and wear, that is,  $(S_F)_P$ ,  $(S_F)_G$ ,  $(S_H)_P$ , and  $(S_H)_G$ . By examining the factors of safety, identify the threat to each gear and to the mesh.
- 14-26** The gearset of Prob. 14-24 needs improvement of wear capacity. Toward this end the gears are nitrided so that the grade 1 materials have hardnesses as follows: The pinion core is 250 and the pinion case hardness is 390 Brinell, and the gear core hardness is 250 core and 390 case. Estimate the power rating for the new gearset.
- 14-27** The gearset of Prob. 14-24 has had its gear specification changed to 9310 for carburizing and surface hardening with the result that the pinion Brinell hardnesses are 285 core and 580–600 case, and the gear hardnesses are 285 core and 580–600 case. Estimate the power rating for the new gearset.
- 14-28** The gearset of Prob. 14-27 is going to be upgraded in material to a quality of grade 2 (9310) steel. Estimate the power rating for the new gearset.
- 14-29** Matters of scale always improve insight and perspective. Reduce the physical size of the gearset in Prob. 14-24 by one-half and note the result on the estimates of transmitted load  $W'$  and power.
- 14-30** AGMA procedures with cast-iron gear pairs differ from those with steels because life predictions are difficult; consequently  $(Y_N)_P$ ,  $(Y_N)_G$ ,  $(Z_N)_P$ , and  $(Z_N)_G$  are set to unity. The consequence of this is that the fatigue strengths of the pinion and gear materials are the same. The reliability is 0.99 and the life is  $10^7$  revolution of the pinion ( $K_R = 1$ ). For longer lives the reducer is derated in power. For the pinion and gear set of Prob. 14-24, use grade 40 cast iron for both gears ( $H_B = 201$  Brinell). Rate the reducer for power with  $S_F$  and  $S_H$  equal to unity.
- 14-31** Spur-gear teeth have rolling and slipping contact (often about 8 percent slip). Spur gears tested to wear failure are reported at  $10^8$  cycles as Buckingham's surface fatigue load-stress factor  $K$ . This factor is related to Hertzian contact strength  $S_C$  by

$$S_C = \sqrt{\frac{1.4K}{(1/E_1 + 1/E_2) \sin \phi}}$$

where  $\phi$  is the normal pressure angle. Cast iron grade 20 gears with  $\phi = 14\frac{1}{2}^\circ$  and  $20^\circ$  pressure angle exhibit a minimum  $K$  of 0.56 and 0.77 MPa, respectively. How does this compare with  $S_C = 2.2H_B$  MPa?

- 14-32** You've probably noticed that although the AGMA method is based on two equations, the details of assembling all the factors is computationally intensive. To reduce error and omissions, a computer program would be useful. Write a program to perform a power rating of an existing gearset, then use Prob. 14-24, 14-26, 14-27, 14-28, and 14-29 to test your program by comparing the results to your longhand solutions.
- 14-33** In Ex. 14-5 use nitrided grade 1 steel (4140) which produces Brinell hardnesses of 250 core and 500 at the surface (case). Use the upper fatigue curves on Figs. 14-14 and 14-15. Estimate the power capacity of the mesh with factors of safety of  $S_F = S_H = 1$ .
- 14-34** In Ex. 14-5 use carburized and case-hardened gears of grade 1. Carburizing and case-hardening can produce a 550 Brinell case. The core hardnesses are 200 Brinell. Estimate the power capacity of the mesh with factors of safety of  $S_F = S_H = 1$ , using the lower fatigue curves in Figs. 14-14 and 14-15.

- 14-35** In Ex. 14-5, use carburized and case-hardened gears of grade 2 steel. The core hardnesses are 200, and surface hardnesses are 600 Brinell. Use the lower fatigue curves of Figs. 14-14 and 14-15. Estimate the power capacity of the mesh using  $S_F = S_H = 1$ . Compare the power capacity with the results of Prob. 14-34.
- 14-36\*** The countershaft in Prob. 3-72, p. 152, is part of a speed reducing compound gear train using  $20^\circ$  spur gears. A gear on the input shaft drives gear *A*. Gear *B* drives a gear on the output shaft. The input shaft runs at 2400 rev/min. Each gear reduces the speed (and thus increases the torque) by a 2 to 1 ratio. All gears are to be of the same material. Since gear *B* is the smallest gear, transmitting the largest load, it will likely be critical, so a preliminary analysis is to be performed on it. Use a module of 4 mm, a face-width of 4 times the circular pitch, a Grade 2 steel through-hardened to a Brinell hardness of 300, and a desired life of 11 kW with a 95 percent reliability. Determine factors of safety for bending and wear.
- 14-37\*** The countershaft in Prob. 3-73, p. 152, is part of a speed reducing compound gear train using  $20^\circ$  spur gears. A gear on the input shaft drives gear *A* with a 2 to 1 speed reduction. Gear *B* drives a gear on the output shaft with a 5 to 1 speed reduction. The input shaft runs at 1800 rev/min. All gears are to be of the same material. Since gear *B* is the smallest gear, transmitting the largest load, it will likely be critical, so a preliminary analysis is to be performed on it. Use a module of 18.75 mm/tooth, a face-width of 4 times the circular pitch, a Grade 2 steel through-hardened to a Brinell hardness of 300, and a desired life of 12 kh with a 98 percent reliability. Determine factors of safety for bending and wear.
- 14-38\*** Build on the results of Prob. 13-40, p. 720, to find factors of safety for bending and wear for gear *F*. Both gears are made from Grade 2 carburized and hardened steel. Use a face-width of 4 times the circular pitch. The desired life is 12 kh with a 95 percent reliability.
- 14-39\*** Build on the results of Prob. 13-41, p. 721, to find factors of safety for bending and wear for gear *C*. Both gears are made from Grade 2 carburized and hardened steel. Use a face-width of 4 times the circular pitch. The desired life is 14 kh with a 98 percent reliability.